数理解析研究所講究録
第 194 巻 1973 年 41-56



# Note on shape theory

Yukihiro Kodama

Department of Mathematics, Tokyo University of Education

## §1. Shape of compacta.

In [2], [3] K.Borsuk introduced the notion of shapes of metric compacta. Let X and Y be compacta lying in the Hilbert cube Q $(= \prod_{n=1}^{\infty} I_n$, $I_n$ a copy of the interval $I = [0,1]$, $n \in N$, where N is the set of positive integers). A sequence $\underline{f} = \{f_n\}$ of maps (= continuous maps) $f_n : Q \to Q$, $n \in N$, is said to be a <u>fundamental sequence</u> of X to Y if for each neighborhood V of Y (in Q) there is a neighborhood U of X such that $f_n|U \simeq f_{n+1}|U$ in V for almost all n, that is, there is a homotopy $H : U \times I \to V$ such that $H(x,0) = f_n(x)$ and $H(x,1) = f_{n+1}(x)$ for $x \in U$. We write $\underline{f} : X \to Y$. Setting $i_n(x) = x$ for each $x \in Q$, for each compactum $X \subset Q$ $\underline{i}_X = \{i_n\} : X \to X$ is a fundamental sequence which is called the <u>fundamental identity sequence</u> of X.

Two fundamental sequences $\underline{f}$, $\underline{g} : X \to Y$ are said to be <u>homotopic</u> if for each neighborhood V of Y there is a neighborhood U of X such that $f_n|U \simeq g_n|U$ in V for almost all n. We denote it by $\underline{f} \simeq \underline{g}$. The collection of all fundamental sequences homotopic to a given fundamental sequence $\underline{f}$ is said to be the fundamental class with the representative $\underline{f}$ and it is denoted by $[\underline{f}]$.

The composition $\underline{h} = \underline{gf} : X \to Z$ of fundamental sequences $\underline{f}$ : $X \to Y$ and $\underline{g} : Y \to Z$ is defined as the fundamental sequence consisting of maps $h_n = g_n f_n : Q \to Q$. If $\underline{f} \simeq \underline{f}' : X \to Y$ and $\underline{g} \simeq \underline{g}'$: $Y \to Z$, then $\underline{gf} \simeq \underline{g}'\underline{f}' : X \to Z$.

Each map $f : X \to Y$ defines a fundamental sequence $\underline{f} : X \to Y$ as follows. Take any extension $h : Q \to Q$ of f and put $f_n = h$ for each n. Then $\underline{f} = \{f_n\}$ is a fundamental sequence of X to Y. We call $\underline{f}$ the fundamental sequence induced by f.

Proposition 1. *Let X and Y be 0-dimensional compacta. Then every fundamental sequence $\underline{f} : X \to Y$ is induced by a map $f$ : $X \to Y$ and f is uniquely determined by $\underline{f}$.*

Proof. Let $\underline{f} = \{f_n\} : X \to Y$. From the definition of a fundamental sequence and the compactness of Q the sequence $\{f_n(x)\}$, $x \in X$, converges some point $f(x)$ of Y. Obviously the correspondence $x \to f(x)$, $x \in X$, defines a map $f : X \to Y$ and it induces $\underline{f}$.

Compacta X and Y in Q are said to be fundamentally equivalent if there exist two fundamental sequences $\underline{f} : X \to Y$ and $\underline{g}$ : $Y \to X$ such that $\underline{gf} \simeq \underline{i}_X$ and $\underline{fg} \simeq \underline{i}_Y$. Then we write $X \simeq_F Y$. If we assume only that the relation $\underline{gf} \simeq \underline{i}_X$ holds, then we say that Y fundamentally dominates X and we write $Y \geq_F X$. If X and Y are homeomorphic, then $X \simeq_F Y$, because if $\underline{f} : X \to Y$ and $\underline{g}$ : $Y \to X$ are fundamental sequences induced by f and $g = f^{-1}$ then

$\underline{gf} \simeq \underline{i}_X$ and $\underline{fg} \simeq \underline{i}_Y$. Also, if X and Y are homotopically equivalent, then $X \simeq_F Y$.

It is known that the relation of the fundamental equivalence and the relation of the fundamental domination have an absolute character, that is, they do not depend on locations of compacta X and Y in Q. Since the relation of the fundamental equivalence is equivalence relation, the class of all compacta decomposes into mutually disjoint classes of compacta, called shapes. We denote by Sh(X) the class containing X and we call it the shape of X. Also we write $Sh(X) \geq Sh(Y)$ if $X \geq_F Y$. If X and Y are ANR's (= compact ANR's for metric spaces), then it is known that $Sh(X) \geq Sh(Y)$ if and only if X dominates homotopically Y and S(X) = Sh(Y) if and only if X and Y have same homotopy type. The shape of a space consisting of only one point is said to be trivial and denote by Sh(1). If X is contractible, then it is obvious Sh(X) = Sh(1).

Let X be a compactum contained in a compactum Y. A fundamental sequence $\underline{f} = \{f_n\} : Y \to X$ is said to be a fundamental retraction if $\underline{f}\underline{j}_X \simeq \underline{i}_X$, where $\underline{j}_X : X \to Y$ is a fundamental sequence induced by the inclusion $j_X : X \subset Y$. If there is a fundamental retraction $\underline{f} : Y \to X$, then we call X a fundamental retract of Y. If there is a fundamental retraction $\underline{f} : Y \to X$ such that $\underline{f} \simeq \underline{i}_Y$, then X is a fundamental deformation retract of Y. A compactum X is said to be a fundamental absolute retract ( a fundamental absolute neighborhood retract), written as FAR (FANR), if X is a fundamental retract of an AR (an ANR).

The following theorem characterizes a compactum with trivial shape ([4],[8],[18]).

Theorem 1. (K.Borsuk, D.M.Hyman, S.Mardešić) *For a compactum* X *the followings are equivalent.*

(1) X *is of trivial shape*.

(2) X *is an* FAR.

(3) *For a certain imbedding* $X \subset Q$ *there is a sequence* $\{X_n\}$ *of neighborhoods of* X *such that each* $X_n$ *is homeomorphic to* Q, $X_{n+1} \subset \text{Interior } X_n$, $n \in N$, *and* $\bigcap_n X_n = X$.

For a compactum X, Borsuk defined Fd(X), the *fundamental dimension* of X, as the minimum of dimensions of all compacta Y with $Sh(Y) \geq Sh(X)$ :

$$Fd(X) = \min_{Sh(Y) \geq Sh(X)} \dim Y$$

Obviously it holds that if $Sh(X) \leq Sh(Y)$ then $Fd(X) \leq Fd(Y)$ and if X and Y are compacta and $Y \neq \emptyset$ then $Fd(X) \leq Fd(X \times Y) \leq Fd(X) + Fd(Y)$.

Let X be a compactum. A closed subset Y of X is said to be a *fundamental* k-*skeleton* of X if $\dim Y \leq k$ and the homomorphisms : $\check{H}_n(Y:G) \to \check{H}_n(X:G)$ and $\underline{\pi}_n(Y,y_0) \to \underline{\pi}_n(X,y_0)$ induced by the inclusion $(Y,y_0) \subset (X,y_0)$, $y_0$ a point of Y, are isomorphisms for $0 \leq n < k$ and an epimorphism for $n = k$, where $\check{H}_n(X:G)$ is the n-dimensional Čech homology group of X with coefficients in G and $\underline{\pi}_n(X,y_0)$ is the n-dimensional fundamental group of $(X,y_0)$ defined by Borsuk [3].

We do not know whether every compactum has a fundamental 0-skeleton or not. If $X$ is a solenoid of Van Dantzig, then $X$ has a fundamental 0-skeleton which homeomorphic to a Cantor discontinuum. (See Corollary of Theorem 5).

## §2. Approach to shapes by Mardesic and Segal.

By an ANR-sequence we imply an inverse sequence $\underline{X} = \{X_n, \pi_{n\,n+1}\}$ over the set of positive ingers N, where $X$ is an ANR and $\pi_{n\,n+1}$: $X_{n+1} \to X_n$ is a map, $n \in N$ ($\pi_{nm} = \pi_{n\,n+1} \cdots \pi_{m-1\,m}$ for $n < m$). Let $X = \varprojlim \underline{X}$ and let $\pi_n : X \to X_n$ be the projection. A map $\underline{f} : \underline{X} \to \underline{Y} = \{Y_n, \mu_{n\,n+1}\}$ consists of an increasing function $f : N \to N$ and of a collection of maps $f_n : X_{f(n)} \to Y_n$ such that

$$f_n \pi_{f(n)\,f(n')} \simeq \mu_{nn'} f_{n'} \qquad \text{for } n \leq n',\ n,n' \in N.$$

Two maps $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ are said to be homotopic, $\underline{f} \simeq \underline{g}$, if for each $n \in N$ there is an $n' \in N$, $n' \geq f(n), g(n)$, such that

$$f_n \pi_{f(n)\,n'} \simeq g_n \pi_{g(n)\,n'} .$$

The composite $\underline{gf} : \underline{X} \to \underline{Z}$ of $\underline{f} : \underline{X} \to \underline{Y}$ and $\underline{g} : \underline{Y} \to \underline{Z} = \{Z_n, \nu_{n\,n+1}\}$ is a map of sequences $\underline{h} : \underline{X} \to \underline{Z}$, where $h = fg : N \to N$ and $h_n = g_n f_{g(n)} : X_{fg(n)} \to Z_n$. The identity map of sequences $\underline{i_X} : \underline{X} \to \underline{X}$ is given by the identity $1_N : N \to N$ and the map $i_{X_n} : X_n \to X_n$ $n \in N$. Two compacta $X$ and $Y$ are said to be of the same shape in the sense of ANR-systems, written as $\overline{Sh}(X) = \overline{Sh}(Y)$, provided there exist ANR-systems $\underline{X}$ and $\underline{Y}$ with $X = \varprojlim \underline{X}$ and $Y = \varprojlim \underline{Y}$ and maps $\underline{f} : \underline{X} \to \underline{Y}$ and $\underline{g} : \underline{Y} \to \underline{X}$ such that $\underline{gf} \simeq \underline{i_X}$ and $\underline{fg} \simeq \underline{i_Y}$.

Mardešić and Segal [15,16] gave the following useful characterization of shapes.

Theorem 2. (S.Mardešić and J.Segal) Let X and Y be compacta. Then Sh(X) = Sh(Y) if and only if $\overline{Sh}(X) = \overline{Sh}(Y)$.

§2. Shape of decomposition spaces.

According to Borsuk [4,p.266], a compactum X is said to be approximatively k-connected if for a certain imbedding $X \subset Q$ and for every neighborhood V of X in Q there is a neighborhood U of X such that every map of a k-sphere $S^k$ into U is null homotopic in V. It is known that the approximative k-connectedness is the shape invariant.

Theorem 3. (Kodama) Let f be a map of a compactum X onto a compactum Y with dim $Y \leq n$ such that for each $y \in Y$ $f^{-1}(y)$ is approximatively k-connected, k = 0,1,..,n. Then $Sh(X) \geq Sh(Y)$. Moreover, if dim $X \leq n$ then Sh(X) = Sh(Y).

In the proof of Theorem 3 ([11]) an argument in the proof of Theorem of [9] is used essentially.

The following corollary is a generalization of Borsuk [4, Theorem (6.1)].

Corollary 1. An n-dimensional compactum X is of trivial shape if and only if X is approximatively k-connected for k = 0,1,..,n.

For the proof it is sufficient to apply Theorem 3 to the case where Y is a space consisting of one point.

Corollary 2. (R.B.Sher) If X and Y are finite dimensional and f is a map of X onto Y such that $f^{-1}(y)$ is of trivial shape

This is an immediate consequence of Theorems 1 and 3.

For a compactum X, denote by $\square(X)$ the set of all components of X. We consider $\square(X)$ as the decomposition space of X. Then it is a compactum. As an application of Theorem 3, we obtain the following theorem by Borsuk [3, Theorem (8.1)].

Corollary 3. (Borsuk) Let X, Y be compacta in Q. Then for every fundamental sequence $\underline{f} : X \to Y$ there is a unique (continuous) map $\Lambda_f : \square X \to \square Y$ such that for each component $X_0$ of X $\underline{f} : X_0 \to \Lambda_f(X_0)$ is a fundamental sequence. Moreover $\Lambda_f$ depends only on the fundamental class $\underline{f}$ and this dependence is covariant, that is, if $\underline{g} : Y \to Z$ is a fundamental sequence then $\Lambda_{gf} = \Lambda_g \Lambda_f$.

Proof. Let $\pi_X : X \to \square X$ and $\pi_Y : Y \to \square Y$ be the decomposition maps. Since $\pi_X^{-1}(x)$ is a continuum for each $x \in \square X$, it is approximatively 0-connected. Since $\dim \square X = 0$, by Theorem 3 there is a fundamental sequence $\underline{h} : \square X \to X$ such that $\underline{\pi}_X \underline{h} \simeq \underline{i}_{\square X}$. Consider $\underline{\pi}_Y \underline{fh} : \square X \to \square Y$. By Proposition 1, $\underline{\pi}_Y \underline{fh}$ is induced by a map $\Lambda_f : \square X \to \square Y$. It is obvious that $\Lambda_f$ satisfies Corollary 3.

The following generalizes Sher [21, Theorem 12] and it is given by a similar method as in a proof of Theorem 3 (cf. [11]).

Corollary 4. Let $(X,x_0)$ and $(Y,y_0)$ be pointed compacta. Let f be a map of $(X,x_0)$ onto $(Y,y_0)$. If $f^{-1}(y)$ is approximatively k-connected for each $y \in Y$ and $k = 0,1,..,n$, then the induced homomorphism $f_* : \underline{\pi}_k(X,x_0) \to \underline{\pi}_k(Y,y_0)$ is an isomorphism for $k = 0,1,..,n$, where $\underline{\pi}_k$ is the k-dimensional fundamental group of Borsuk [3].

Corollary 5. Let f be a map of a compactum X onto an n-dimensional compactum Y such that $f^{-1}(y)$ is approximatively k-connected for each $y \in Y$ and $k = 0,1,..,n$. Then $Fd(X) \geq Fd(Y)$.

4. $\Delta$-spaces and fundamental dimension.

A compactum X is said to be a $\Delta$-space if there is an inverse sequence $\{K_n, \pi_{n\,n+1}\}$ of finite simplicial complexes such that $X = \varprojlim \{K_n\}$ and each bonding map $\pi_{n\,n+1}: K_{n+1} \to K_n$ is simplicial.

Theorem 4. (Kodama) (1) Every 0-dimensional compactum and every finite polytope are $\Delta$-spaces.

(2) There is a 1-dimensional AR with property ($\Delta$) which is not a $\Delta$-space.

(3) Every $\Delta$-space is dimensionally full-valued for paracompact spaces (cf. [14]).

In the shape category every compactum has a $\Delta$-space as its representative as shown by the following.

Theorem 5. (Kodama) For each compactum X there is a $\Delta$-space X' such that Sh(X) = Sh(X') and Fd(X) = Fd(X').

Corollary. For every compactum X there is a compactum X' such that X' contains X as a fundamental deformation retract and X' has a fundamental k-skeleton for each $k = 0,1,2,..$ .

We only give a proof of Corollary. For a given compactum X, find a $\Delta$-space Y by Theorem 5 such that Sh(X) = Sh(Y). By Moszyńska [20] there is a compactum X' such that both X and Y are fundamental deformation retracts of X'. Let $\{K_n, \pi_{n\,n+1}\}$ be an inverse sequence of finite simplicial complexes such that Y = $\varprojlim \{K_n\}$ and each $\pi_{n\,n+1}$ is simplicial. Denote by $K_n^i$ the i-skeleton of $K_n$, i = 0,1,... . Then $\{K_n^i, \pi_{n\,n+1}\}$ forms an inverse sequence. Put $Y_i = \varprojlim \{K_n^i\}$, i = 0,1,... . Then it is obvious that $Y_i$ is a fundamental i-skeleton of X'.

As shown in the above, every $\Delta$-space has a fundamental k-skeleton for each k = 0,1,2,... . On the other hand, consider the 2-dimensional continuum $Q(\sigma)$ constructed in [10,p.390]. It is easy to know that $Q(\sigma)$ has no fundamental 1-skeleton. Also, we can see that every ANR has a fundamental i-skeleton for i = 0, 1. The following example is a trivial modification of the example constructed by Borsuk [1].

Example. There is an infinite dimensional ANR X which does not have a fundamental k-skeleton for each k = 2,3,... .

To find such an ANR X, let $S^2$ be a 2-sphere and let A be an arc in $S^2$. Take a map f from A onto the Hilbert cube Q and let X be the adjunction space obtained by $S^2$, Q and f. Then X is an infinite dimensional ANR. If $X_k$ is a fundamental k-skeleton of X for $k \geq 2$, then $X_k$ has to contain a subset $S^2-A$ of X. Since $S^2-A$ is dense in X, we have $X_k = X$. Thus there is no fundamental k-skeleton of X, k = 2,3,... .

Proposition 2. <u>If</u> X <u>is a compactum in an</u> n-<u>dimensional euclidean space</u> $R^n$, <u>then</u> $Fd(X) \leq n - 1$.

Proof. Take a sequence $\{K_k\}$ of triangulable neighborhoods of X in $R^n$ such that $K_{k+1} \subset K_k$, $k = 1,2,\ldots$ , and $\bigcap_k K_k = X$. Since $K_k$ is an n-dimensional polyhedron in $R^n$, there is a sub-plyhedron $L_k$ of $K_k$ such that $L_k$ is a strong deformation retract of $K_k$ and dim $L_k \leq n-1$. By induction, we can find a simplicial subdivision $\widetilde{L}_k$ of $L_k$ and a simplicial map $\pi_{kk+1}: \widetilde{L}_{k+1} \to \widetilde{L}_k$ such that $j_k \pi_{kk+1} \simeq i_{k+1} j_{k+1} | \widetilde{L}_{k+1}$ in $K_k$ for $k = 1,2,\ldots$ , where $j_k$ : $L_k \to K_k$, $j_{k+1}$ : $L_{k+1} \to K_{k+1}$ and $i_{k+1}$ : $K_{k+1} \to K_k$ are the inclusions. Consider the inverse sequence $\{\widetilde{L}_k, \pi_{kk+1}\}$ and $X' = \varprojlim \widetilde{L}_k$. It is known by Theorem 2 that Sh(X) = Sh(X'). Since dim $X' \leq n-1$, we know $Fd(X) \leq n-1$.

Let $\mathcal{C} = \{X_\alpha | \alpha \in \Lambda\}$ be a collection of compacta. A member $X_0$ of $\mathcal{C}$ is said to be <u>majorant</u> for the shapes of members of $\mathcal{C}$ if $Sh(X_0) \geq Sh(X_\alpha)$ for each $X_\alpha \in \mathcal{C}$. For example, let $\mathcal{C}$ be the collection of all 0-dimensional compacta Y such that $Sh(Y) \leq Sh(X)$ for a given compactum X. Then the decomposition space $\square X$ of X consisting of all components of X is majorant for the shapes of members of $\mathcal{C}$. This follows from Corollary 3 of Theorem 3.

Proposition 3. (Watanabe) <u>For the collection $\mathcal{R}$ of all compacta in $R^1$ a Cantor discontinuum is majorant for the shapes of members of $\mathcal{R}$.</u>

This is a consequence of Proposition 2.

Theorem 6. (1) (S.Spież) <u>There is a compactum in $R^2$ which is majorant for the shapes of all compacta in $R^2$.</u>

(2) (K.Borsuk and W.Holsztyński) <u>For the collection of</u>

<u>all solenoids</u> $\mathcal{T}$ <u>no compactum</u> $X_0$ <u>satisfies the condition</u> $Sh(X) \leq Sh(X_0)$ <u>for every</u> $X \in \mathcal{T}$. <u>Hence, if</u> $\mathcal{C}$ <u>is the collection of all compacta in</u> $R^3$, <u>then there is no compactum which is majorant for the shapes of members of</u> $\mathcal{C}$.

Problem 1. Let X be a compactum and let $\mathcal{C}_X$ be the collection of all compacta Y such that $Sh(X) > Sh(Y)$. Does there exist a compactum which is majorant for the shapes of members of $\mathcal{C}_X$ ?

The following problem is raised by Borsuk [3].

Problem 2. (Borsuk) Let X and Y be compacta. If $Fd(Y) > 0$, then does it hold $Fd(X \times Y) \geq Fd(X) + 1$ ?

It is likely true that the following holds. However it does not know yet.

Problem 3. For every compactum X, does it hold that $Fd(X \times S^1) = Fd(X) + 1$ ? Here $S^1$ is a 1-sphere.

§5. Movable compacta.

According to Borsuk [3,5], a compactum X in Q is said to be <u>movable</u> if for each neighborhood U of X there is a neighborhood V of X such that for every neighborhood W of X there is a homotopy $H : V \times I \rightarrow U$ satisfying the condition:

$$H(x,0) = x \quad \text{and} \quad H(x,1) \in W \quad \text{for each } x \in V.$$

A compactum X is said to be k-<u>movable</u> if for every neighborhood U of X there is a neighborhood V of X such that for every compactum $A \subset V$ with $\dim A \leq k$ and for every neighborhood W of X there is a homotopy $H : A \times I \rightarrow U$ satisfying the condition:

$$H(x,0) = x \quad \text{and} \quad H(x,1) \in W \qquad \text{for } x \in A.$$

Mardešić and Segal [17] gave a characterization of movable compacta in terms of ANR sequences.

Theorem 7. (Mardešić and Segal) A compactum X is movable if and only if there is an ANR sequence $\{X_n, \pi_{n\,n+1}\}$ satisfying the following condition: $X = \varprojlim \{X_n\}$ and for each $n \in N$ there is an n', $n' \geq n$, such that every $n'' \geq n$ there is a map $\mu_{n''n'}: X_{n'} \to X_{n''}$ satisfying the homotopy relation $\mu_{n''n'} \pi_{nn''} \simeq \pi_{nn'}$.

For movable compacta, the followings are known.

Theorem 8. (Borsuk) (1) Let X and Y be compacta with $Sh(X) \geq Sh(Y)$. If X is movable (k-movable), then Y is movable (k-movable).

(2) If X is movable (k-movable), then the suspension $\Sigma X$ of X is movable (k-movable).

(3) Every compactum in $R^2$ is movable.

(4) If $X_i$ is a movable compactum for $i = 1,2,\ldots$, then $\prod_i X_i$ is movable.

(5) Every FANR is movable.

Theorem 9. (Kodama and Watanabe) An n-dimensional and n-movable compactum is movable.

Theorem 10. (1) (Mardešić) An n-dimensional $LC^{n-1}$ compactum is movable.

(2) (Borsuk) An $LC^{n-1}$ compactum is n-movable.

Let X be a $\Delta$-space. As we know from the proof of Corollary of Theorem 5, for each $k = 0,1,\ldots$, there is a fundamental k-skeleton $X_k$ of X. It is easy to see $X_k$ is i-movable for $i = 0,1,..,k-1$, if X is movable.

Problem 4. Let X be a movable $\Delta$-space. For each $k = 1, 2,..$ , does there exist a fundamental k-skeleton $X_k$ of X which is movable ?

K.Borsuk[5] raised the following problems:

(1) Is it true that if X is m-movable and Y is n-movable then $X \times Y$ is (m+n)-movable ?

(2) Does there exist, for each $n = 1,2,..$ , a continuum which is n-movable, but is not (n+1)-movable ?

(3) Does there exist a non-movable compactum which is n-movable for every $n = 1,2,..$ ?

These were solved by Kodama and Watanabe [12].

Theorem 11. (Kodama and Watanabe) (1) *For compacta* X *and* Y, $X \times Y$ *is* k-*movable if and only if both* X *and* Y *are* k-*movable*.

(2) *If* X *is* k-*movable, then* $\Sigma X$ *is* (k+1)-*movable*.

(3) *There is a continuum* X *such that* X *is* k-*movable for every* $k = 1,2,..$ , *but not movable*.

To show (3) of Theorem 11, we remark that there is a non-movable continuum $X_0$ such that $\Sigma X_0$ is homeomorphic to $X_0$ [7]. Since an n-fold suspension of a compactum X is (n-1)-movable by (2) of Theorem 11, the continuum $X_0$ mensioned above is k-movable for every $k = 1,2,..$ . Borsuk proved every solenoid is not 1-movable. It is known that a suspension of a solenoid is 1-movable but not 2-movable.

It is known that every 2-dimensional ANR is dimensionally full-valued (cf.[14]).

Problem 5. Is every 2-dimensional movable compactum dimensionally full-valued ?

Let X be a compactum with metric d. Let K be a finite simplicial complex and let V(K) be the set of vertices of K. For a map $f : V(K) \to X$, we mean by mesh f the maximum of diameters of $f(s \cap V(K))$ for every simplex s of K. Let $\epsilon > 0$. For maps $f,g : V(K) \to X$ with max (mesh f, mesh g) $< \epsilon$, by $f \sim_\epsilon g$ we imply that there is a sequence of maps $h_i : V(K) \to X$, $i = 0,1,..,n$, such that $f = h_o$, $g = h_n$, mesh $h_i < \epsilon$, $i = 0,1,..,n$, and $\max \{d(h_i(v), h_{i+1}(v)) : v \in V(K)\} < \epsilon$, $i = 0,1,..,n-1$.

Proposition 4. A compactum X is movable if and only if for every $\epsilon > 0$ there is a $\delta > 0$ satisfying the following conditions: For every finite simplicial complex K, every map $f : V(K) \to X$ with mesh $f < \delta$ and every $\nu > 0$ there is a subdivision K' of K and a map $g : V(K') \to X$ such that mesh $g < \nu$ and $f\pi \sim_\epsilon g$, where $\pi : V(K') \to V(K)$ is a map defined by any projection of K' to K.

This proposition gives a simple proof of Theorem 10. In a similar form to Proposition 4 we can obtain a necessary and sufficient condition for a compactum X in order that X be an FANR.

## References

[1] K.Borsuk, Some remarks concerning the position of sets in a space, Bull.Acad.Polon.Sci.,Sér.Sci.Math.,Astronom., 8(1960), 609-613.

[2] K.Borsuk, Concerning homotopy properties of compacta, Fund.Math., 62(1968), 223-254.

[3] ———, Theory of shape, Aarhus University (1971), Lecture Nore.

[4] ———, A note on the theory of shape of compacta, Fund. Math., 68(1970), 265-278.

[5] ———, On the n-movability, Bull.Acad.Polon.Sci.,Sér. Sci.Math.,Astronom.,20(1972), 859-864.

[6] K.Borsuk and W.Holsztyński, Concerning the ordering of compacta, Fund.Math., 68(1970), 107-115.

[7] D.Handel and J.Segal, An acyclic continuum with non-movable suspensions, Bull.Acad.Polon.Sci.,Sér.Sci.Math. Astronom.Phys., 21(1973), 171-172.

[8] D.M.Hyman, On decreasing sequences of compact absolute retracts, Fund.Math., 64(1969), 91-97.

[9] Y.Kodama, On a closed mapping between ANR's, Fund.Math., 45(1958), 217-228.

[10] ———, On a problem of Alexandroff concerning the dimension of product spaces I, J.Math.Soc.Japan, 10(1958), 380-404.

[11] ———, On the shape of decomposition spaces, to appear.

[12] ——— and Watanabe, A note on Borsuk's n-movability, to appear.

[13] Y.Kodama, On $\Delta$-spaces and fundamental dimension of K. Borsuk, to appear.

[14] ———, Note on cohomological dimension for non-compact spaces, J.Math.Soc.Japan, 18(1966), 343-359.

[15] S.Mardešić and J.Segal, Shapes of compacta and ANR-systems, Fund.Math.,72(1971), 41-59.

[16] ———, Equivalence of Borsuk and the ANR-system approach to shapes, Fund.Math., 72(1971), 61-68.

[17] ———, Movable compacta and ANR-systems, Bull. Acad.Polon.Sci., Sér.Sci.Math.Astronom.Phys., 18(1970),

649-654.

[18] S.Mardešić, Decreasing sequences of cubes and compacta of trivial shape, Gen.top. and its appli., 2(1972), 17-23.

[19] ——————, n-dimensional $LC^{n-1}$ compacta are movable, Bull. Polon.Sci.,Sér.Sci.Math.Astr.Phys., 19(1971), 505-509.

[20] M.Moszyńska, On shape and fundamental deformation retracts II, Fund.Math., 77(1973), 235-240.

[21] R.B.Sher, Realizing cell-like maps in Euclidean space, Gen.top.and its appli.,2(1972), 75-89.

[22] S.Spież, On a plane compactum with the maximal shape, Fund.Math., 78(1973), 145-156.