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# An Accurate Model of Bipolar Transistors for Computer-Aided Circuit Analysis

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Abstract

A compact and accurate large-signal model of bipolar transistors is derived from the original Ebers-Moll model, taking account of the computing efficiency and simplicity. Since the proposed model including the Early effect and the nonlinearity of current-gain factor ($\alpha$) has good agreement with the measured current-voltage relationship, it may be suitable for computer-aided circuit design.

## 1. Introduction

The successful computer simulation of integrated and discrete circuits requires a compact and accurate model of transistors. By adding the Early effect [1] to the Ebers-Moll formulation [2], we can obtain a new accurate model of bipolar transistors.

Many models of bipolar transistors presented until now are based on the exponential dependence of junction current. However, we can grasp no essential meaning of exponential functions from the output characteristics of common-emitter transistors, because they play important role implicitly, that is, the exponential cancellation occurs. Moreover, sufficient careful attentions should be paid for the computer program with exponential functions, because they have extraordinary large values or small ones in numerical computations. Since all the exponential functions are eliminated from the model equations in the operating range of common-emitter current-voltage characteristics, the proposed model is very suitable for computer-aided circuit design program.

## 2. Large-Signal Model of Transistors

### 2.1 Output Characteristics

The Ebers-Moll representation of n-p-n transistor is given by the following equations [2].

$$\left.\begin{aligned} I_E &= -I_{ES}\left( e^{-\lambda V_{EB}} - 1 \right) + \alpha_R I_{CS}\left( e^{-\lambda V_{CB}} - 1 \right) \\ I_C &= \alpha_F I_{ES}\left( e^{-\lambda V_{EB}} - 1 \right) - I_{CS}\left( e^{-\lambda V_{CB}} - 1 \right) \end{aligned}\right\} \quad (1)$$

where $\lambda = q/(NkT)$ and N is an effective emission coefficient with values typically between 1 and 2.

Eliminating $I_E$ and $V_{EB}$ from (1) and introducing $y = \exp(-\lambda V_{CE})$, we can obtain the following relation;

$$I_C = \frac{(\alpha_F I_{ES} - I_{CS}y)I_B + (1-y)\varepsilon_{FR}I_{ES}I_{CS}}{I_F + I_R y}$$

where

$$I_F = (1-\alpha_F)I_{ES}$$

$$I_R = (1-\alpha_R)I_{CS}$$

$$\varepsilon_{FR} = 1-\alpha_F\alpha_R$$

The exponential functions which are derived from an ideal p-n junction model as a first approximation by taking account of the recombination generation centers [3] would be unsuitable for obtaining circuit characteristics by numerical computations. For this reason, we introduce the following approximation for y.

$$y = e^{-\lambda V_{CE}} = \delta(V_{CE} + \delta)^{-1} - E$$

where E is an approximation error and $\delta$ is a small constant. Owing to the very large decaying constant $\lambda$ ($\simeq 38.46$ volts$^{-1}$ for N = 1 at 25 degrees C.) of exponential functions, the error E is a small monotone decreasing function of $V_{CE}$ in the operating region of transistors as shown in Fig. 1.

Next we consider the Early effect which is formulated by the Early voltage $V_A$ [1]. By multiplying the collector current in the Ebers-Moll formulation by the factor $[1+(V_{CE}/V_A)]$ and using the boundary condition $I_C = 0$ at $V_{CE} = 0$, we can obtain the following collector characteristics;

$$I_C = \frac{f(I_B)\ V_{CE}}{V_{CE} + \varepsilon} + g(I_B)\ V_{CE} \tag{2}$$

where

$$f(I_B) = V_A\ g(I_B) - \delta(I_B + I_F + I_R)\eta$$
$$g(I_B) = \beta_F I_B/V_A + \eta I_F$$
$$\beta_F = \alpha_F/(1-\alpha_F)$$
$$\theta = 1+I_R/I_F$$
$$\varepsilon = \theta\delta$$
$$\eta = \frac{\beta_F \varepsilon_{FR} I_{CS}}{\alpha_F I_F V_A}$$

Strictly speaking, the parameters $\alpha_F$, $\alpha_R$ and $V_A$ are functions of $I_B$. Therefore, $f(I_B)$ and $g(I_B)$ should be approximated by the following equations on the basis of experimental results.

$$\left.\begin{aligned} f(I_B) &= \sum_{i=1}^{n} a_i I_B^{n-i} \\ g(I_B) &= \sum_{j=1}^{m} b_j I_B^{m-j} \end{aligned}\right\} \qquad (3)$$

In practice, they can be independently determined, if we make the best use of the small quantity $\varepsilon$.

2.2 Input Characteristics

By using the relation $I_B = -(I_E + I_C)$ and (1), we obtain

$$V_{BE} = \frac{1}{\lambda} \log\left(\frac{I_B}{I_F + I_R} + 1\right) - \log\frac{I_F + I_R e^{-\lambda V_{CE}}}{I_F + I_R} \qquad (4)$$

The following small argument logarithm expansion is applied to (4) for small $I_B$;

$$\log(x+1) = 2\left[\left(\frac{x}{x+2}\right) + \frac{1}{3}\left(\frac{x}{x+2}\right)^3 + \dots\right]$$

Eliminating the exponential function as the same manner mentioned in the output characteristics, we can obtain

$$V_{BE} = \frac{1}{\lambda}\left[\frac{2I_B}{I_B + 2(I_F+I_R)} - \frac{2\delta I_R}{2I_F V_{CE} + \delta(I_R+2I_F)} - \log\frac{I_F}{I_F+I_R}\right]$$

However, in the large $I_B$ region the forward-biased current is limited by a series resistance $R_S$ of p-n junction[2]. Therefore, the following input characteristics can be derived.

$$V_{BE} = R_S I_B + \frac{2}{\lambda}\frac{I_B}{I_B + 2\theta I_F} + h(V_{CE}) \qquad (5)$$

where

$$h(V_{CE}) = \frac{1}{\lambda} \log \theta + \frac{\delta(1-\theta)}{V_{CE} + \delta(\theta+1)/2}$$

$$= h_1 + \frac{h_3}{V_{CE} + h_2}$$

## 3. Experimental Results

As a typical example of modern transistors, we have measured the current-voltage relationship of n-p-n silicon epitaxial transistor 2SC2003 fabricated in NEC. The calculation of the fit parameters is performed only once by a pre-processor program at the beginning of the circuit analysis. Provided that the condition $\varepsilon \to 0$ is satisfied, the first approximation of the modeling parameters can be obtained. Starting from the first approximation, an iterative nonlinear least squares algorithm is used to get the final parameter values which are listed in Table I. The input characteristics are shown by the solid lines in Fig. 2. And the output characteristics are shown by the solid lines in Fig. 3. The theoretical results have reasonable agreement with the measured data which are shown by the small circles in the two figures.

## 4. Conclusion

The proposed large-signal model has the simplified structure for representing the static characteristics of bipolar transistors. As the model without exponential function has good agreement with the experimental results, it may be suitable for numerical computations in computer-aided circuit analysis and design.

The determination of the modeling parameters reduces to one-dimentional curve-fitting problem. Therefore, we can easily obtain the more accurate expression of transistors by increasing the parameters of the approximated functions.

REFERENCES

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Table I

Results of Model Parameter Determination

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $a_i$ | 6.307 | -63.306 | 232.176 | -358.341 | 146.460 | 174.709 | 0.0 |
| $b_i$ | -0.152 | 2.099 | -9.545 | 12.444 | 11.853 | 0.0 | 0.0 |
| $h_i$ | 0.361 | 0.038 | 0.110 | | | | |

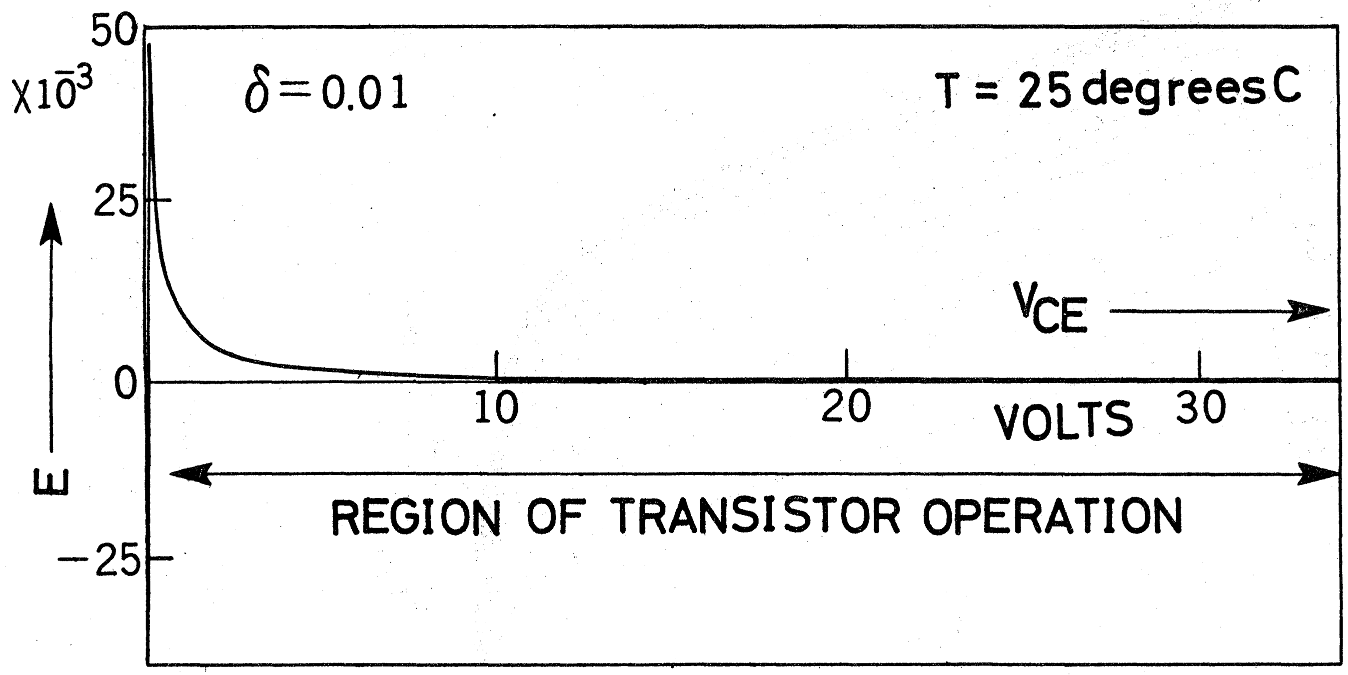


Fig. 1 The approximated error E versus collector voltage $V_{CE}$.

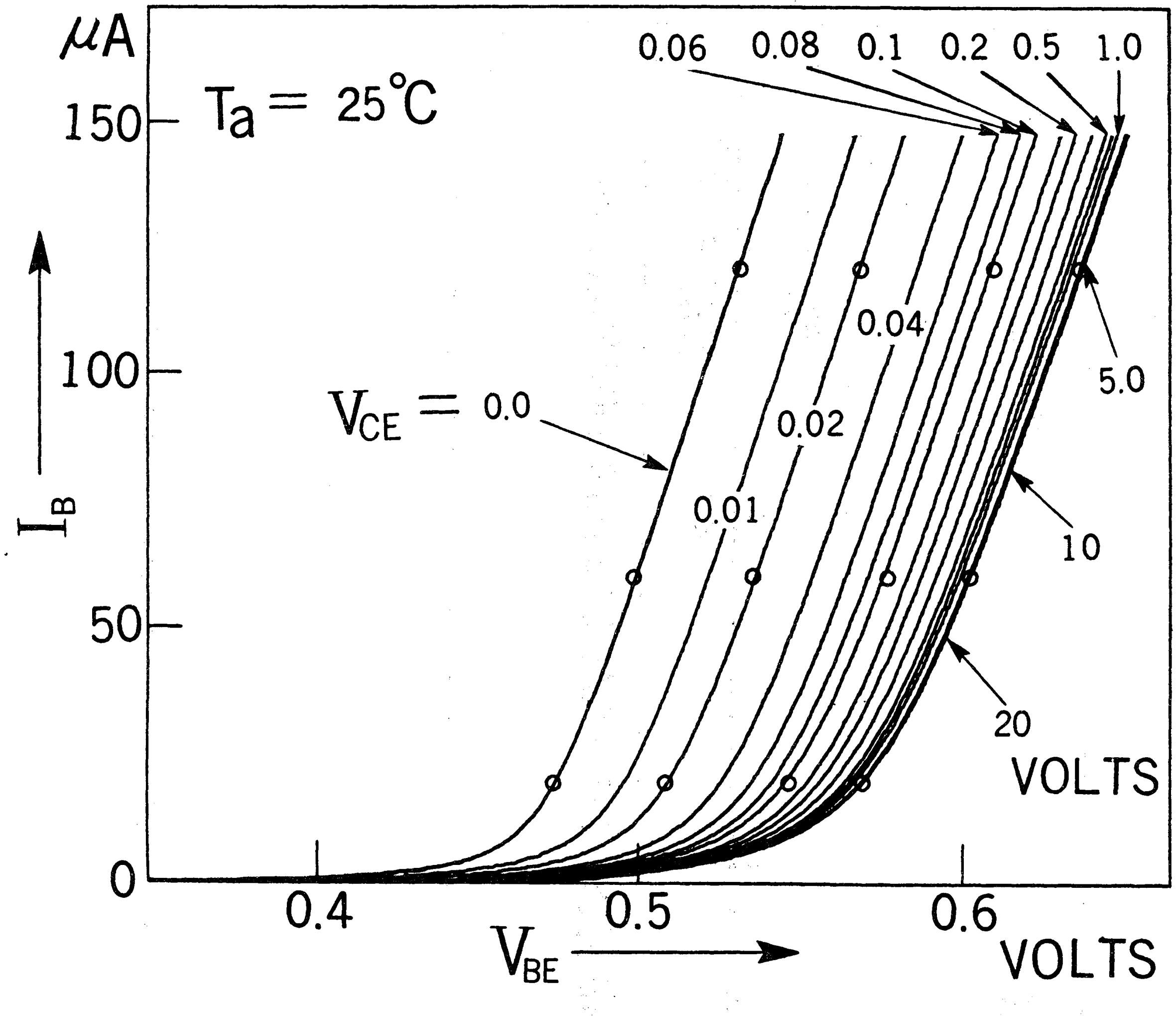


Fig. 2 Input characteristics of a bipolar transistor. Lines represent theory, small circles experimental results. ($R_S$=0.5, 2/λ=0.113, 2θ$I_F$=0.004)

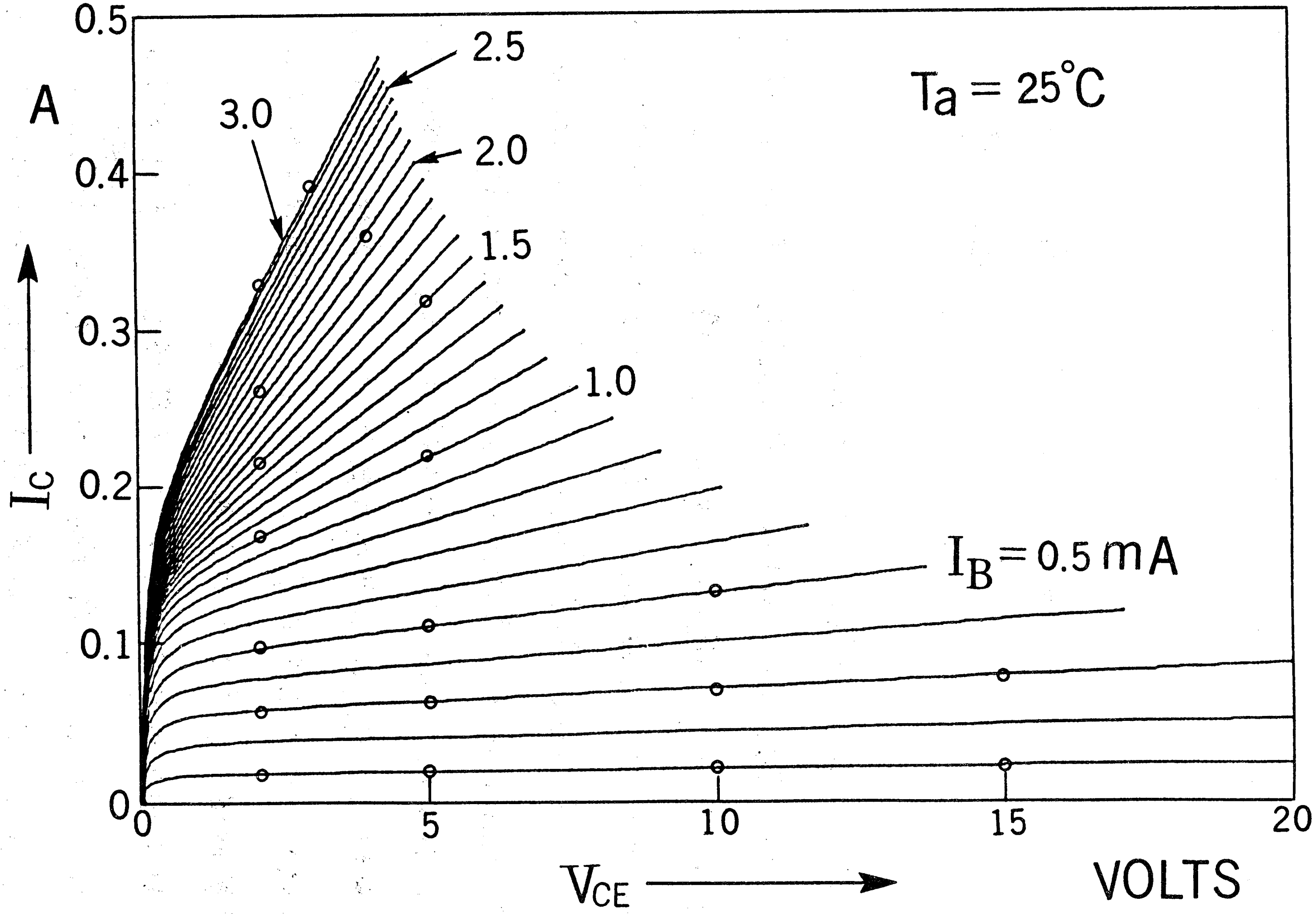


Fig. 3 Output characteristics of a bipolar transistor. Lines represent theory, small circles experimental results. ($m=n=7$, $\varepsilon=0.08$)