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Wave Equation with Wentzell's Boundary Condition and a Related Semigroup on the Boundary

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Here, the problem is to solve the wave equation

(1) $$\frac{\partial^2 u}{\partial t^2}(x) = Au(x), \quad x \in D$$

on a compact domain $\bar{D}$, with Wentzell's boundary condition

(2) $$Lu(x) = \sum_1^{N-1} \alpha_{ij}(x)\frac{\partial^2 u}{\partial\xi_i\partial\xi_j}(x) + \sum_1^{N-1}\beta_i(x)\frac{\partial u}{\partial\xi_i}(x) + \gamma(x) + \delta(x)Au(x)$$
$$+ \int_{\bar{D}}(u(y) - u(x) - \sum_1^{N-1}\frac{\partial u}{\partial\xi_i}(x)\,\xi_i(x, y))\,\nu(x, dy) = 0, \ x \in \partial D,$$

which is, in a sense, the most general boundary condition for diffusion equation. The solution is given as a group of operators on a function space.

Another group of operators is obtained, which corresponds to

(3) $$\frac{\partial^2 \varphi}{\partial t^2}(x) = \overline{LH}\,\varphi(x), \qquad x \in \partial D,$$

where $\overline{LH}$ is a closure of LH : $(LH)\varphi(x) = L(H\varphi)(x)$. Here, $H\varphi(x)$ is the solution of the Dirichlet problem $Au(x) = 0$, $x \in D$, with the boundary condition $u(x) = \varphi(x)$, $x \in \partial D$.

Equation (3) is expected to describe the wave propagation through the boundary with mass distribution $\delta(x)dx$ and the vibration term $\sum_1^{N-1}\alpha_{ij}(x)\frac{\partial^2}{\partial\xi_i\partial\xi_j}$ of the boundary.

The concerte results are contained in the article with the same title in Proc. Japan Acad., vol. 49, 1973.