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Lecture No. 3

Liouville and Phragmen-Lindelöf type theorems for elliptic and parabolic systems.

The results for the systems of partial differential equations which are described in this lecture, are all obtained by the identical procedure, using the analyticity of solutions of some auxiliary systems of partial differential equations. This approach can be described as follows. We introduce into a system of partial differential equations a new, additional independent variable x_0, in such a way that all solutions of the resulting system are analytic functions of x_0. Then we estimate an analytic continuation of the solution of the auxiliary system for complex values of the independent variable x_0, and as a consequence of this a priori estimate we obtain theorems for the original system of partial differential equations.

In this way one can prove theorems on the behaviour of solutions of elliptic and parabolic systems in unbounded domains and, in particular, Liouville and Phragmen-Lindelöf type theorems, uniqueness theorems for the solutions of the boundary value problems in unbounded domains in the classes of growing functions for elliptic and parabolic systems, the uniqueness theorems for the Cauchy problem, the asymptotic behaviour of fundamental solutions and the Green functions for parabolic systems, theorems on eigenfunctions and on solutions of systems of differential equations depending on a parameter and also some other theorems (see [1]-[9]).

Some of these results for parabolic systems were considered in Lecture 2. All mentioned results are based on modifications of the theorem on the Banach spaces of analytic functions in a real domain, which is given in Lecture 1.

For solutions of elliptic and parabolic systems of partial differential equations the estimate of the form (1) (see Lecture 1) can be proved using well-known a priori estimates for solutions and Morrey-Nirenberg method for proving the analyticity of solutions of elliptic equations. In this way one can study the dependence of constants δ_0 and C_0 which appear in the estimate (1) of Lecture 1, on the coefficients of the systems, the boundary conditions and the domain G. Some applications of this method can be done for the linearized system of the Navier-Stokes equations and for the system of the theory of elasticity.

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