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The definition of the set Pr^k needs to be fixed in the case that $\mathfrak{g}$ is of type C_l and q is even. It should be defined as the set of admissible weights λ such that $\widehat{\Delta}(\lambda) = y(\widehat{\Delta}(k\Lambda_0))$ for some $y \in \widetilde{W}$. This set coincides with the set of admissible weights λ such that $\widehat{\Delta}(\lambda) \cong \widehat{\Delta}(k\Lambda_0)$ if $\mathfrak{g}$ is not of type C_l or q is odd. However, in type C_l when q is even, there are admissible weights λ that $\widehat{\Delta}(\lambda) \cong \widehat{\Delta}(k\Lambda_0)$ but are not of the form $y(\widehat{\Delta}(k\Lambda_0))$ for any $y \in \widetilde{W}$, which correspond to the case $\sigma_l(\Pi^\vee)$ in Table 1 of [KW2]. Accordingly, Main Theorem on page 68 should be fixed as the following.

Main Theorem. *Let k be an admissible number, λ a weight of $\widehat{\mathfrak{g}}$ of level k. Then $L(\lambda)$ is a module over $L(k\Lambda_0)$ if and only if it is an admissible representation such that $\widehat{\Delta}(\lambda) = y(\widehat{\Delta}(k\Lambda_0))$ for some $y \in \widetilde{W}$. In particular, any $\widehat{\mathfrak{g}}$-module from the category $\mathcal{O}$ is an $L(k\Lambda_0)$-module if and only if it is a direct sum of such admissible representations of $\widehat{\mathfrak{g}}$ of level k.*

We are grateful to Jethro van Ekeren for pointing out this error.

Also, on page 85, Proposition 4.5 and its proof should be replaced by the following.

Proposition 4.5. *Suppose that $L(\lambda)$ is an $L(k\Lambda_0)$-module. Then there exsits $\mu \in P^\vee$ such that $\widehat{\Delta}(\lambda) = t_{-\mu}(\widehat{\Delta}(k\Lambda_0))$.*

Proof. By Lemma 4.4, $\langle \lambda + \rho, \alpha_i^\vee \rangle \in \frac{2}{(\alpha_i|\alpha_i)q}\mathbb{Z}$ for all $i = 1, \cdots l$. Hence there exists $n_i \in \mathbb{Z}$ for each $i = 1, \dots, l$ such that

$$\alpha_i + n\delta \in \widehat{\Delta}(\lambda) \iff n \equiv n_i \begin{cases} (\mathrm{mod}\ \ q\mathbb{Z}) \text{ if } (q, r^\vee) = 1 \text{ or } \alpha_i \text{ is a long root,} \\ (\mathrm{mod}\ \ \frac{q}{r^\vee}\mathbb{Z}) \text{ if } (q, r^\vee) = r^\vee \text{ and } \alpha_i \text{ is a short root.} \end{cases}$$

Set $\mu = \sum_{i=1}^l n_i \varpi_i^\vee \in P^\vee$, where $\varpi_i^\vee$ is the i-th fundamental coweight. Then

$$\alpha_i + n\delta \in \widehat{\Delta}(t_\mu \circ \lambda) \iff n \equiv 0 \begin{cases} (\mathrm{mod}\ \ q\mathbb{Z}) \text{ if } (q, r^\vee) = 1 \text{ or } \alpha_i \text{ is a long root,} \\ (\mathrm{mod}\ \ \frac{q}{r^\vee}\mathbb{Z}) \text{ if } (q, r^\vee) = r^\vee \text{ and } \alpha_i \text{ is a short root.} \end{cases}$$

It follows that

$$\widehat{\Delta}(t_\mu \circ \lambda) = \{\alpha + nq\delta \mid \alpha \in \Delta,\ n \in \mathbb{Z}\} \quad \text{if} \quad (q, r^\vee) = 1,$$

and

$$\widehat{\Delta}(t_\mu \circ \lambda)^\vee = \{\alpha^\vee + nq\delta \mid \alpha \in \Delta,\ n \in \mathbb{Z}\} \quad \text{if} \quad (q, r^\vee) = r^\vee.$$

That is, $\widehat{\Delta}(t_\mu \circ \lambda) = \widehat{\Delta}(k\Lambda_0)$, and we get that $\widehat{\Delta}(\lambda) = t_{-\mu}(\widehat{\Delta}(k\Lambda_0))$. □

On page 87, in the proof of the "only if" part of Main Theorem, the sentense "By Proposition 4.5, $\widehat{\Delta}(\lambda) \cong \widehat{\Delta}(k\Lambda_0)$" should be replaced by "By Proposition 4.5, $\widehat{\Delta}(\lambda) = t_{-\mu}(\widehat{\Delta}(k\Lambda_0))$ for some $\mu \in P_+$".

References

[KW2] V. G. Kac and M. Wakimoto. Classification of modular invariant representations of affine algebras. In *Infinite-dimensional Lie algebras and groups (Luminy-Marseille, 1988)*, volume 7 of *Adv. Ser. Math. Phys.*, pages 138–177. World Sci. Publ., Teaneck, NJ, 1989.

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