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Finite groups with Sylow 2-subgroups of type A_{16}

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A 2-group is said to be of *type* X if it is isomorphic to a Sylow 2-subgroup of the group X. If G is a group with a Sylow 2-subgroup S of type X, we say that G has the *involution fusion pattern* of X if for some isomorphism θ of S onto a Sylow 2-subgroup of X, two involutions a, b of S are conjugate in G if and only if the involutions $\theta(a)$, $\theta(b)$ of $\theta(S)$ are conjugate in X. Also we say that a group G is *fusion-simple* if $G = O^2(G)$ and $O(G) = Z(G) = 1$.

Now we have obtained the following:

THEOREM A. *Let* G *be a fusion-simple finite group with Sylow* 2-*subgroups of type* A_{16}. *Then one of the following holds:*

(1) $G \cong A_{16}$ *or* A_{17},

(2) $G \cong A_9 \cdot E_{256}$, *the split extension of an elementary abelian group* E_{256} *of order* 256 *by* A_9 *with the action afforded by the* 8-*dimensional irreducible* GF(2)-*representation*, *or*

(3) G *has the involution fusion pattern of* $\Omega_9(3)$.

Here $\Omega_9(3)$ denotes the orthogonal commutator group of degree 9 over the field of 3-elements and A_m the alternating group on m-letters.

In the process of proving Theorem A we obtain the following characterization.

THEOREM B. *Let* G *be a finite group with Sylow* 2-*subgroups of type* A_{16}. *If* G *has the involution fusion pattern of* A_{16}, *then* $G/O(G) \cong A_{16}$ *or* A_{17}.

Proof of the Theorem A is obtained in the following way which appears to be rapidly becoming standard(cf. Gorenstein-Harada[5], [6], Solomon[9]). Let S be a Sylow 2-subgroup of G and A be the unique elementary abelian subgroup of S of order 256. At first we show that the fusion of elements of S is controlled by $N_G(A)$ and $N_G(Z_2(S))$ where $Z_2(S)$ is the second center of S, using results of Alperin[1] and Goldschmidt[2] on conjugation family. Since S/A is of type A_8, the structure of $N_G(A)/C_G(A)$ which is isomorphic to a subgroup of $GL(8,2)$ is determined by theorems of Harada[7] and Gorenstein-Harada[5],[6]. Then the fusion possibilities of involutions follow immediately. Here we can prove that if A is strongly closed in S with respect to G, then $G = N_G(A) \cong A_9 \cdot E_{256}$ by a recent result of Goldschmidt [4]. Characterization theorems of Gorenstein-Harada[5],[6] and Solomon[9] permit the determination of $C_G(a)/O(C_G(a))$ for all involution a in S. Now O is an A-signalizer functor and a signalizer functor theorem[3] implies that $W_A = \langle O(C_G(a)); a \in A^{\#} \rangle$ has odd order. It follows that $N_G(W_A)$ is strongly embedded in G provided $W_A \neq 1$. Since G has more than one conjugacy class of involutions, $W_A = 1$. Therefore $O(C_G(a)) = 1$ and Kondo's characterization theorem[8] implies that $G \cong A_{16}$ or A_{17}.

References

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