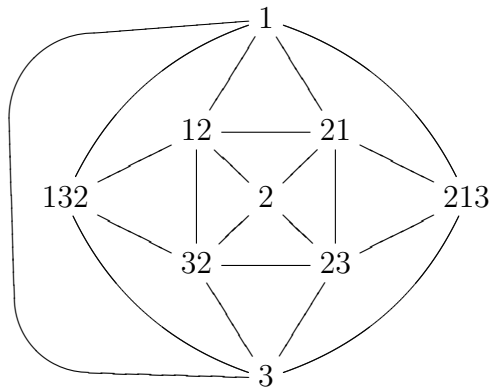


EXAMPLES

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1. CLUSTERS

1.1. Case A_3 .



q -center 321, 2132, 123

14 clusters,

The number of cluster variables : 12 = 9 (exchangeable) + 3 (frozen)

$M \setminus N$	1	2	3	12	21	23	32	213	132	123	321	2132
1	\	1	0	1	-1	1	1	-1	1	1	-1	0
2	1	\	1	-1	1	1	-1	2	0	0	0	0
3	0	1	\	1	1	-1	1	-1	1	-1	1	0
12	-1	1	1	\	0	2	0	1	-1	1	-1	0
21	1	-1	1	0	\	0	0	1	1	1	-1	0
23	1	-1	1	0	0	\	0	1	1	-1	1	0
32	1	1	-1	0	2	0	\	1	-1	-1	1	0
213	1	0	1	1	-1	-1	1	\	2	0	0	0
132	-1	2	-1	1	1	1	1	0	\	0	0	0
123	-1	0	1	-1	-1	1	1	0	0	\	0	0
321	1	0	-1	1	1	-1	-1	0	0	0	\	0
2132	0	0	0	0	0	0	0	0	0	0	0	\

TABLE 1. List of $\Lambda(M, N)$

$M \setminus N$	1	2	3	12	21	23	32	213	132	123	321	2132
1	1	0	0	1	0	0	0	0	1	1	0	0
2	0	1	0	0	1	1	0	1	0	0	0	1
3	0	0	1	0	0	0	1	0	1	0	1	0
12	0	1	0	1	1	1	0	1	0	1	0	1
21	1	0	0	1	1	0	0	1	1	1	0	1
23	0	0	1	0	0	1	1	1	1	0	1	1
32	0	1	0	0	1	1	1	1	0	0	1	1
213	1	0	1	1	0	0	1	1	2	1	1	1
132	0	1	0	1	1	1	1	1	1	1	1	1
123	0	0	1	0	0	1	1	1	1	1	1	1
321	1	0	0	1	1	0	0	1	1	1	1	1
2132	0	1	0	1	1	1	1	1	1	1	1	2

TABLE 2. List of $\tilde{\Lambda}(M, N)$

$$0 \rightarrow 21 \rightarrow 1 \circ 2 \rightarrow 12 \rightarrow 0$$

$$0 \rightarrow 2132 \rightarrow 32 \circ 21 \rightarrow 2 \circ 321 \rightarrow 0$$

$$0 \rightarrow 32 \circ 12 \rightarrow 2 \circ 132 \rightarrow 2132 \rightarrow 0$$

$$0 \rightarrow 1 \circ 2132 \rightarrow 132 \circ 21 \rightarrow 12 \circ 321 \rightarrow 0$$

$$0 \rightarrow 132 \rightarrow 12 \circ 3 \rightarrow 123 \rightarrow 0$$

$$0 \rightarrow 2132 \circ 1 \circ 3 \rightarrow 132 \circ 213 \rightarrow 123 \circ 321 \rightarrow 0$$

$$132 \circ 213 \rightarrow 123 \circ 321 \quad \begin{array}{ccccccc} 1 & 3 & 2 & & 2 & 3 & 1 \\ | & / & \backslash & & / & \backslash & | \\ 1 & 2 & 3 & & 3 & 2 & 1 \end{array}$$

Right dual

$$1 \rightarrow 23 \rightarrow 12 \rightarrow 3 \rightarrow 21 \rightarrow 32 \rightarrow 1$$

$$2 \rightarrow 132 \rightarrow 213 \rightarrow 2$$

Right dual of clusters

$$\{1, 12, 21\} \longleftrightarrow \{3, 23, 32\}$$

$$\{2, 12, 32\} \rightarrow \{1, 3, 132\} \rightarrow \{21, 23, 213\} \rightarrow \{2, 12, 32\}$$

$$\{2, 21, 23\} \rightarrow \{12, 32, 132\} \rightarrow \{1, 3, 213\} \rightarrow \{2, 21, 23\}$$

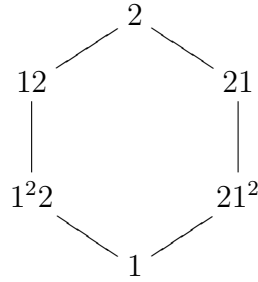
$$\begin{aligned} \{2, 12, 21\} &\rightarrow \{3, 32, 132\} \rightarrow \{1, 21, 213\} \rightarrow \{2, 23, 32\} \rightarrow \\ &\rightarrow \{1, 12, 132\} \rightarrow \{3, 23, 213\} \rightarrow \{2, 12, 21\} \end{aligned}$$

1.2. Case C_2 .

$$\begin{array}{ccc} \circ & \longleftarrow & \circ \\ 1 & & 2 \end{array} \quad 1 : \text{short root}, \quad 2 : \text{long root.}$$

$$(\alpha_1, \alpha_1) = 2, \quad (\alpha_1, \alpha_2) = -2, \quad (\alpha_2, \alpha_2) = 4$$

$$\begin{aligned} Q_{12}(u, v) &= u^2 - v, \\ \overline{Q}_{12}(u, v, w) &= u + w, \quad \overline{Q}_{2,1} = -1. \end{aligned}$$



True center : 121, 21²2

$M \setminus N$	1	2	12	21	1 ² 2	21 ²
1	\	2	2	0	2	-2
2	2	\	-2	2	0	4
12	0	2	\	2	-2	2
21	2	-2	0	\	2	2
1 ² 2	-2	4	2	2	\	0
21 ²	2	0	2	-2	4	\

TABLE 3. List of $\Lambda(M, N)$

$M \setminus N$	1	2	12	21	1 ² 2	21 ²
1	1	0	1	0	2	0
2	0	2	0	2	0	2
12	0	2	1	2	0	2
21	1	0	1	1	2	2
1 ² 2	0	2	2	2	2	2
21 ²	2	0	2	0	4	2

TABLE 4. List of $\tilde{\Lambda}(M, N)$

$$L(1^2 2) \circ L(21^2) \twoheadrightarrow L(121) \circ L(121) \quad \begin{array}{ccccccc} 1 & 1 & 2 & 2 & 1 & 1 \\ & \diagdown & / & \diagdown & / & \diagdown \\ & 1 & 2 & 1 & 1 & 2 & 1 \end{array}$$

Right dual

$$\begin{aligned} 1 &\rightarrow 21 \rightarrow 12 \rightarrow 1 \\ 2 &\rightarrow 1^2 2 \rightarrow 21^2 \rightarrow 2 \end{aligned}$$

Right dual of clusters

$$\begin{aligned} \{1, 1^2 2\} &\rightarrow \{21, 21^2\} \rightarrow \{12, 2\} \rightarrow \{1, 1^2 2\} \\ \{1, 21^2\} &\rightarrow \{21, 2\} \rightarrow \{12, 12^2\} \rightarrow \{1, 21^2\} \end{aligned}$$

1.3. Case A_4 .

The number of cluster modules $40 = 36 + 4$ (q-centers)

q-centers $C_1 = \langle 4321 \rangle$, $C_2 = \langle \overset{\bullet}{3}214\overset{\bullet}{3}2 \rangle = \langle \overset{\bullet}{3}4231\overset{\bullet}{2} \rangle$, $C_3 = \langle \overset{\bullet}{2}1324\overset{\bullet}{3} \rangle = \langle \overset{\bullet}{2}3412\overset{\bullet}{3} \rangle$, $C_4 = \langle 1234 \rangle$

C_3 5-dim $\{213243, 231243, 213423, 231423, 234123\}$

$\langle 1 \rangle$, $\langle 2 \rangle$, $\langle 3 \rangle$, $\langle 4 \rangle$

$\langle 12 \rangle$, $\langle 23 \rangle$, $\langle 34 \rangle$, $\langle 21 \rangle$, $\langle 32 \rangle$, $\langle 43 \rangle$

$\langle 123 \rangle$, $\langle 132 \rangle$, $\langle 213 \rangle$, $\langle 321 \rangle$, $\langle 234 \rangle$, $\langle 324 \rangle$, $\langle 243 \rangle$, $\langle 432 \rangle$

$\langle 2132 \rangle$, $\langle 3243 \rangle$,

$\langle \widehat{4\{12\}3} \rangle$, $\langle \widehat{1\{43\}2} \rangle$, $\langle \widehat{3\{4\}21} \rangle$, $\langle \widehat{2\{1\}34} \rangle$

$\langle \widehat{4\{12\}3} \rangle$ 3-dim $\{4123, 1423, 1243\}$

commuting with 1, 2, 4, simply linked with 3

$\langle \widehat{2413} \rangle$, $\langle \widehat{3124} \rangle$

$\langle \widehat{2413} \rangle$ 5-dim $\{2413, 4213, 2431, 4231, 2143\}$

commuting with 1, 4, simply linked with 2, 3

$\langle \widehat{3142\overset{\bullet}{3}} \rangle$, $\langle \widehat{2413\overset{\bullet}{2}} \rangle$, $\langle \widehat{\overset{\bullet}{3}2413} \rangle$, $\langle \widehat{\overset{\bullet}{2}3142} \rangle$

$\langle \widehat{3142\overset{\bullet}{3}} \rangle = E_2 C_3$ 5-dim $\{31423, 13423, 31243, 13243, 34123\}$

commuting with 1, 3, 4, simply linked with 2

$\langle 1342^2 3 \rangle$, $\langle 4213^2 2 \rangle$, $\langle 32^2 431 \rangle$, $\langle 23^2 124 \rangle$

$\langle 1342^2 3 \rangle$ 25-dim,

$$\widehat{132^2 43} + \widehat{1342^2 3} + 3412^2 3 + \widehat{132432}$$

$$+123\widehat{243} + 321\widehat{243} + 341232 + 3\widehat{241}23 + (q + q^{-1})\widehat{132423}$$

commuting with 2, 4, simply linked with 1, 3

$$E_1\langle 1342^23 \rangle = \langle 3423 \rangle \circ \langle 2 \rangle$$

$$\langle 1342^23 \rangle, \varepsilon = (1, 0, 1, 0), \varepsilon^* = (0, 1, 1, 0)$$

$$\langle 4213^22 \rangle, \varepsilon = (0, 1, 0, 1), \varepsilon^* = (0, 1, 1, 0)$$

$$\langle 32^2431 \rangle, \varepsilon = (0, 1, 1, 0), \varepsilon^* = (1, 0, 1, 0)$$

$$\langle 23^2124 \rangle, \varepsilon = (0, 1, 1, 0), \varepsilon^* = (0, 1, 0, 1)$$

$$\langle 132^2431 \rangle \text{ 70-dim. stable by } e(\nu_1, \dots, \nu_7) \leftrightarrow e(\nu_7, \dots, \nu_1)$$

commuting with 1, 2, 4, simply linked with 3

$$\varepsilon = (1, 0, 1, 0), \varepsilon^* = (1, 0, 1, 0)$$

$$\langle 423^2124 \rangle \text{ 70-dim. stable by } e(\nu_1, \dots, \nu_7) \leftrightarrow e(\nu_7, \dots, \nu_1)$$

commuting with 1, 3, 4, simply linked with 2

$$\varepsilon = (0, 1, 0, 1), \varepsilon^* = (0, 1, 0, 1)$$

$$0 \longrightarrow \langle \widehat{3142\dot{3}} \rangle \longrightarrow \langle 412\dot{3} \rangle \circ \langle 3 \rangle \longrightarrow \langle 123 \rangle \circ \langle 43 \rangle \longrightarrow 0$$

$5\text{-dim} \qquad\qquad\qquad 15\text{-dim} \qquad\qquad\qquad 10\text{-dim}$

$$0 \longrightarrow \langle 21 \rangle \circ \langle 243 \rangle \longrightarrow \langle 421\dot{3} \rangle \circ \langle 2 \rangle \longrightarrow \langle 2413\dot{2} \rangle \longrightarrow 0$$

$20\text{-dim} \qquad\qquad\qquad 25\text{-dim} \qquad\qquad\qquad 5\text{-dim}$

$$0 \longrightarrow \langle \dot{3}2413 \rangle \longrightarrow \langle 421\dot{3} \rangle \circ \langle 3 \rangle \longrightarrow \langle 43 \rangle \circ \langle 213 \rangle \longrightarrow 0$$

$5\text{-dim} \qquad\qquad\qquad 25\text{-dim} \qquad\qquad\qquad 20\text{-dim}$

$$0 \longrightarrow \langle 234123 \rangle \longrightarrow \langle 13423 \rangle \circ \langle 2 \rangle \longrightarrow \langle 1342^23 \rangle \longrightarrow 0$$

$5\text{-dim} \qquad\qquad\qquad 30\text{-dim} \qquad\qquad\qquad 25\text{-dim}$

$$0 \longrightarrow \langle 1342^23 \rangle \longrightarrow \langle 123 \rangle \circ \langle 324 \rangle \longrightarrow \langle 1234 \rangle \circ \langle 32 \rangle \longrightarrow 0$$

$25\text{-dim} \qquad\qquad\qquad 40\text{-dim} \qquad\qquad\qquad 15\text{-dim}$

$$0 \longrightarrow \langle 1234 \rangle \circ \langle 32 \rangle \longrightarrow \langle 324 \rangle \circ \langle 123 \rangle \longrightarrow \langle 1342^23 \rangle \longrightarrow 0$$

$15\text{-dim} \qquad\qquad\qquad 40\text{-dim} \qquad\qquad\qquad 25\text{-dim}$

$$0 \longrightarrow \langle 32 \rangle \circ \langle \widehat{3142\dot{3}} \rangle \longrightarrow \langle 1342^23 \rangle \circ \langle 3 \rangle \longrightarrow \langle 123 \rangle \circ \langle \dot{3}24\dot{3} \rangle \longrightarrow 0$$

$105\text{-dim} \qquad\qquad\qquad 175\text{-dim} \qquad\qquad\qquad 70\text{-dim}$

$$0 \longrightarrow \langle 12 \rangle \circ \langle \widehat{3142\dot{3}} \rangle \longrightarrow \langle 1342^2\dot{3} \rangle \circ \langle 1 \rangle \longrightarrow \langle 1342^2\dot{3}1 \rangle \longrightarrow 0$$

$105\text{-dim} \qquad\qquad\qquad 175\text{-dim} \qquad\qquad\qquad 70\text{-dim}$

$$0 \longrightarrow \langle 1 \rangle \circ \langle 23412\dot{3} \rangle \longrightarrow \langle 3124\dot{3} \rangle \circ \langle 21 \rangle \longrightarrow \langle 1342^2\dot{3}1 \rangle \longrightarrow 0$$

$35\text{-dim} \qquad\qquad\qquad 105\text{-dim} \qquad\qquad\qquad 70\text{-dim}$

$$0 \longrightarrow \langle 123 \rangle \circ \langle 4321 \rangle \longrightarrow \langle 321 \rangle \circ \langle 1243 \rangle \longrightarrow \langle 1342^2\dot{3}1 \rangle \longrightarrow 0$$

$35\text{-dim} \qquad\qquad\qquad 105\text{-dim} \qquad\qquad\qquad 70\text{-dim}$

$$0 \longrightarrow \langle 1234 \rangle \circ \langle 321 \rangle \longrightarrow \langle 3421 \rangle \circ \langle 123 \rangle \longrightarrow \langle 1342^2\dot{3}1 \rangle \longrightarrow 0$$

$35\text{-dim} \qquad\qquad\qquad 105\text{-dim} \qquad\qquad\qquad 70\text{-dim}$

$$0 \longrightarrow \langle 1 \rangle \circ \langle 32143\dot{2} \rangle \longrightarrow \langle 12 \rangle \circ \langle 3241\dot{3} \rangle \longrightarrow \langle 1342^2\dot{3}1 \rangle \longrightarrow 0$$

$35\text{-dim} \qquad\qquad\qquad 105\text{-dim} \qquad\qquad\qquad 70\text{-dim}$

$$0 \longrightarrow \langle 321 \rangle \circ \langle 3142\dot{3} \rangle \longrightarrow \langle 1342^2\dot{3}1 \rangle \circ \langle 3 \rangle \longrightarrow \langle \dot{3}241\dot{3} \rangle \circ \langle 123 \rangle \longrightarrow 0$$

$280\text{-dim} \qquad\qquad\qquad 560\text{-dim} \qquad\qquad\qquad 280\text{-dim}$

$$0 \longrightarrow \langle 21 \rangle \circ \langle 123 \rangle \circ \langle 32143\dot{2} \rangle \longrightarrow \langle 2132 \rangle \circ \langle 1342^2\dot{3}1 \rangle$$

$11\cdot 700\cdot 3\text{-dim} \qquad\qquad\qquad 11\cdot 700\cdot 6\text{-dim}$

$$\longrightarrow \langle 12 \rangle \circ \langle 321 \rangle \circ \langle 23412\dot{3} \rangle \longrightarrow 0$$

$11\cdot 700\cdot 3\text{-dim}$

$$0 \longrightarrow \langle 12 \rangle \circ \langle 21324\dot{3} \rangle \longrightarrow \langle 123 \rangle \circ \langle 2314\dot{2} \rangle$$

$140\text{-dim} \qquad\qquad\qquad 280\text{-dim}$

$$\longrightarrow \langle 1234 \rangle \circ \langle 2132 \rangle \longrightarrow 0$$

140-dim

Right dual

$$1 \longrightarrow 234 \longrightarrow 123 \longrightarrow 4 \longrightarrow 321 \longrightarrow 432 \longrightarrow 1$$

$$2 \longrightarrow 31423 \longrightarrow 23142 \longrightarrow 3 \longrightarrow 21432 \longrightarrow 32143 \longrightarrow 2$$

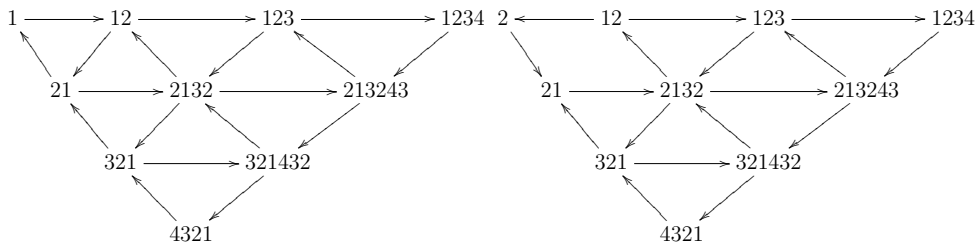
$$12 \longrightarrow 34 \longrightarrow 2132 \longrightarrow 43 \longrightarrow 21 \longrightarrow 3243 \longrightarrow 12$$

$$23 \longrightarrow 1423 \longrightarrow 3214 \longrightarrow 32 \longrightarrow 1432 \longrightarrow 2314 \longrightarrow 23$$

$$132 \longrightarrow 423^2124 \longrightarrow 213 \longrightarrow 243 \longrightarrow 132^2431 \longrightarrow 324 \longrightarrow 132$$

$$2413 \longrightarrow 32^2431 \longrightarrow 132^2431 \longrightarrow 1324 \longrightarrow 23^2124 \longrightarrow 423^2124 \longrightarrow 2413$$

Clusters



- {1, 12, 21, 123, 2132, 321},
- {2, 12, 21, 123, 2132, 321},
- {1, 213, 21, 123, 2132, 321},
- {1, 12, 132, 123, 2132, 321},
- {1, 12, 21, $\overset{\bullet}{2} \widehat{3142}$, 2132, 321},
- {1, 12, 21, 123, $132^2 431$, 321},
- {1, 12, 21, 123, 2132, $\widehat{24132} \overset{\bullet}{}$ },

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