

標本調査法の記号について

千葉大 理 浅井 晃

標本調査法ではその内容の性格上他の分野に比べてかなり多くの記号を定義する必要がある。その発達当初から各著者は記号の選定に苦勞したことがそれぞれの著書とみると窺える。特異な例として Yates [17] は活字の型を渡える方法を用い、一時期幾つかの論文にも利用されたが、現在は殆ど使われていない。本来数学的記号の定義は自由であると言っても、一般には不文律的に統一されているものが多い。しかし標本調査法では不統一が甚だしい。例えば x_n を確率変数とする書物もあれば母数とちるものもある等である。論文の場合は、本来全文を通読すべきものであるから、自由な定義があっても支障ないが、成書の場合は、部分的利用のためにも、初心者の混乱を防ぐ意味から、是非とも統一した記号を採用して欲しいものである。この意味から、筆者が過去に通読した成書と比較検討し、一方一般数理統計学の常識から

逸脱しないことを念頭に、標本調査法の記号統一の素案を作ってみた。それは必ずしも頻度の高いものとも限らず、記号のもつ意味から決めたものもある。比較検討した事物の主なものを文献欄に示した（他の教種および邦書は割愛した）。以下の記号表には参考として有名または最近の成書（文献番号に*を付したもの）にある記号も示した。なお、記号統一を詳細に追まるとかえって弊害もありうるので、基本的なもの、主要抽出方法、に限定した。最後に JIS のような意味での統一を企てる意志のないことを付記しておく。

	Proposed	[1] Cochran	[4] Deming	[5] Hansen ...	[9] Konijn	[10] Murthy	[11] Sukhatme
[General]							
Expectation	$E[\hat{\theta}]$	$E(\hat{\theta})$	$E \hat{\theta}$	$E \hat{\theta}$	$E \hat{\theta}$	$E(\hat{\theta})$	$E(\hat{\theta})$
Variance	$V[\hat{\theta}]$	$V(\hat{\theta})$	$Var \hat{\theta}$	$\sigma_{\hat{\theta}}^2$	$V(\hat{\theta})$	$V(\hat{\theta})$	$V(\hat{\theta})$
Standard error	$d[\hat{\theta}]$	$\sigma_{\hat{\theta}}$	$\sigma_{\hat{\theta}}$	$\sigma_{\hat{\theta}}$	-	$\sigma(\hat{\theta})$	Est. $V(\hat{\theta})$
Coef. of variation	$cv[\hat{\theta}]$	c.v.	$C_{\hat{\theta}}$	$V_{\hat{\theta}}$	-	$C(\hat{\theta})$	S.E. $(\hat{\theta})$
Bias	$b[\hat{\theta}]$	bias in $\hat{\theta}$	$B \hat{\theta}$	-	-	$b(\hat{\theta})$	-
Mean square error	$mse[\hat{\theta}]$	$MSE(\hat{\theta})$	$\{M \hat{\theta}\}^2$	$MSE_{\hat{\theta}}$	-	$M(\hat{\theta})$	M.S.E. $(\hat{\theta})$
Covariance	$cov[\hat{\theta}, \hat{\theta}']$	$Cov(\hat{\theta}, \hat{\theta}')$	$Cov \hat{\theta}, \hat{\theta}'$	$S_{\hat{\theta}\hat{\theta}'}$	-	$Cov(\hat{\theta}, \hat{\theta}')$	$Cov(\hat{\theta}, \hat{\theta}')$
Correlation coef.	$r[\hat{\theta}, \hat{\theta}']$	$\rho_{\hat{\theta}\hat{\theta}'}$	$\rho_{\hat{\theta}\hat{\theta}'}$	$\rho_{\hat{\theta}\hat{\theta}'}$	$\rho(\hat{\theta}, \hat{\theta}')$	ρ	ρ
Summation	$\sum_{i=1}^n a_i$	$\sum_{i=1}^n y_i$	$\sum_{i=1}^n a_i$	$\sum_{i=1}^n x_i$	$\sum_{i=1}^n y_i$	$\sum_{i=1}^n y_i$	$\sum_{i=1}^n y_i$
Conditional E & V	$V[\hat{\theta}] = E[V[\hat{\theta}]] + V[E[\hat{\theta}]]$	-	$\sum_{i=1}^n x_i$	$E\sigma_{\hat{\theta} [n]}^2 + \sigma_{E[\hat{\theta}]}^2$	$E[V(\hat{\theta})] + V[E(\hat{\theta})]$	$E[V(\hat{\theta})] + V[E(\hat{\theta})]$	$E[V(\hat{\theta})] + V[E(\hat{\theta})]$
[Simple Random Samp.]							
No. of s. units	N	N	N	N	N	N	N
Variate	a_i	y_i	x_i	x_i	y_i	y_i	y_i
Mean	\bar{a}	\bar{y}	\bar{x}	\bar{x}	\bar{y}	\bar{y}	\bar{y}
Total	A	Y	X	X	$N\bar{y}$	Y	$N\bar{y}$
Variance (para.)	$\sigma^2 = \frac{1}{N} \sum (a_i - \bar{a})^2$	σ^2	σ^2	σ^2	σ^2	σ^2	σ^2
"	$S^2 = \frac{1}{N-1} \sum (a_i - \bar{a})^2$	S^2	$\hat{\sigma}^2$	S^2	S^2	S^2	S^2
C.V. (parameter)	γ	c.v.	-	v	-	-	C_y
Proportion	P, Q	P, Q	\hat{p}, \hat{q}	P, Q	P, Q	P, Q	P, Q
Sampling fraction	f	f	n/N	f	f	f	n/N
Sampling interval	d	k	N/n	k	k	k	K

[Ratio & Regression]	Proposed	[1] Cochran	[4] Deming	[5] Hansen ...	[9] Konijn	[10] Murthy	[11] Sukhatme
Variates	x_i, y_i	y_i, x_i	x_i, y_i	x_i, y_i	y_i, x_i	y_i, x_i	y_i, x_i
Mean	\bar{a}, \bar{b}	\bar{y}, \bar{x}	\bar{a}, \bar{b}	\bar{X}, \bar{Y}	\bar{Y}, \bar{X}	\bar{y}, \bar{x}	\bar{y}_N, \bar{x}_N
Total	A, B	Y, X	A, B	X, Y	Y, X	Y, X	Y, X
Ratio	$\hat{R} = A/B$	\hat{R}	ϕ	R	R	\hat{R}	R_N
Mean in subclass	\bar{x}_D	\bar{y}_j	f	\bar{X}_g	\bar{Y}_i	\bar{y}''	-
Variance	$\sigma^2(x)$	-	σ_x^2	σ_x^2	$\sigma^2(y)$	σ_y^2	-
"	$S^2(x)$	S_y^2	σ_x^2	S_x^2	$S^2(y)$	σ_y^2	S_y^2
C.V.	$\gamma(x)$	C_{yy}	C_x	V_x	$C(yg)$	-	C_y
Covariance	$\sigma(x, y)$	S_{yx}	-	S_{xy}	$S(ygx)$	σ_{yx}	S_{yx}
Rel-covariance	$\gamma(x, y)$	C_{yx}	\hat{C}_{xy}	V_{xy}	$C(ygx)$	-	C_{yx}
Correlation coef.	$\rho_{xy} f(x, y)$	ρ	ρ	ρ	$\rho(ygx)$	-	-
Var. of differ.	$S^2(R) = \frac{1}{N-1} \sum (a_i - R b_i)^2$	S_d^2	-	S_z^2	$S^2(d)$	-	-
Size of subclass	N_D	N_j	-	N_g	N_i	-	-
Var. in subclass	S_D^2	S^2	-	S_g^2	S_i^2	$\sigma^{''2}$	-
Ratio est. (mean)	\bar{x}_R	\hat{Y}_R	\bar{x}'	x''	\hat{Y}_R	\bar{y}_R	\bar{y}_R
" (total)	X_R	\hat{Y}_R	X'	x''	\hat{Y}_R	\hat{Y}_R	-
Regres. est. (total)	X_{LR}	\hat{Y}_{LR}	-	-	\hat{Y}_R	\hat{Y}_T	$N \bar{y}_e$
Regression coef.	β	b	-	k, β	k	β	β
[Stratified Sampling]							
No. of strata	L	L	-	L	L	K	K
No. of units in st.	N_k	N_k	N_i	N_k	N_k	N_o	N_i

	Proposed	[1] Cochran	[4] Deming	[5] Hansen ...	[9] Konijn	[10] Murthy	[11] Sukhatme
Samp. fraction in st.	f_k	f_k	n_i/N_i	f_k	n_k/N_k	n_o/N_o	n_i/N_i
Stratum weight	W_k	W_k	N_i/N	N_k/N	N_k/N	W_o	p_i
Variate	a_{ki}	y_{ki}	a_{ij}	x_{ki}	y_{ki}	y_{oi}	y_{ij}
Stratum mean	\bar{a}_k	\bar{y}_k	\bar{a}_i	\bar{x}_k	\bar{y}_k	\bar{y}_o	\hat{y}_o
Stratum total	A_k	X_k	A_i	X_k	X_k	\bar{Y}	\hat{Y}_{st}
Population mean	\bar{a}	\bar{y}_{st}	\bar{a}	\bar{x}	\bar{y}_{st}	\bar{Y}	\bar{y}_{st}
Population total	A	\bar{Y}_{st}	A	X	Y	Y	\hat{Y}_{st}
Variance	$\sigma_k^2 = \frac{1}{N_k} \sum (a_{ki} - \bar{a}_k)^2$	-	σ_i^2	σ_k^2	-	σ_o^2	-
"	$S_k^2 = \frac{1}{N_k-1} \sum (a_{ki} - \bar{a}_k)^2$	S_k^2	σ_i^2	S_k^2	S_k^2	σ_o^2	S_i^2
"	$\sigma_{WS}^2 = \sum \frac{N_k}{N} \sigma_k^2$	-	$\sigma_{w'}^2$	-	-	-	-
"	$S_{WS}^2 = \sum \frac{N_k}{N} S_k^2$	$S_{w'}^2$	-	$S_{w'}^2$	-	-	-
"	$\bar{\sigma}_{WS}^2 = (\sum \frac{N_k}{N} \sigma_k^2)$	-	$\bar{\sigma}_{w'}^2$	-	-	-	$\bar{S}_{w'}^2$
"	$\bar{S}_{WS}^2 = (\sum \frac{N_k}{N} S_k^2)$	-	-	-	-	-	$\bar{S}_{w'}^2$
Cost function	$K = k_o + \sum k_k n_k$	$C = c_o + \sum c_k n_k$	-	$C = \sum C_k n_k$	$C = c_o + \sum c_k n_k$	$C = c_o + \sum C_o n_o$	$C = \sum c_i n_i$
[Cluster & Two-stage]							
No. of clusters	M	N	M	m	N	N	N
Cluster size	N_i	M_i	N_i	N_i	M_i	M_i	M_i
Samp. size in cluster	n_i	m_i	n_i	n_i	m_i	m_i	m_i
Prob. for selection	P_i	p_i	-	P_i	p_i	p_i	-
Inclusion prob.	π_i	π_i	-	-	π_i	π_i	-
Sampling fractions	f, f_1, f_{s1}, f_2	f, f_1, f_{s1}, f_2	-	f, f_1, f_{s1}, f_2	-	-	-
Variate of SSU	a_{ij}	y_{ij}	a_{ij}	x_{ij}	y_{ij}	y_{ij}	y_{ij}

	Proposed	[1] Cochran	[4] Deming	[5] Hansen ...	[9] Konijn	[10] Murthy	[11] Sukhatme
Cluster mean	\bar{a}_i	\bar{y}_i	\bar{a}_i	\bar{x}_i	\bar{y}_i	\bar{y}_i	$\bar{y}_{i.}$
Cluster total	A_i	y_i	A_i	X_i	Y_i	Y_i	Y_i
Mean per cluster	\bar{A}	\bar{y}	\bar{A}	-	-	-	-
Population mean	\bar{a}	\bar{y}	\bar{a}	\bar{x}	\bar{Y}	\hat{Y}	$\bar{y}_{..}$
Population total	A	Y	A	X	Y	Y	Y
Var. (between clus.)	$\sigma_c^2 = \frac{1}{M} \sum (A_i - \bar{A})^2$	-	-	-	-	-	-
"	$S_c^2 = \frac{1}{M-1} \sum (A_i - \bar{A})^2$	-	-	$\frac{1}{M} S_{cX}^2$	$F^2 S_c^2$	-	-
"	$\sigma_{B1}^2 = \frac{1}{M} \sum \frac{N_i}{N} (a_i - \bar{a})^2$	-	σ_b^2	σ_b^2	-	-	-
"	$S_{B1}^2 = \frac{1}{M-1} \sum \frac{N_i}{N} (a_i - \bar{a})^2$	-	-	-	-	σ_b^2	-
"	$\sigma_{B2}^2 = \frac{1}{M} \sum (\frac{N_i}{N} a_i - \bar{a})^2$	-	-	-	-	σ_b^2	-
"	$S_{B2}^2 = \frac{1}{M-1} \sum (\frac{N_i}{N} a_i - \bar{a})^2$	-	-	S_{1X}^2	-	σ_b^2	S_b^2
"	$\sigma_{B3}^2 = \frac{1}{M} \sum (\frac{N_i}{N})^2 (a_i - \bar{a})^2$	-	-	-	-	-	-
"	$S_{B3}^2 = \frac{1}{M-1} \sum (\frac{N_i}{N})^2 (a_i - \bar{a})^2$	-	-	-	-	-	S_b^2
"	$\sigma_{Pc}^2 = \frac{1}{M} \sum P_i (\frac{A_i}{P_i} - A)^2$	-	-	S_x^2	-	-	-
"	$S_{Pc}^2 = \frac{1}{M-1} \sum P_i (\frac{A_i}{P_i} - A)^2$	-	-	-	-	-	-
"	$\sigma_{PB2}^2 = \frac{1}{M} \sum P_i (\frac{N_i}{P_i N} a_i - \bar{a})^2$	-	-	-	-	-	-
"	$S_{PB2}^2 = \frac{1}{M-1} \sum P_i (\frac{N_i}{P_i N} a_i - \bar{a})^2$	-	-	-	-	-	-
"	$\sigma_{PB3}^2 = \frac{1}{M} \sum P_i (\frac{N_i}{P_i N})^2 (a_i - \bar{a})^2$	-	-	-	-	-	-
"	$S_{PB3}^2 = \frac{1}{M-1} \sum P_i (\frac{N_i}{P_i N})^2 (a_i - \bar{a})^2$	-	-	-	-	-	-
Var. (within clus.)	$\sigma_i^2 = \frac{1}{N_i} \sum (a_{ij} - \bar{a}_i)^2$	-	σ_i^2	-	-	σ_{wi}^2	-
"	$S_i^2 = \frac{1}{N_i-1} \sum (a_{ij} - \bar{a}_i)^2$	S_{2i}^2	-	S_{2i}^2	$\frac{M_i}{M} S_{W(i)}^2$	σ_{wi}^2	S_i^2
"	$\sigma_w^2 = \frac{1}{M} \sum \frac{N_i}{N} \sigma_i^2$	-	σ_w^2	σ_w^2	S_W^2	σ_w^2	-
"	$S_w^2 = \frac{1}{M-1} \sum \frac{N_i}{N} S_i^2$	-	-	S_{2X}^2	-	-	S_w^2
Intracluster correl.	ρ_c	ρ_w	-	δ	r	ρ_c	ρ
Cost function	$K = k_0 + k_1 m + k_2 n$	$C = C_1 n + C_2 nm$	$K = C_1 m + C_2 m n$	$C = C_1 m + C_2 m n$	$C = C_0 + C_1 n + C_2 nm$	$C = C_0 + C_1 n + C_2 nm$	$C = C_1 n + C_2 nm$

[文献]

- *[1] Cochran, W. G. (1963) : *Sampling Techniques*, J. Wiley, New York.
- [2] Dalenius, T. (1957) : *Sampling in Sweden*, Almqvist & Wiksell, Stockholm.
- [3] Deming, W. E. (1950) : *Some Theory of Sampling*, J. Wiley, New York.
- *[4] Deming, W. E. (1960) : *Sample Design in Business Research*, J. Wiley, New York.
- *[5] Hansen, M. H., Hurwitz, W. N., & Madow, W. G. (1953) : *Sample Survey Methods and Theory - Vols. 1 & 2*, J. Wiley, New York.
- [6] Hendrics, W. A. (1956) : *The Mathematical Theory of Sampling*, Scarecrow Press, New Brunswick.
- [7] Kendall, M. G., & Stuart, A. (1966) : *Advanced Theory of Statistics - Vol. 3*, C. Griffin, London.
- [8] Kish, I. (1965) : *Survey Sampling*, J. Wiley, New York.
- *[9] Konijn, H. S. (1973) : *Statistical Theory of Sample Survey Design and Analysis*, North-Holland, Amsterdam.
- *[10] Murthy, M. N. (1967) : *Sampling Theory and Methods*, Statistical Pub. Soc., Calcutta.
- [11] Raj, D. (1968) : *Sampling Theory*, Prentice-Hall, New York.
- [12] Raj, D. (1972) : *The Design of Sample Surveys*, McGraw-Hill, New York.
- [13] Sampford, M. R. (1962) : *An Introduction to Sampling Theory with Application to Agriculture*, Oliver & Boyd, Edimburgh.
- [14] Stuart, A. (1962) : *Basic Ideas of Scientific Sampling*, C. Griffin, London.
- *[15] Sukhatme, P. V., & Sukhatme, B. V. (1970) : *Sampling Theory of Surveys with Applications*, Asia Pub. House, Bombay.
- [16] Yamane, T. (1967) : *Elementary Sampling Theory*, Prentice-Hall, Englewood Cliffs.
- [17] Yates, F. (1960) : *Sampling Methods for Census and Surveys*, C. Griffin, London.