Resurgence of parabolic curves in $\mathbb{C}^2$
(Joint work with Vassili Gelfreich)

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We study parabolic germs of holomorphic transformations of $(\mathbb{C}^2, 0)$ under some non-degeneracy hypothesis. They always admit invariant curves which may be called stable and unstable manifolds, and can be obtained as Borel sums of a single formal expansion involving powers of z^{-1} and $z^{-1} \log z$, where z is a large variable. This formal curve is in fact the separatrix of the formal infinitesimal generator of the parabolic germ, i.e. a formal vector field with nilpotent linear part, the time-1 map of which coincides with the given germ.

Generically, the formal separatrix is divergent and its two sums are not the analytic continuation one of the other; the leading order of their difference, which is exponentially small, is determined by a pair of complex constants. We analyse this phenomenon in the framework of Resurgence theory and prove that these constants depend analytically on parameters. They vanish for the time-1 map of a holomorphic vector field but not for a generic germ. We also study the "formal integral", which to corresponds to the invariant foliation induced by the formal infinitesimal generator, and derive the so-called Bridge Equation in order to describe completely the resurgent structure.