数理解析研究所講究録
第 369 巻 1979 年 82-97

Ribbon Knots and Ribbon Disks

Kouhei ASANO (Kinki Univ.),
Yoshihiko MARUMOTO (Osaka City Univ.)
and Takaaki YANAGAWA (Kobe Univ.)

Given a ribbon knot, we will define, in § 1, the ribbon disk pair associated with it. On the other hand, J.F.P.Hudson and D.W.Sumners gave a method to construct a disk pair [3], [14]. In §1 and 2, we will generalize this construction and show that a ribbon disk pair is obtained by this construction and vice versa.

In [11], C.D.Papakyriakopoulos proved that the complement of a classical knot is aspherical. As an analogy of this, we will prove, in §3, that the complement of a ribbon disk is aspherical, and it follows from this that the fundamental group of a ribbon knot complement has no element of finite order. In the final section, we will calculate the higher homotopy groups of a higher-dimensional ribbon knot complement, and in Theorem 4.3 we show that a ribbon n-knot for $n \geqq 3$ is unknotted if the fundamental group of the knot complement is the infinite cyclic group. This result is proved independently by A.Kawauchi and T.Matumoto [6].

Throughout the paper, we work in the piecewise-linear category although the results remain valid in the smooth category.

§ 1. Preliminaries

1.1. By S^n denote an n-sphere, and by B^n or D^n an n-disk.

By ∂M, int M and cl M denote the boundary, the interior and the closure of a manifold M respectively. In this paper, every submanifold in a manifold is assumed to be locally flat. If $\partial M \neq \emptyset$, by $\mathcal{D}M$ we mean the double of M, i.e. $\mathcal{D}M$ is obtained from the disjoint union of two copies of M by identifying their boundaries via the identity map. For a subcomplex C in a manifold M, $N(C; M)$ is a regular neighbourhood of C in M. By a pair (M, W) denote a manifold M and a proper submanifold W in M, i.e. $W \cap \partial M = \partial W$. Two pairs (M_1, W_1) and (M_2, W_2) are equivalent if there exists a homeomorphism from M_1 to M_2 which maps W_1 to W_2. Let $\partial(M, W) = (\partial M, \partial W)$ and $\mathcal{D}(M, W) = (\mathcal{D}M, \mathcal{D}W)$.

An n-knot will mean a pair (S^{n+2}, K^n) with K^n homeomorphic to an n-sphere, and we will ususlly refer to the n-knot K^n. An n-knot is unknotted if it bounds an (n+1)-disk in S^{n+2}. For a proper disk D^n in a manifold M, (M, D^n) is unknotted, or D^n is unknotted in M, if there exists an (n+1)-disk D^{n+1} in M such that $D^{n+1} \cap \partial M$ is an n-disk in ∂D^{n+1} and $\mathrm{cl}(\partial D^{n+1} \cap \mathrm{int}\, M) = D^n$. For terminologies in handle theory, we refer the readers to [12], and for knot theory, to [15].

1.2. Let $S^n_0, S^n_1, \ldots, S^n_m$ be mutually disjoint n-spheres in a q-manifold M^q for $n \geq 1$ and $q \geq 3$. Suppose that an embedding β of $B^n \times I$ into M^q, where $I = [0, 1]$, satisfies the following :

(1) $\beta(B^n \times I) \cap (S^n_0 \cup \ldots \cup S^n_m) = \beta(B^n \times \partial I)$, and

(2) the orientation of $\beta(B^n \times I)$ is coherent with that of $S^n_0 \cup \ldots \cup S^n_m$.

Then we call β or $\beta(B^n \times I)$ a band compatible with $S^n_0 \cup \ldots \cup S^n_m$.

Let $\beta_1, \ldots, \beta_m$ be bands compatible with $S^n_0 \cup \ldots \cup S^n_m$ such that

(1) $\beta_i(B^n \times I) \cap \beta_j(B^n \times I) = \emptyset$ if $i \neq j$, and

(2) $\bigcup\{S_i^n;\ 0 \leq i \leq m\} \cup \bigcup\{\beta_j(B^n \times I);\ 1 \leq j \leq m\}$ is connected.

Then

$$(\bigcup\{S_i^n;\ 0 \leq i \leq m\} - \bigcup\{\beta_j(B^n \times \partial I);\ 1 \leq j \leq m\}) \cup \bigcup\{\beta_j(\partial B^n \times I);\ 1 \leq j \leq m\}$$

is an n-sphere, and denoted by

$$\mathcal{F}(S_0^n, \ldots, S_m^n; \beta_1, \ldots, \beta_m).$$

Suppose that $M^q = S^{n+2}$ and there exist mutually disjoint (n+1)-disks $B_0^{n+1}, \ldots, B_m^{n+1}$ with $\partial B_i^{n+1} = S_i^n$ for $0 \leq i \leq m$. Then

$$K^n = \mathcal{F}(S_0^n, \ldots, S_m^n; \beta_1, \ldots, \beta_m)$$

is called a ribbon n-knot of type $(\beta_1, \ldots, \beta_m)$.

Our definition of a ribbon n-knot is equivalent to that of [2], [20].

1.3. Remark. In 1.2, it is easily seen that we can deform each band isotopically so that

$$\beta_i(B^n \times I) \cap S_j^n = \begin{cases} \beta_i(B^n \times 0) & \text{if } j = 0, \\ \beta_i(B^n \times 1) & \text{if } j = i, \text{ and} \\ \emptyset & \text{otherwise.} \end{cases}$$

1.4. Let D^{n+3} be obtained from the disjoint union of $S^{n+2} \times I$ and B^{n+3} by identifying $S^{n+2} \times 1$ and ∂B^{n+3}. Let K^n be a ribbon n-knot of type $(\beta_1, \ldots, \beta_m)$, then we can construct an (n+1)-disk L^{n+1} in D^{n+3} which bounds $K^n \times 0$ as follows : Let $D_i^{n+1} = (S_i^n \times [0, 3/4]) \cup (B_i^{n+1} \times 3/4)$ in $S^{n+2} \times I$ for $0 \leq i \leq m$, where B_i^{n+1} and S_i^n are as in 1.2. For $1 \leq j \leq m$, let $\overline{\beta}_j : B^n \times I \times I \to S^{n+2} \times I$ be the product of β_j and a map from I into I which takes t to $t/2$, i.e.

$$\overline{\beta}_j(x, y, t) = (\beta_j(x, y), t/2)$$

for $x \in B^n$ and $y, t \in I$. Then

$$L^{n+1} = (\bigcup\{D_i^{n+1};\ 0 \leq i \leq m\} - \bigcup\{\overline{\beta}_j(B^n \times \partial I \times I);\ 1 \leq j \leq m\})$$
$$\cup \bigcup\{\overline{\beta}_j(\partial B^n \times I \times I) \cup \overline{\beta}_j(B^n \times I \times 1);\ 1 \leq j \leq m\}$$

is an $(n+1)$-disk and bounds $K^n \times 0$ in D^{n+3}. Note that the section of L^{n+1} by $S^{n+2} \times t$ is

(1) $K^n \times t$ if $0 \leqq t < 1/2$,

(2) $(\bigcup\{S_i^n;\ 0 \leqq i \leqq m\} \cup \bigcup\{\beta_j(B^n \times I);\ 1 \leqq j \leqq m\}) \times 1/2$ if $t = 1/2$,

(3) $(S_0^n \cup \ldots \cup S_m^n) \times t$ if $1/2 < t < 3/4$,

(4) $(B_0^n \cup \ldots \cup B_m^n) \times 3/4$ if $t = 3/4$,

(5) $\emptyset$ if $3/4 < t \leqq 1$.

(See Fig. 1.1.)

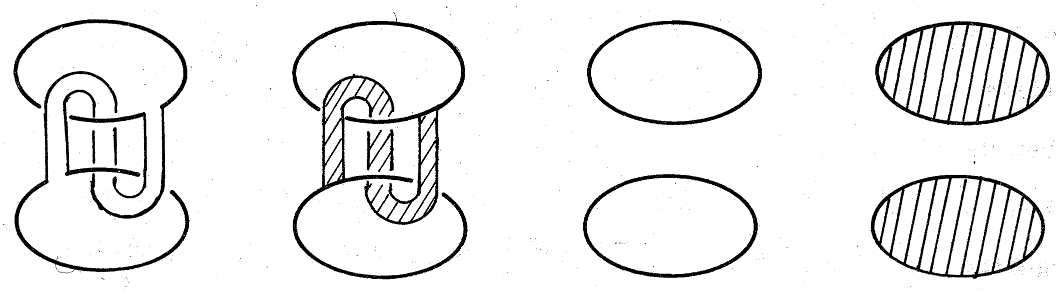

Fig. 1.1

We call L^{n+1} in D^{n+3} the <u>ribbon</u> $(n+1)$-<u>disk associated with a ribbon</u> n-<u>knot</u> K^n, or (D^{n+3}, L^{n+1}) the <u>ribbon</u> $(n+1)$-<u>disk pair associated with</u> K^n.

The double $\mathcal{D}(D^{n+3}, L^{n+1})$ of a ribbon $(n+1)$-disk pair is an $(n+1)$-knot in the $(n+3)$-sphere $\mathcal{D}D^{n+3}$. Since $\mathcal{D}(D_0^{n+1} \cup \ldots \cup D_m^{n+1})$ is trivial $(n+1)$-link and each $\mathcal{D}(\beta_i(B^n \times I \times I))$ is a band, L^{n+1} is a ribbon $(n+1)$-knot. We say that $\partial(D^{n+3}, L^{n+1})$ is an <u>equatorial knot</u> of $\mathcal{D}(D^{n+3}, L^{n+1})$. (See [20].)

1.5. We will generalize the construction of $(n+1)$-disk in [3] and [14], for $n \geqq 1$. Let D_0^{n+1} be an unknotted $(n+1)$-disk in B^{n+3}. Adding m 1-handles $h^1_1, \dots, h^1_m$ to B^{n+3} such that $h^1_i \cap D^{n+1}_0 = \emptyset$ for each i, we obtain an (n+3)-disk with m 1-handles, say V. We take mutually disjoint oriented 1-spheres $\alpha_1, \dots, \alpha_m$ on $\partial V - \partial D^{n+1}_0$ such that α_i intersects the belt sphere of h^1_i at only one point and $\alpha_i \cap h^1_j = \emptyset$ for $i \neq j$. Then we call α_i a <u>standard curve</u> on ∂V. Let Δ_0 be a proper 2-disk in $N(\partial D^{n+1}_0; \partial V)$ such that Δ_0 intersects ∂D^{n+1}_0 at only one point, then we call Δ_0 a <u>meridian disk</u> of ∂D^{n+1}_0 in ∂V and $\alpha_0 = \partial \Delta_0$ a <u>meridian</u> of ∂D^{n+1}_0 in ∂V, where we give an orientation to α_0. Let u_i be a simple closed curve in $\partial V - \partial D^{n+1}_0$ for $1 \leqq i \leqq m$ such that there exists an ambient isotopy of ∂V which carries u_i to α_i for all i. Then we add m 2-handles $h^2_1, \dots, h^2_m$ to V along $u_1, \dots, u_m$ such that $h^2_i \cap D^{n+1}_0 = \emptyset$ for each i. By the handle cancelling theorem, h^2_i cancels h^1_i for each i. Thus $V \cup h^2_1 \cup \dots \cup h^2_m$ is an (n+3)-disk D^{n+3}. In general, D^{n+1}_0 is not unknotted in D^{n+3}, so we rewite D^{n+1}_0 in D^{n+3} as L^{n+1}. We say that the pair (D^{n+3}, L^{n+1}) is <u>of S-type</u>, or simply L^{n+1} is <u>of S-type</u> when no confusion can arise.

Let Δ_{0i}, for $1 \leqq i \leqq m$, be mutually disjoint meridian disks of ∂D^{n+1}_0 in ∂V, and γ_i a band in ∂V compatible with α_i and $\alpha_{0i} = \partial \Delta_{0i}$ such that (1) $\gamma_i(B^1 \times I) \cap \gamma_j(B^1 \times I) = \emptyset$ for $i \neq j$, and

(2) $\gamma_i(B^1 \times I) \cap N(\partial D^{n+1}_0; \partial V) = \gamma_i(B^1 \times 0)$ for $1 \leqq i \leqq m$.

Then there exists an ambient isotopy of ∂V which carries v_i to α_i for $1 \leqq i \leqq m$, where $v_i = \mathcal{F}(\alpha_i, \alpha_{0i}; \gamma_i)$ for each i. Thus the (n+3)-manifold obtained from V by adding m 2-handles with v_i, for $1 \leqq i \leqq m$, as the attaching spheres is an (n+3)-disk which contains D^{n+1}_0 as a proper (n+1)-disk, then this disk pair is said to be <u>of S^*-type</u>. Clearly, a disk pair of S^*-type is of S-type.

1.6. Remark. In 1.5, if $n \geq 2$, then the attaching sphere u_i of h_i^2 is uniquely determined up to ambient isotopy by its homotopy class $[u_i]$ in $\pi_1(\partial V - \partial D_0^{n+1})$. For $n = 1$, this is false.

1.7. Let C_0 be a bouquet of $m+1$ 1-spheres $e_0^1, e_1^1, \dots, e_m^1$. Let z_i be the element of $\pi_1(C_0)$ represented by e_i^1 for $0 \leq i \leq m$. By C denote a 2-dimensional cell complex obtained from C_0 by attaching 2-cells $e_1^2, \dots, e_m^2$ with the boundary formulae corresponding to $w_1, \dots, w_m$, where $w_i = w_i(z_0, z_1, \dots, z_m)$ is an element of $\pi_1(C_0)$ such that $w_i(1, z_1, \dots, z_m) = z_i$ for $1 \leq i \leq m$. Then we call C a cell complex of S-type.

In 1.5, $cl(B^{n+3} - N(D_0^{n+1}; B^{n+3}))$ has a 1-dimensional spine. Hence, by the assumption on the attaching spheres u_i of h_i^2, we have the following :

1.8. Proposition. Let (D^{n+3}, L^{n+1}) be an $(n+1)$-disk pair of S-type for $n \geq 1$. Then $cl(D^{n+3} - N(L^{n+1}; D^{n+3}))$ collapses to a cell complex of S-type.

§ 2. Ribbon disks and disk pairs of S-type

2.1. Lemma. Let $w = w(z_0, z_1, \dots, z_m)$ be a word in F, the free group on $z_0, z_1, \dots, z_m$. Then $w(1, z_1, \dots, z_m) = z_i$ in F if and only if there exist a word t_j in F and an integer ε_j such that $w = \prod_j (t_j z_0 t_j^{-1})^{\varepsilon_j} \cdot z_i$.

Proof. The sufficiency is trivial. The necessity is proved by easy calculation, see [1].

2.2. Lemma. Let D_0^{n+1}, V, α_i and u_i be as in 1.5. Then there exist disjoint meridian disks Δ_{0ij}'s of ∂D_0^{n+1} in ∂V, a band γ_{ij} in ∂V compatible with α_i and $\tilde{\alpha}_{ij} = \partial \Delta_{0ij}$ for $1 \leq i \leq m$ and $1 \leq j \leq r(i)$, and an ambient isotopy $\{\varphi_t;\ 0 \leq t \leq 1\}$ of ∂V such that

(1) $\gamma_{ij}(B^1 \times I) \cap \gamma_{k\ell}(B^1 \times I) = \emptyset$ if $(i, j) \neq (k, \ell)$,

(2) $\gamma_{ij}(B^1 \times I) \cap N(\partial D_0^{n+1};\ \partial V) = \gamma_{ij}(B^1 \times 0)$,

(3) $\gamma_{ij}(B^1 \times I) \cap \alpha_k = \emptyset$ if $i \neq k$,

(4) $\varphi_t | \partial D_0^{n+1}$ is the identity map for $0 \leq t \leq 1$,

(5) φ_0 is the identity map, and

(6) $\varphi_1(u_i) = \mathcal{F}(\alpha_i, \tilde{\alpha}_{i1}, \ldots, \tilde{\alpha}_{ir(i)};\ \gamma_{i1}, \ldots, \gamma_{ir(i)})$ for $1 \leq i \leq m$. (See Fig. 2.1.)

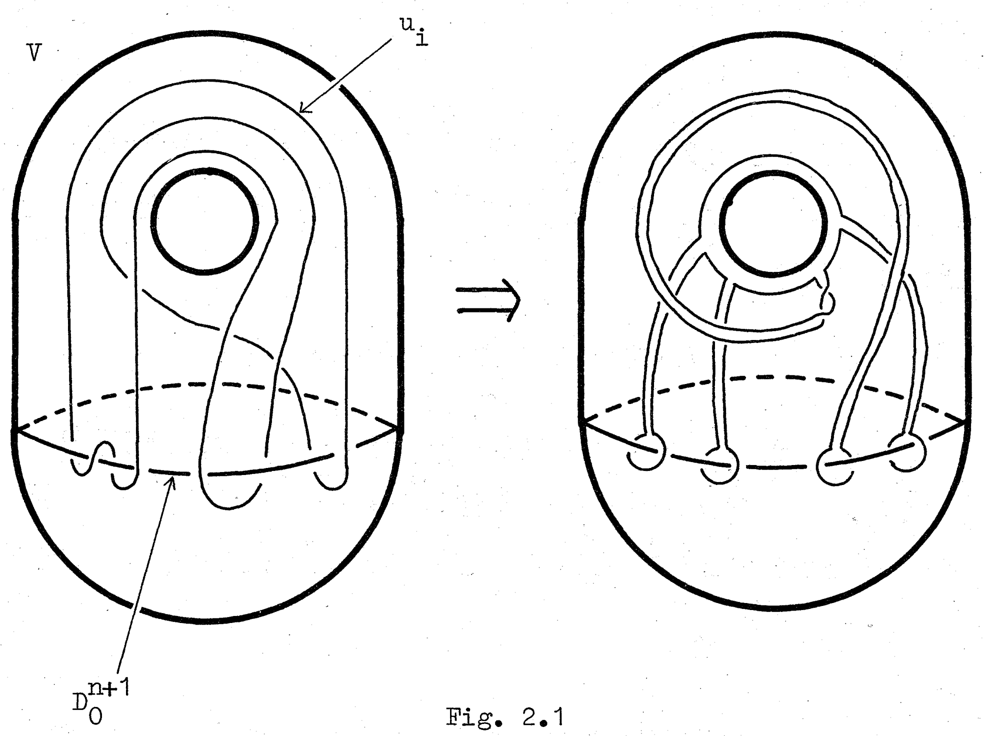

Fig. 2.1

Proof of Lemma 2.2. For $n = 1$, the assertion is easily shown by the modification as in Fig. 2.2.

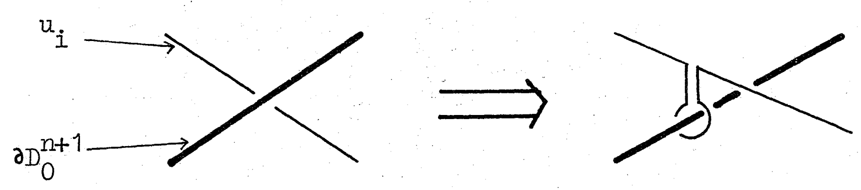

Fig. 2.2

For $n \geq 2$, the assertion is proved by using Remark 1.6 and Lemma 2.1. For details, see [1].

Using Lemma 2.2, we have the following Proposition 2.3, and we omit the proof [1].

2.3. Proposition. For $n \geq 1$, an $(n+1)$-disk pair of S-type is of S^*-type. (See Fig. 2.3.)

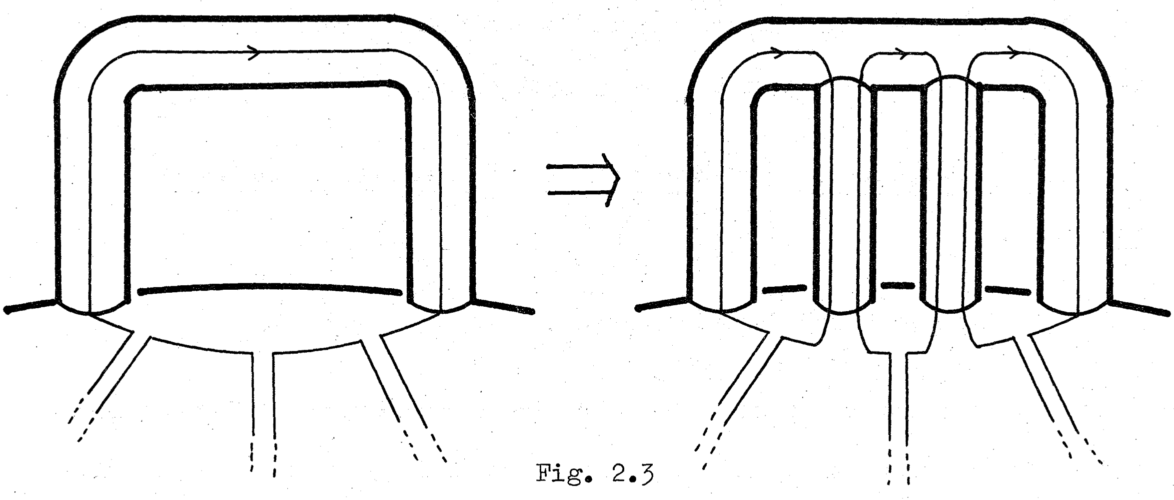

Fig. 2.3

Then we have the following theorem :

2.4. Theorem. Suppose $n \geq 1$. Then a ribbon $(n+1)$-disk pair is of S-type, and conversely an (n+1)-disk pair of S-type is a ribbon disk pair. (For the proof, see [1].)

2.5. Remark. A.Omae [9] proved that the boundary pair of a 3-disk pair of S-type is a ribbon 2-knot for a special case, and L.R.Hitt [2] announced that he proved that the boundary pair of an (n+1)-disk pair of S-type is a ribbon n-knot and the converse.

By Proposition 1.8, Lemma 2.1, Lemma 2.2 and Theorem 2.4, we have the following Corollary 2.6 :

2.6. Corollary. Let (D^{n+3}, L^{n+1}) be a ribbon (n+1)-disk pair for $n \geqq 1$, then $cl(D^{n+3} - N(L^{n+1}; D^{n+3}))$ collapses to a cell complex of S-type. Conversely, let C be a cell complex of S-type. Then there exists a ribbon (n+1)-disk pair (D^{n+3}, L^{n+1}) for $n \geqq 1$ such that C is a spine of the exterior of L^{n+1} in D^{n+3}.

In [20], the third author proved the following Proposition 2.7, and using Theorem 2.4 we can give an alternative proof [1] :

2.7. Proposition. Every ribbon n-knot has an equatorial knot, for $n \geqq 2$.

§ 3. Asphericity of ribbon disks

In this section, we will show that the complement of a higher dimensional ribbon disk is aspherical which is an analogy to the case of classical knots [11].

3.1. Regarding S^4 as a one-point compactification of R^4, we may consider that a 2-knot is in R^4. Let R^3_t be the hyperplane of R^4

whose 4-th coordinate is t, R^4_+ the half space of R^4 whose 4-th coordinate is non-negative, and $R^4_- = cl(R^4 - R^4_+)$. By Proposition 2.7, we can assume that a ribbon 2-knot K^2 satisfies the followings :

(1) K^2 is symmetric with respect to R^3_0,

(2) $K^2 \cap R^4_+$ has elliptic critical points only in R^3_2, and

(3) $K^2 \cap R^4_+$ has hyperbolic critical points only in R^3_1 (Fig. 3.1).

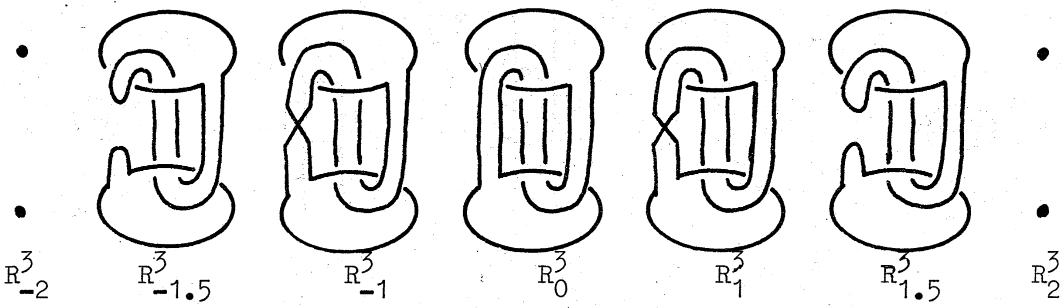

Fig. 3.1

On the other hand, a ribbon 2-knot K^2 can be described as follows :

(1) all elliptic critical points occur at R^3_2 or R^3_{-2}, and

(2) all hyperbolic critical points occur at R^3_0 (Fig. 3.2).

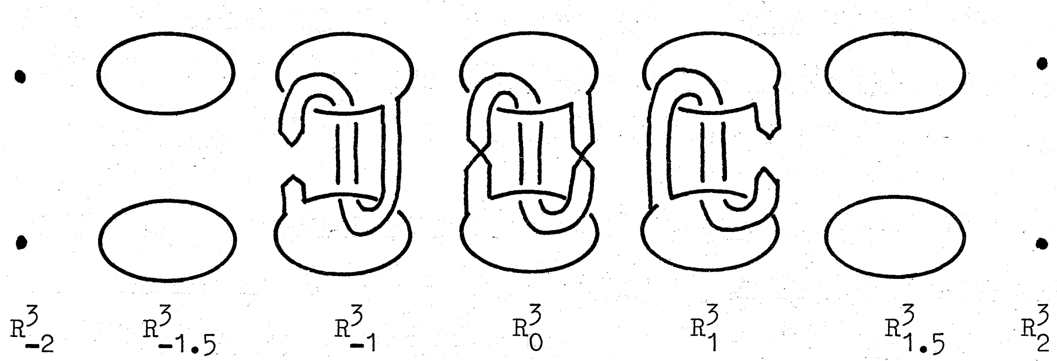

Fig. 3.2

The latter description of a 2-knot is adopted by S.J.Lomonaco [8], then he has stated the following in Theorem 3.2 in [8] :

3.2. Proposition. Let K^2 be a ribbon 2-knot of type $(\beta_1, \dots, \beta_m)$, and d_i^3 a 3-disk in $N(\beta_i; S^4)$ such that the intersection of d_i^3 and $\beta_i(B^2 \times I)$ is $\beta_i(B^2 \times 1/2)$ for $1 \leq i \leq m$. Let $[\partial d_i^3]$ be the element of $\pi_2(S^4 - K^2)$ represented by ∂d_i^3. Then $\pi_2(S^4 - K^2)$ is generated by $[\partial d_1^3], \dots, [\partial d_m^3]$ as a $Z\pi_1$-module, where $Z\pi_1$ is the integral group ring of $\pi_1 = \pi_1(S^4 - K^2)$. (Fig. 3.3)

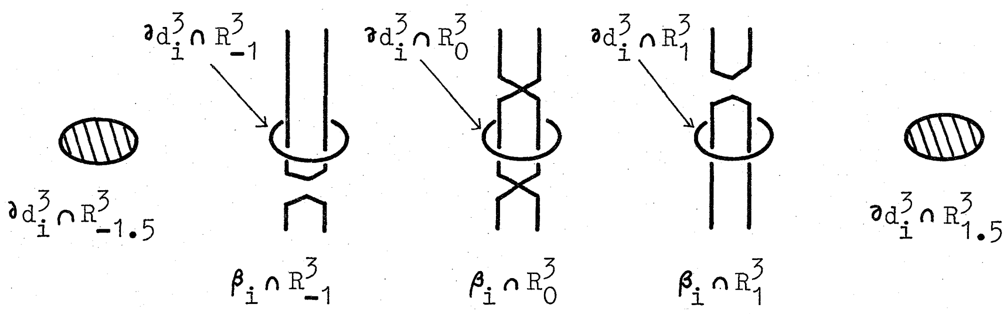

Fig. 3.3

The following Proposition 3.3 has been proved by the third author [21] using the Van-Kampen theorem :

3.3. Proposition. Let (D^{n+3}, L^{n+1}) be the ribbon (n+1)-disk pair associated with a ribbon n-knot K^n, and $(S^{n+3}, K^{n+1}) = \mathcal{D}(D^{n+3}, L^{n+1})$. Then $\pi_1(S^{n+2} - K^n) \cong \pi_1(D^{n+3} - L^{n+1}) \cong \pi_1(S^{n+3} - K^{n+1})$ provided $n \geqq 2$.

3.4. Lemma. Let (D^{n+3}, L^{n+1}) be the ribbon (n+1)-disk pair associated with a ribbon n-knot K^n. If $n \geqq 2$, then the inclusion from $S^{n+2} - K^n$ into $D^{n+3} - L^{n+1}$ induces an onto-homomorphism $\pi_2(S^{n+2} - K^n) \longrightarrow \pi_2(D^{n+3} - L^{n+1})$ as $Z\pi_1$-modules.

Proof. Let K^n be of type $(\beta_1, \dots, \beta_m)$, then we will use the notations in 1.4.

Note that, by the general position arguments, we can choose a base for

$\pi_2(D^{n+3} - L^{n+1})$ consisting of embedded 2-spheres. Let Σ^2 be a 2-sphere in $D^{n+3} - L^{n+1}$, then we may assume that Σ^2 is in $S^{n+2} \times I - L^{n+1}$. We can move Σ^2 into general position so that Σ^2 does not intersect $p_i \times [3/4, 1]$ where $p_i \in \text{int } B_i^{n+1}$. Since $B_i^{n+1} \times [3/4, 1]$ is regarded as a regular neighbourhood of $p_i \times [3/4, 1]$ in $S^{n+2} \times [3/4, 1]$, we have $\Sigma^2 \cap B_i^{n+1} \times [3/4, 1] = \emptyset$ for $0 \leq i \leq m$. This implies that Σ^2 can be pushed into $S^{n+2} \times [0, 3/4] - L^{n+1}$. For $q \in \text{int } B^n$, let $W_i = \beta_i(q \times I) \times [1/2, 3/4]$. Again by the general position arguments, we can have $\Sigma^2 \cap W_i = \emptyset$ for each i. Hence Σ^2 does not intersect $\beta_i(B^n \times I) \times [1/2, 3/4]$ for each i, i.e. we can deform Σ^2 isotopically into $S^{n+2} \times [0, 1/2] - L^{n+1}$. Therefore Σ^2 can be moved isotopically into $S^{n+2} \times 0 - L^{n+1}$, i.e. into $S^{n+2} - K^n$. By this fact and Proposition 3.3 the required result is implied.

3.5. Lemma. <u>For a ribbon 3-disk pair</u> (D^5, L^3), <u>we have</u> $\pi_2(D^5 - L^3) = 0$.

<u>Proof</u>. Let (D^5, L^3) be associated with a ribbon 2-knot of type $(\beta_1, \ldots, \beta_m)$. We will use the notations in 1.4 for $n = 2$ and in 3.2. The 2-sphere ∂d_i^3 bounds the 3-disk $(\partial d_i^3 \times [0, 3/4]) \cup (d_i^3 \times 3/4)$ in $D^5 - L^3$ for each i. This implies that $\pi_2(D^5 - L^3) = 0$ by Lemma 3.4.

The following Theorem 3.6 is a generalization of [19] :

3.6. <u>Theorem</u>. <u>Let</u> (D^{n+3}, L^{n+1}) <u>be a ribbon</u> (n+1)-<u>disk pair with</u> $n \geq 1$, <u>then</u> $D^{n+3} - L^{n+1}$ <u>is aspherical</u>.

<u>Proof</u>. By Corollary 2.6, there exists a cell complex C of S-type such that $D^{n+3} - L^{n+1}$ is homotopy equivalent to C. Again by 2.6, there exists a ribbon 3-disk pair (D^5, L^3) such that $D^5 - L^3$ is homotopy

equivalent to C. It follows from Lemma 3.5 that $\pi_2(C) = 0$. Let $\tilde{C}$ be the universal covering space of C. Then $H_i(\tilde{C}) = 0$ for $i \geq 3$, since $\tilde{C}$ is 2-dimensional. Thus, by Hurewicz theorem, $\tilde{C}$ is aspherical, because $\pi_2(\tilde{C}) \cong \pi_2(C) = 0$. Therefore C is aspherical. This completes the proof.

3.7. *Remark.* A cell complex of S-type is a subcomplex of a contractible 2-complex, hence the proof of Theorem 3.6 gives a partial answer to a problem of J.H.C.Whitehead : Is any subcomplex of an aspherical 2-complex aspherical ?

3.8. *Corollary.* *Let* K^n *be a ribbon n-knot for* $n \geq 1$, *then* $\pi_1(S^{n+2} - K^n)$ *has no element of finite order.*

Proof. For $n = 1$, the assertion is a special case of [11]. For $n \geq 2$, this is true by Proposition 3.3, Theorem 3.6 and a result due essentially to P.A.Smith (p.216 in [4]), namely : The fundamental group of an aspherical polyhedron of finite dimension has no element of finite order.

T.Yajima characterized the knot group of ribbon 2-knots in [17], then by Corollary 3.8 and [17] we have the following :

3.9. *Corollary.* *Let* G *be a finitely presented group having a Wirtinger presentation of deficiency* 1 *with* $G/G' \cong Z$. *Then* G *has no element of finite order.*

§ 4. Unknotting ribbon knots

4.1. *Theorem.* *Let* K^n *be a ribbon n-knot for* $n \geq 3$, *then we have* $\pi_i(S^{n+2} - K^n)$ *for* $2 \leq i \leq n-1$.

Proof. By Proposition 2.7, there exists a ribbon n-disk pair

(D^{n+2}, L^n) such that $\mathcal{D}(D^{n+2}, L^n) = (S^{n+2}, K^n)$. Let $(D^{n+2}_\varepsilon, L^n_\varepsilon)$ be a copy of (D^{n+2}, L^n) for $\varepsilon = \pm$ such that (D^{n+2}, L^n) is obtained from the disjoint union of (D^{n+2}_+, L^n_+) and (D^{n+2}_-, L^n_-) by identifying their boundaries via the identity map. Let $(S^{n+1}, K^{n-1}_0) = (D^{n+2}_+, L^n_+)$. Let $\tilde{X}$ be the universal covering space of $S^{n+2} - K^n$, $\tilde{X}_\varepsilon$ the lift of $D^{n+2}_\varepsilon - L^n_\varepsilon$ in $\tilde{X}$ for $\varepsilon = \pm$, and $\tilde{X}_0$ the lift of $S^{n+1} - K^{n-1}_0$ in $\tilde{X}$. By Proposition 3.3, all of $\tilde{X}_+$, $\tilde{X}_-$ and $\tilde{X}_0$ are also universal covering spaces. By the Mayer-Vietoris theorem, we have the following exact sequence :

$$\cdots \to H_j(\tilde{X}_+) \oplus H_j(\tilde{X}_-) \to H_j(\tilde{X}) \to H_{j-1}(\tilde{X}_0) \to H_{j-1}(\tilde{X}_+) \oplus H_{j-1}(\tilde{X}_-) \to \cdots .$$

By Theorem 3.6, $H_j(\tilde{X}_\varepsilon) = 0$ for $j \geq 1$ and $\varepsilon = \pm$. Therefore it follows that $H_j(\tilde{X}) \cong H_{j-1}(\tilde{X}_0)$ for $j \geq 2$.

Suppose $n = 3$, then $\pi_2(S^5 - K^3) \cong H_2(\tilde{X}) \cong H_1(\tilde{X}_0) = 0$. By induction on the dimension n, it is easily seen that the fact $H_j(\tilde{X}) \cong H_{j-1}(\tilde{X}_0)$ and $H_1(\tilde{X}_0)$ imply $H_i(\tilde{X}) = 0$ for $1 \leq i \leq n-1$. This implies the required result.

The following Proposition 4.2 is due to A.Kawauchi ([5] or p.331 in [15]) :

4.2. Proposition. For a 2-knot K^2, $S^4 - K^2$ is homotopy equivalent to S^1 if $\pi_1(S^4 - K^2) \cong Z$.

4.3. Theorem. Let K^n be a ribbon n-knot for $n \geq 3$. If $\pi_1(S^{n+2} - K^n) \cong Z$, then K^n is unknotted.

Proof. We can use the notations in the proof of Theorem 4.1. In the proof of Theorm 4.1, we have $H_j(\tilde{X})$ $H_{j-1}(\tilde{X}_0)$ for $j \geq 2$.

Suppose $n = 3$ and $\pi_1(S^5 - K^3) \cong Z$, then by Proposition 3.3 it follows that $\pi_1(S^4 - K^2_0) \cong Z$, where K^2_0 is an equatorial knot of K^3. By Proposition 4.2, we have $H_i(\tilde{X}_0) = 0$ for all $i \geq 1$. This implies that

$H_j(\tilde{X}) = 0$ for $j \geqq 1$. Therefore $S^5 - K^3$ is homotopy equivalent to S^1, hence by [7], [13] and [16], K^3 is unknotted. Similarly, for $n \geqq 4$, it is easy to see the assertion is true by induction on the dimension n. This completes the proof.

Recently A.Kawauchi and T.Matumoto have obtained independently the same result as Theorem 4.3 [6].

The following is obtained by Proposition 3.3 and Theorem 4.3 :

4.5. Corollary. *For a ribbon n-knot K^n with $n \geqq 4$, any equatorial knot of K^n is unknotted if K^n is unknotted.*

For $n = 2$, Corollary 4.5 is false. For example, Kinoshita-Terasaka knot is an equatorial knot of the unknot [9]. The case $n = 3$ still remains open.

References

[1] Asano, K., Marumoto, Y. & Yanagawa, T. : Ribbon knots and ribbon disks, (preprint).

[2] Hitt, L.R. : Characterization of ribbon n-knots, Notices Amer. Math. Soc. 26 (1979), A-128.

[3] Hudson, J.F.P. & Sumners, D.W. : Knotted ball pairs in unknotted sphere pairs, J. London Math. Soc. 41 (1966), 717-722.

[4] Hurewicz, W. : Beiträge zur Topologie der Deformationen IV , Proc. Akad. Amsterdam 39 (1936), 215-224.

[5] Kawauchi, A. : On partial Poincaré duality and higher dimensional knots with $\pi_1 = Z$, Master's thesis, Kobe Univ., 1974.

[6] Kawauchi, A. & Matumoto, T. : An estimate of infinite cyclic coverings and knot theory, (preprint).

[7] Levine, J. : Unknotting spheres in codimension two, Topology 4 (1965), 9-16.

[8] Lomonaco, Jr., S.J. : The homotopy groups of knots, II. A solution to Problem 36 of R.H.Fox. (to appear).

[9] Marumoto, Y. : On ribbon 2-knots of 1-fusion, Math. Sem. Notes Kobe Univ. 5 (1977), 59-68.

[10] Omae, A. : A note on ribbon 2-knots, Proc. Japan Acad. 47 (1971), 850-853.

[11] Papakyriakopoulos, C.D. : On Dehn's lemma and asphericity of knots, Ann. of Math. 66 (1957), 1-26.

[12] Rourke, C. & Sanderson, B. : Introduction to Piecewise-Linear Topology, Ergeb. der Math. 69, Springer, (1972).

[13] Shaneson, J.L. : Embeddings of spheres in spheres of codimension two and h-cobordism of $S^1 \times S^3$, Bull. Amer. Math. Soc. 74 (1968), 972-974.

[14] Sumners, D.W. : Homotopy torsion in codimension two knots, Proc. Amer.. Math. Soc. 24 (1970), 229-240.

[15] Suzuki, S. : Knotting problems of 2-spheres in 4-sphere, Math. Sem. Notes Kobe Univ. 4 (1976), 241-371.

[16] Wall, C.T.C. : Unknotting tori in codimension one and spheres in codimension two, Proc. Camb. Phil. Soc. 61 (1965), 659-664.

[17] Yajima, T. : On a characterization of knot groups of some spheres in R^4, Osaka J. Math. 6 (1969), 447-464.

[18] Yanagawa, T. : On ribbon 2-knots, The 3-manifold bounded by the 2-knots., Osaka J. Math. 6 (1969), 447-464.

[19] Yanagawa, T. : On ribbon 2-knots II, The second homotopy group of the complementary domain, Osaka J. Math. 6 (1969), 465-473.

[20] Yanagawa, T. : On cross sections of higher dimensional ribbon-knots, (to appear).

[21] Yanagawa, T. : Knot-groups of higher dimensional ribbon knots, (to appear).