数理解析研究所講究録 311

デザインの構成と解析

京都大学数理解析研究所

1977年10月

デザインの構成と解析
研究会報告集

1977年8月29日～8月31日

研究代表者　浜田昇

目　次

SYMPOSIUM ON CONSTRUCTION AND ANALYSIS OF DESIGNS

Place : Research Institute for Mathematical Sciences, Kyoto University, Kyoto, Japan

Date : August 29 - 31, 1977

Organizer : Noboru Hamada, Department of Mathematics, Faculty of Science, Hiroshima University

PROGRAMME AND ABSTRACT

1. N. Hamada and F. Tamari
(Hiroshima Univ. and Fukuoka Univ. of Education)
Construction of maximal t-linearly independent sets using spreads in a finite projective geometry

Abstract : Recently, B. R. Gulati and E. G. Kounias (J. Combinatorial Theory (A) 15 (1973), 54-65) and N. Hamada and F. Tamari (Essays in Probability and Statistics (1976), 41-55) have shown that it is sufficient to solve a linear programming in order to obtain a maximal t-linearly independent set. In this paper, a solution of the above linear programming is given using the concept of a min·hyper (or a max·hyper) and flats and spreads in a finite projective geometry.

2. I. Takahashi and E. Sugimoto
(Waseda University)
New construction methods of indexing polynomials (Primitive irreducible polynomials)

Abstract : It takes much amounts of computations to find indexing polynomials of degree n over GF(p) (being p prime). We clarify relations between coefficients of indexing polynomials and decompositions of semi simple rings generated from sums of Frobenius cycles. In case of p = 2 we may apply general algorithms of orthogonalizations to our problem, but we propose a more efficient cyclic algorithm for special cases. In case of p > 2 we can not yet find concrete algorithms but it is not so difficult for us to find algorithms based on our theories.

3. H. Enomoto
(Tokyo University)
On tight designs

Abstract : This is a survey article on tight designs: generalization of Fisher's inequality, intersection numbers, association schemes, recent progress in the classification problem of tight designs. "Classification of tight 4-designs" is still open, but recently the following theorem is proved:

There exist at most finitely many possibilities of nontrivial tight 4-designs.

Complete classification of tight 4-designs may not be too far off.

4. R. Noda
(Osaka University)
On some t-$(2k,k,\lambda)$ designs

Abstract : We consider t-$(2k,k,\lambda)$ designs with $t \geqq 2$ having the property that (*) the complement of each block is a block. The following theorems hold:

Theorem 1. Let a t-$(2k,k,\lambda)$ design have the property, together with (*) above, that if A and B are a complementary pair of blocks then $|A \cap C| = |B \cap C| \pm u$ $(u > 0)$ holds for any block C distinct from A and B. Then we have $t \leqq 3$, and if $t = 3$, then $k = u(2u+1)$ and $\lambda_3 = u(2u^2+u-2)$.

Theorem 2. Let a t-$(2k,k,\lambda)$ design have the property, together with (*) above, that if A and B are a complementary pair of blocks then $|A \cap C| = |B \cap C|$ or $|A \cap C| = |B \cap C| \pm u$ $(u > 0)$ holds for any block C distinct from A and B. Then we have $t \leqq 5$ and if $t = 5$ then one of the following holds:

(a) $k = 6$ and $\lambda_5 = 1$,

(b) $k = u(2u+1)/3$ and $\lambda_5 = u(2u^2+u-9)(2u^2+u-12)/54$,

(c) $k = u^2$ and $\lambda_5 = (u^2-3)(u^2-4)/4$.

5. E. Bannai
(Gakushuin University)
Tight spherical designs

Abstract : Let Ω_d be the unit sphere in the real Euclidean space $\mathbb{R}^d$. A finite subset X in Ω_d is said to be a spherical t-design if

$$\sum_{\xi\in X} f(\xi) = 0$$

for all homogeneous harmonic polynomials f of degree 1,2,...,t. Delsarte-Goethals-Seidel has obtained the following inequalities, which is analogous to the generalized Fisher's inequality for ordinary t-designs:

$$|X| \geqslant \binom{d+s-1}{d-1} + \binom{d+s-2}{d-1} \quad , \text{ for } \quad t = 2s$$

$$|X| \geqslant 2 \cdot \binom{d+s-1}{s-1} \quad , \text{ for } \quad t = 2s+1 \ .$$

A design is said to be <u>tight</u> if an equality holds in the above.

<u>Theorem</u> (Bannai-Damerell) Suppose that $d \geqslant 3$. (i) There exist no tight spherical 2s-designs with $s \geqslant 3$. (ii) Except for some small values of s, there exist no tight spherical (2s+1)-designs.

6. Y. Kobayashi and N. Hamada
(Hiroshima University)
A necessary condition for the existence of a quasi-residual BIB design (v,b,r,k,λ) with a pair of blocks intersecting in more than λ varieties

Abstract : It is well known that any quasi-residual design (v,b,r,k,λ) with a pair of blocks intersecting in more than λ treatments is not embeddable to a corresponding SBIB design. The purpose of this paper is to give a necessary condition for the block size k that there exists a quasi-residual design (v,b,r,k,λ) with a pair of blocks intersecting in more than λ treatments for a given integer $\lambda \geqq 3$. As the special case, it is shown that (i) if $k \neq \lambda(2\lambda-1)$, there is no quasi-residual design (v,b,r,k,λ) with a pair of blocks intersecting in $2\lambda-1$ treatments and (ii) if $k > 2(\lambda-1)(\lambda^2-\lambda+1)$, there is no quasi-residual design (v,b,r,k,λ) with a pair of blocks intersecting in more than λ treatments. The result (ii) coincides with the result due to R. C. Bose, S. S. Shrikhande and N. M. Singhi (Teorie Combinatorie Tomo I (1973), 49-82).

7. S. Kageyama and T. Tsuji

(Hiroshima University)

General upper bound for the number of blocks having a given number of treatments common with a given block

Abstract : The purpose of this paper is systematically to derive the general upper bound for the number of blocks having a given number of treatments common with a given block of certain incomplete block designs. The approach adopted here is based on the spectral decomposition of N'N for the incidence matrix N of a design, where N' is the transpose of the matrix N. This approach will lead us to upper bounds for incomplete block designs, in particular for a large number of partially balanced incomplete block (PBIB) designs, which are not covered with the standard approach (Shah (1966)) of using well known relations between blocks of the designs and their association schemes.

8. T. Shirakura

(Kobe University)

Balanced arrays of strength 2ℓ and balanced fractional 2^m factorial designs of resolution 2ℓ

Abstract : Consider a balanced array of 2 symbols, strength 2ℓ, m ($\geqq 2\ell$) constraints and indices μ_i ($i = 0,1,\cdots,2\ell$) as a fractional design. We shall give a necessary and sufficient condition for such an array to be a balanced fractional 2^m factorial design of resolution 2ℓ, in which the main effects, two-factor interactions,$\cdots$, (ℓ-1)-factor interactions are estimable ignoring (ℓ+1)-factor and higher order interactions, and the covariance matrix of their estimates is invariant under any permutation of m factors. This condition includes sufficient conditions given in earlier works of Shirakura.

9. S. Tazawa and S. Yamamoto

(Hiroshima College of Economics and Hiroshima Univ.)

Partite-claw-decomposition of a complete multi-partite graph - II

Abstract : A partite-claw of degree c is a claw or star of degree c which is a subgraph of a multi-partite graph such that no pair of points lies in the same set of points of the multi-partite graph. A multi-

partite graph is called partite-claw decomposable if it can be decomposed into a union of line-disjoint partite-claws of the same degree. A necessary and sufficient condition with respect to the decomposability of a complete m-partite graph $K_m(n,n,\cdots,n)$, which is a multi-partite graph defined on m sets of n points each, into a union of line-disjoint partite-claws of degree c is given.

10. K. Yamamoto
(Tokyo Woman's Christian College)
The Gordon-Mills-Welch difference sets

Abstract : Let F be a finite field GF(q) of $q = p^h$ elements, where p is a prime, and K an extension $GF(q^n)$ of degree $n \geqslant 2$. We consider K as an n-dimensional vector space over F, and take $f(\xi,\eta) = S_{K/F}(\xi\eta)$ as a fundamental bilinear form in K. Then non-zero elements of the hyperplane $D_{K/F} = \{\xi; S_{K/F}\xi = 0\}$ form a multiplicatively written cyclic difference set, if reduced modulo F^*. This is the classical Singer difference set associated with K/F, alternately defined as the set of zero-points in a maximal-length shift-register sequence $\{S_{K/F}\alpha^m\}_{m=0,1,2,\ldots}$, for a primitive element α of K. The multiplier group is generated by p, provided that $n \geqslant 3$.

If $L = GF(q^m)$ is an intermediate field of K/F and $\bar{\Delta}$ is a cyclic difference set in L^*/F^*, then the Gordon-Mills-Welch convolution $\bar{D}_{K/L} \oplus \bar{\Delta}$ is defined as $D_{K/L} \dot{+} \Delta E_{K/L}$, excluded of 0 and reduced modulo F^*, where $E_{K/L} = \{\xi; S_{K/L}\xi = 1\}$. A typical example is $\bar{D}_{K/K_2} \oplus (\bar{D}_{K_2/K_1} \oplus \bar{D}^{r_1}_{K_1/F})^{r_2}$, or $D_{K/K_2} \dot{+} E_{K/K_2}(D_{K_2/K_1} \dot{+} E_{K_2/K_1} D^{r_1}_{K_1/F})^{r_2}$ consisting of ξ such that $S_{K_1/F}(S_{K_2/K_1}(S_{K/K_2}\xi)^{t_2})^{t_1} = 0$, excluded of 0 and reduced modulo F^*.

The number of non-isomorphic Gordon-Mills-Welch difference sets associated with a tower of fields $F = K_0 \subset K_1 \subset \ldots \subset K_{s-1} \subset K_s = K$, $K_i = GF(q^{m_i})$, $1 = m_0 | m_1 | \ \ldots \ | m_{s-1} | m_s = n$, $m_0 < m_1 < \ldots < m_{s-1} < m_s$ is given by

$$\prod_{i=1}^{s-1} \frac{1}{hm_i} \phi\left(\frac{q^{m_i} - 1}{q - 1}\right),$$

where the factor with $m_1 = 2$, if appears, should be replaced by 1. It can be shown that if $\bar{D}_{K/K_{s-1}} \oplus (\bar{D}_{K_{s-1}/K_{s-2}} \oplus \ldots \oplus (\bar{D}_{K_2/K_1} \oplus \bar{D}^{r_1}_{K_1/F})^{r_2} \ldots)^{r_{s-1}}$ is isomorphic with $\bar{D}_{K/L_{t-1}} \oplus (\bar{D}_{L_{t-1}/L_{t-2}} \oplus \ldots \oplus (\bar{D}_{L_2/L_1} \oplus \bar{D}^{r'_1}_{L_1/F})^{r'_2} \ldots)^{r'_{t-1}}$,

and if $(\dim K/K_{s-1}, \dim K/L_{t-1}) = 1$, then they are isomorphic with a difference set of the form $D_{K/M} \oplus (D_{M/M'} \oplus \ldots)^u$, where $M = K_{s-1} \cap L_{t-1}$.

11. M. Deza
(C.N.R.S., Paris)
On perfect Matroid designs

Abstract : We give a short survey of known results and some new extremal properties of perfect Matroid designs.

12. T. Miwa
(University of Tokyo)
Experimental design for minimizing mean squared error of prediction

Abstract : In the least squares theory, a model is assumed and inference is made about the unknown parameters of the model. However, if the assumed model is not the case, the prediction of a responce using least squares estimates is biased. Hence to seek the optimal experimental design by attention only to variance is not adequate.

We adopt, as a criterion, mean squared error of prediction integrated with weight over the region of interest.

13. M. Kuwada and R. Nishii
(Maritime Safety Academy and Hiroshima University)
On a relation between balanced arrays and balanced fractional s^m factorial designs

Abstract : A general connection between a balanced fractional 2^m factorial (2^m-BFF) design of resolution $2\ell+1$ and a balanced array (B-array) of strength 2ℓ has been established by Yamamoto, Shirakura and Kuwada (1975, Ann. Inst. Statist. Math.). Recently, Kuwada has obtained a connection between a 3^m-BFF design of resolution V and a B-array of strength 4 (1977, submitted to J. Statist. Planning Inf.).

As a generalization of a multidimensional partially balanced association scheme and its algebra, a multidimensional relationship (MDRS) and its algebra is introduced here. By utilizing the property of MDRS and its algebra, it can be proved that a necessary and sufficient condition for a s^m-BFF design T of resolution $2\ell+1$ to be balanced is that T is a

B-array of strength 2ℓ with index set $\{\lambda_{i_0 i_1 \cdots i_{s-1}}\}$, provided the information matrix M_T is non-singular.

14. M. Kuwada

(Maritime Safety Academy)

Characteristic polynomials of the information matrices of balanced fractional 3^m factorial designs of resolution V

Abstract : An explicit expression of the characteristic polynomial of the information matrix M_T of a balanced fractional 3^m factorial design T of resolution V is obtained by using the decomposition of the multi-dimensional relationship algebra $\mathfrak{A}$ into its two-sided ideals.