G.A.G.A. affine by polynomial growth

by

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§0. Introduction

P.Deligne and G.Maltsiniotis studied G.A.G.A. on a non-singular separated scheme of finite type over **C**. Their method is dependent on L^2 estimates for the $\bar{\partial}$ operator by Hörmander. On the other hand, using the Čech theory and Cousin integrals, N.Sasakura studied a polynomial growth cochain complex for a p.g. coherent sheaf. We shall define a polynomial growth cochain complex for a locally free sheaf(i.e.a vector bundle)by his techniques in [2] and give G.A.G.A. similar to the one due to [1]. This is a generalization of the theorem(3.4) in [2].

The contents of the various sections are as follows. In §1 we summarise some results in [2],which we shall use in the last section. In §2 we shall define a polynomial growth cochain complex for locally free sheaf over a smooth affine variety over **C**. In §3 we shall give G.A.G.A.(Th.(3.0)) for a vector bundle over a smooth affine variety.

The idea of this paper is much in debt to Professor N.Sasakura. Professor T.Kori has encouriged me in my studies. I wish to thank them.

§1. Preliminaries

Let X be a smooth affine variety over **C**. Unless we say otherwise, we regard X as the analytic variety (with the underlying analytic structure). Moreover, $\mathcal{O}_X$ = the structure sheaf of the analytic variety. Let $X \overset{i}{\hookrightarrow} \mathbf{C}^N$ be an embedding of X and $g = |z| + 1$ a growth function increasing at infinity, where z is the coordinate of $\mathbf{C}^N$.

Definition(1.0)

By a polynomial growth (or simply p.g.) covering of X of size $\sigma = (\sigma_1, \sigma_2)(\sigma_1 \geqq 1, \sigma_2 \geqq 1)$ with respect to g, we mean the collection as follows:

$\hat{A}_\sigma(X;g)$(or simply,$= \hat{A}_\sigma(X)$) $= \{\tilde{U}_\sigma(Q;g) = X \cap U_\sigma(Q;g); Q \in X\}$, where $U_\sigma(Q;g)$ is the disc,whose center is Q and radius is $\frac{1}{\sigma_1} g(Q)^{-\sigma_2}$.

Definition(1.1)

By a p.g. cochain compex of $\mathcal{O}_X^k$ for $\hat{A}_\sigma(X)$,we mean the subgroup of the cochain compex $C^*(\hat{A}_\sigma(X), \mathcal{O}_X^k)$ as follows:

$C^*(\hat{A}_\sigma(X), \mathcal{O}_X^k;g)_{p.g} = \{\varphi \in C^*(\hat{A}_\sigma(X), \mathcal{O}_X^k); \exists \alpha, \beta \geqq 1, |\varphi| \leqq \alpha(|z| + 1)^\beta$

,where $|\varphi|$ runs through all the components of $\varphi\}$

,where $\mathcal{O}_X^k$ is a direct sum of $\mathcal{O}_X$.

Theorem(1.2)

Let X be as above. Then $H^q(\varprojlim_\sigma C^*(\hat{A}_\sigma(X), \mathcal{O}_X^k)_{p.g}) = 0$ for $q \geqq 1$

$H^0(\varprojlim_\sigma C^*(\hat{A}_\sigma(X), \mathcal{O}_X^k)_{p.g}) = \Gamma(X_{alg.}, \mathcal{O}_{X_{alg.}}^k)$

,where $\mathcal{O}_{X_{alg.}}$ is a sructure sheaf as an algebraic variety.

We shall extend it to a locally free sheaf in the last section.

2. Polynomial growth cochain complexes for locally free sheaves.

(i) Let X, $\mathcal{O}_X$ and g be the same as those in §1.

Proposition(2.1)

Let $X \overset{\mathit{i'}}{\hookrightarrow} C^{N'}$ be another embedding of X and $g = |z'| + 1$ a growth function, where z' is the coordinate of $C^{N'}$. Then

$$\varinjlim_{\sigma} c^*(\hat{\mathcal{R}}_\sigma(X), \mathcal{O}_X^k; g)_{p.g} = \varinjlim_{\sigma} c^*(\hat{\mathcal{R}}_\sigma(X), \mathcal{O}_X^k; g')_{p.g}.$$

In order to prove this proposition we need the following Lemma.

Lemma(2.2)

Let $H(z)$ be a polynomial of $z = (z_1 \cdots \cdots z_N)$, then there exist $\alpha_1 > 0, \alpha_2 > 0, \beta_1 > 0, \beta_2 > 0$ such that

(*) $\alpha_1(|z| + 1)^{\alpha_2} \leq |H(z)| + 1 \leq \beta_1(|z| + 1)^{\beta_2}$.

proof of (2.2)

We may write $H(z) = \sum_{|\nu| \leq n} a_\nu z_1^{\nu_1} \cdots z_N^{\nu_N}$. Then we have

(a) $|H(z)| \leq \sum_{|\nu| \leq n} |a_\nu z_1^{\nu_1} \cdots z_N^{\nu_N}| \leq \underset{|\nu| \leq n}{\text{Max}} |a_\nu| \cdot (|z| + 1)^n$.

We derive the inequality of the right hand side in (*) from (a).

On the other hand we divide the numerator and the denominator of the following function by $|z|^n$,

(b) $\dfrac{(|z| + 1)^n}{\left| \sum_{|\nu| \leq n} a_\nu z_1^{\nu_1} \cdots z_N^{\nu_N} \right| + 1}$

And $|z| \longrightarrow \infty$, then (b) has a non-zero limit, so that there exist $r > 0$ and $M' > 0$ such that

(c) $(|z| + 1)^n \leq M'(|H(z)| + 1)$ for $|z| > r$.

Now we have

(d) $(|z| + 1)^n \leq (r + 1)^n$ for $|z| \leq r$.

Then we get the following inequality by (c) and (d):

(e) $\dfrac{1}{M}(|z| + 1)^n \leq |H(z)| + 1$, where $M = \text{Max}((r + 1)^n, M')$.

Q.E.D

<u>proof of</u> (2.1)

By the assumption of the embeding of X there exist polynomials $f_i(z) \in \mathbb{C}[z_1, \cdots, z_N]$ $(i = 1 \cdots\cdots N)$ such that

(a) $\qquad z_i' = f_i(z) \qquad (i = 1 \cdots\cdots\cdots N')$.

From (a) and Lemma(2.2) we have

(b) $\qquad \alpha_i(|z| + 1)^{\alpha_2} \leqq |z'| + 1 \leqq \beta_i(|z| + 1)^{\beta_2}$ on X.

We take an element $\varphi \in C^*(\hat{\mathcal{A}}_\sigma(X), \mathcal{O}_X^k; g)_{p.g}$. By the definition of $C^*(\hat{\mathcal{A}}_\sigma(X), \mathcal{O}_X^k; g)_{p.g}$ and (b) we get

(c) $\qquad |\varphi| = \gamma_i(|z'| + 1)^{\gamma_2}$.

(c) shows that φ is contained in $C^*(\hat{\mathcal{A}}_\sigma(X), \mathcal{O}_X^k; g')_{p.g}$.

<div align="right">Q.E.D.</div>

Prop.(2.1) shows that p.g cochain complex is independent of any embedding of X.

Let E be an algebraic vector bundle over X and $X = \bigcup_{i \in I} X_{f_i}$ $(= \bigcup_{i \in I} X_i$ simply) an affine open covering,where E is trivial over each X_i and f_i $(i \in I)$ is a polynomial. Taking a subset J of I, X_J denotes $X_{f_1} \cap \cdots \cap X_{f_s} = X_{f_1, \cdots, f_s}$. There is a wellknown diagram:

$$
\begin{array}{ccc}
\mathbb{C}^N - \{f_1 \cdots f_s = 0\} & \longrightarrow & S = \{1 - f_1(z)w_1 = 0 \cdots 1 - f_s(z)w_s = 0\} \subset \mathbb{C}^N \times \mathbb{C}^s \\
(z) & \longmapsto & (z) \times (\frac{1}{f_1(z)}, \cdots, \frac{1}{f_s(z)}) \\
(*) \qquad \cup & & \cup \\
X_J & \longrightarrow & \tilde{X}_J = S \cap \pi^{-1}(X_J)
\end{array}
$$

, where $\pi ; \mathbb{C}^N \times \mathbb{C}^s \longrightarrow \mathbb{C}^N$ is a projection and (w) is the coordinate of $\mathbb{C}^s$. Since X_J and $\tilde{X}_J$ are isomorphic each other in (*),

$g_J = 1 + |z| + \frac{1}{|f_1(z)|} + \cdots + \frac{1}{|f_s(z)|}$ is a growth function on X_J. E is trivial over X_J and we can regard E as an analytic vector bundle, so

(**) $E \mid X_J \cong \mathcal{O}_X^k \mid X_J.$

From (**) we can define the p.g. cochain complex $C^*(\hat{\mathcal{A}}_\sigma(X_J;g_J),E)_{p.g.}$ This p.g. cochain complex is independent of the way of taking any isomorphism in (**) and uniquely determined by Prop.(2.1).

Proposition(2.3)

Let K be a subset of J, then $X_J \subset X_K$ and there is a restriction map:

$$\text{res}_{KJ}; \ C^*(\hat{\mathcal{A}}_\sigma(X_J;g_J),E)_{p.g} \longrightarrow C^*(\hat{\mathcal{A}}_\sigma(X_K;g_K),E)_{p.g.}$$

proof

Because

$$
\begin{cases}
g_K(z) = 1 + |z| + \frac{1}{|f_1(z)|} + \cdots + \frac{1}{|f_s(z)|} \\
\\
g_J(z) = 1 + |z| + \frac{1}{|f_1(z)|} + \cdots + \frac{1}{|f_s(z)|} + \cdots + \frac{1}{f_t(z)},
\end{cases}
$$

we have $g_K(Q) \leqq g_J(Q)$ for $\forall Q \in X_J$. This means that

(a) $U_\sigma(Q;g_J) \supset U_\sigma(Q;g_K)$, where $U_\sigma(Q;g_J)$ is the disc, whose center is Q and radius is $\frac{1}{\sigma_1} g_J^{-\sigma_2}$. We have the following refining map by (a):

(b) $r \ ; \ \hat{\mathcal{A}}_\sigma(X_K;g_K) \longrightarrow \hat{\mathcal{A}}_\sigma(X_J;g_J)$

$\cup$ \qquad\qquad $\cup$

$\tilde{U}_\sigma(Q;g_K) = U_\sigma(Q;g_K) \cap X_K \longmapsto \tilde{U}_\sigma(Q;g_J) = U_\sigma(Q;g_J) \cap X_J.$

(b) induces the following restriction map:

$$\text{res}_{KJ}; \ C^*(\hat{\mathcal{A}}_\sigma(X_J;g_J),E)_{p.g} \longrightarrow C^*(\hat{\mathcal{A}}_\sigma(X_K;g_K),E)_{p.g.}$$

Q.E.D.

(ii) Let X,E and g_J be as above. We make a double complex in the following manner:

$$K^{pq}_{\sigma,p\cdot g} \Longleftarrow \overset{alt.}{\underset{|J|=q+1}{\oplus}} c^p(\widehat{\mathcal{A}}_\sigma(X_J;g_J),E)_{p\cdot g}.$$

Differentials δ_1 and δ_2 can be defined asfollows:

$$\delta_1; \quad K^{pq}_{\sigma,p\cdot g} \longrightarrow K^{p+1q}_{\sigma,p\cdot g}$$
$$\psi \qquad\qquad\qquad \psi$$
$$\varphi \longmapsto \delta_1\varphi$$

where $(\delta_1\varphi)_{j_0\cdots j_{p+1}} = \overset{p+1}{\underset{k=0}{\sum}}(-1)^k res(\varphi_{j_0\cdots \widehat{j}_k\cdots j_{p+1}})$ and $\overset{alt.}{\oplus}$ denotes an alternating sum.

$$\delta_2; \quad K^{pq}_{\sigma,p\cdot g} \longrightarrow K^{pq+1}_{\sigma,p\cdot g}$$
$$\psi \qquad\qquad\qquad \psi$$
$$\varphi \longmapsto \delta_2\varphi$$

,where $(\delta_2\varphi)_{i_0\cdots i_{q+1}} = \overset{q+1}{\underset{k=0}{\sum}}(-1)^k res_{(i_0\cdots i_{q+1})(i_0\cdots \widehat{i}_k\cdots i_{q+1})}(\varphi_{i_0\cdots\widehat{i}_k\cdots i_{q+1}})$

and $res_{(i_0\cdots i_{q+1})(i_0\cdots \widehat{i}_k\cdots i_{q+1})}$ is the restriction map in Prop.(2.3). On the other hand we have the following refining map:

(a) $\quad r_i; \quad \widehat{\mathcal{A}}_\sigma(X_i;g_i) \longrightarrow \mathcal{A}_\sigma(X;g)$
$$\psi \qquad\qquad\qquad\qquad \psi$$
$$X_i \cap U_\sigma(Q;g_i) \lhook\joinrel\longrightarrow X \cap U_\sigma(Q;g),$$

where $g_i = 1 + |z| + \dfrac{1}{|f_i(z)|}$ and $g = 1 + |z|$. From (a) we have the following diagram:

$$c^p(\widehat{\mathcal{A}}_\sigma(X;g),E) \xrightarrow{\quad r = \oplus r_i \quad} \underset{i}{\oplus} c^p(\widehat{\mathcal{A}}_\sigma(X_i;g_i),E)$$

$$\Big\downarrow \delta_2$$

$$\underset{i_0 i_1}{\overset{alt.}{\oplus}} c^p(\widehat{\mathcal{A}}_\sigma(X_{i_0 i_1};g_{i_0 i_1}),E)$$

,where $c^p(\widehat{\mathcal{A}}_\sigma(X;g),E)$, $c^p(\widehat{\mathcal{A}}_\sigma(X_i;g_i),E)$ and $c^p(\widehat{\mathcal{A}}_\sigma(X_{i_0 i_1};g_{i_0 i_1}),E)$ are the usual cochain complexes and

$$g_{i_0 i_1} = 1 + |z| + \dfrac{1}{|f_{i_0}(z)|} + \dfrac{1}{|f_{i_1}(z)|}.$$

Proposition(2.3)

We take the inductive limit of the above diagram:

$$\varinjlim_{\sigma} C^p(\hat{\mathcal{A}}_\sigma(X),E) \xrightarrow{\ r \ = \ \oplus \ r_i \ } \bigoplus_i \varinjlim_{\sigma} C^p(\hat{\mathcal{A}}_\sigma(X_i),E)$$

$$\downarrow \delta_2$$

$$\overset{alt.}{\bigoplus_{i_0 i_1}} \varinjlim_{\sigma} C^p(\hat{\mathcal{A}}_\sigma(X_{i_0 i_1}),E)$$

Then $\operatorname{Ker} \delta_2 = \operatorname{Im} r$.

proof

$\operatorname{Ker} \delta_2 \supset \operatorname{Im} r$ is trivial.

First we show that $\operatorname{Ker} \delta_2 \subset \operatorname{Im} r$ in case p=0. We take an element

$\varphi = (\varphi_i) \in \bigoplus_i C^0(\hat{\mathcal{A}}_\sigma(X_i),E)$. If we can take $\sigma' \geq \sigma$ $(\sigma'_1 \geq \sigma_1, \sigma'_2 \geq \sigma_2)$

which is independent of $Q \in X$ such that there exists $i = i_Q \in I$ for $\forall Q \in X$ and

(a) $\tilde{U}_{\sigma'}(Q;g) \subset \tilde{U}_\sigma(Q;g_i)$, we may take $\Psi = \{\Psi_Q\}_Q \in C^0(\hat{\mathcal{A}}_\sigma(X),E)$ such

that $\Psi_Q = \varphi_{i_Q} \big| \tilde{U}_{\sigma'}(Q;g)$. On the other hand for another correspondence

$j = j_Q$ as above we can define Ψ'_Q as $\varphi_{j_Q} \big| \tilde{U}_{\sigma'}(Q;g_j)$, then there exists

$i \in I$ such that $\varphi_{i_Q} = \varphi_{j_Q}$ on $\tilde{U}_\sigma(X_{ij};g_{ij})$ which is a cocycle condi-

tion (i.e. $\delta_2\varphi = 0$). This means that $\Psi'_Q = \Psi_Q$. Therefore the

$\{\Psi_Q\}_Q \in C^0(\hat{\mathcal{A}}_\sigma(X),E)$ is independent of the above correspondence,
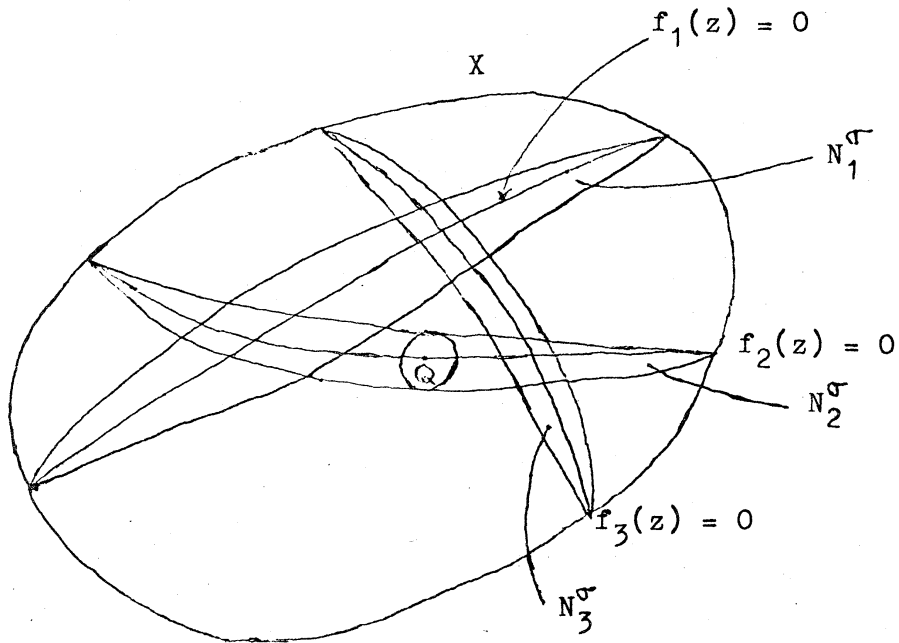
$X \ni Q \longmapsto i = i_Q \in I$. Then we shall define the following manner:

$$X \ni Q \longrightarrow i_Q \in I,$$

where this i_Q is i such that $g_i(Q) = \min_{j \in I}\{g_j(Q)\}$. Since I is finite,

we can choose it. The existence of $\sigma' \geq \sigma$ which is independent of

$Q \in X$ is derived from the following figure:

, where $N_i^\sigma = \bigcup_{Q \in \{f_i = 0\}} U_o(Q;g)$ and $g = 1 + |z|$. When we choose sufficiently large σ, $X = \bigcup_{i \in I} X - N_i^\sigma$. We can prove Ker $\delta_2 \subset$ Im r for the p-th cochain complex $C^p(\hat{A}_\sigma(X), E)$ in the same manner.

<div align="right">Q.E.D.</div>

Now we can define a p.g. cochain complex $C^*(X, \dot{E})_{p.g}$ for X.

Definition(2.4)

$$C^p(X, E)_{p.g} = \varinjlim_\sigma (\text{Im } r)_{p.g}^{p, \sigma},$$

where $(\text{Im } r)_{p.g}^{p, \sigma} \subset \bigoplus_i C^p(\hat{A}_\sigma(X_i; g_i), E)_{p.g}$. Since $(\text{Im } r)^{p, \sigma} \subset \bigoplus_i C^p(\hat{A}_\sigma(X_i; g_i), E)$ and $C^p(\hat{A}_\sigma(X_i; g_i), E)_{p.g}$ has defined, we can attach $(\text{Im } r)^{p, \sigma}$ to p.g. condition.

Remark(2.5)

As r is injective, Im r is uniquely defined. (i.e. $C^p(X, E)_{p.g}$ is independent of an affine open covering $X = \bigcup_{i \in I} X_i$.)

Proposition(2.6)

$C^*(X, E)_{p.g}$ is a complex in the usual manner.

This is trivial by the definition.

§3. The comparison and vanishing theorem

Let X and E be as above and $C^*(X,E)_{p.g}$ the one in the def.(2.4).
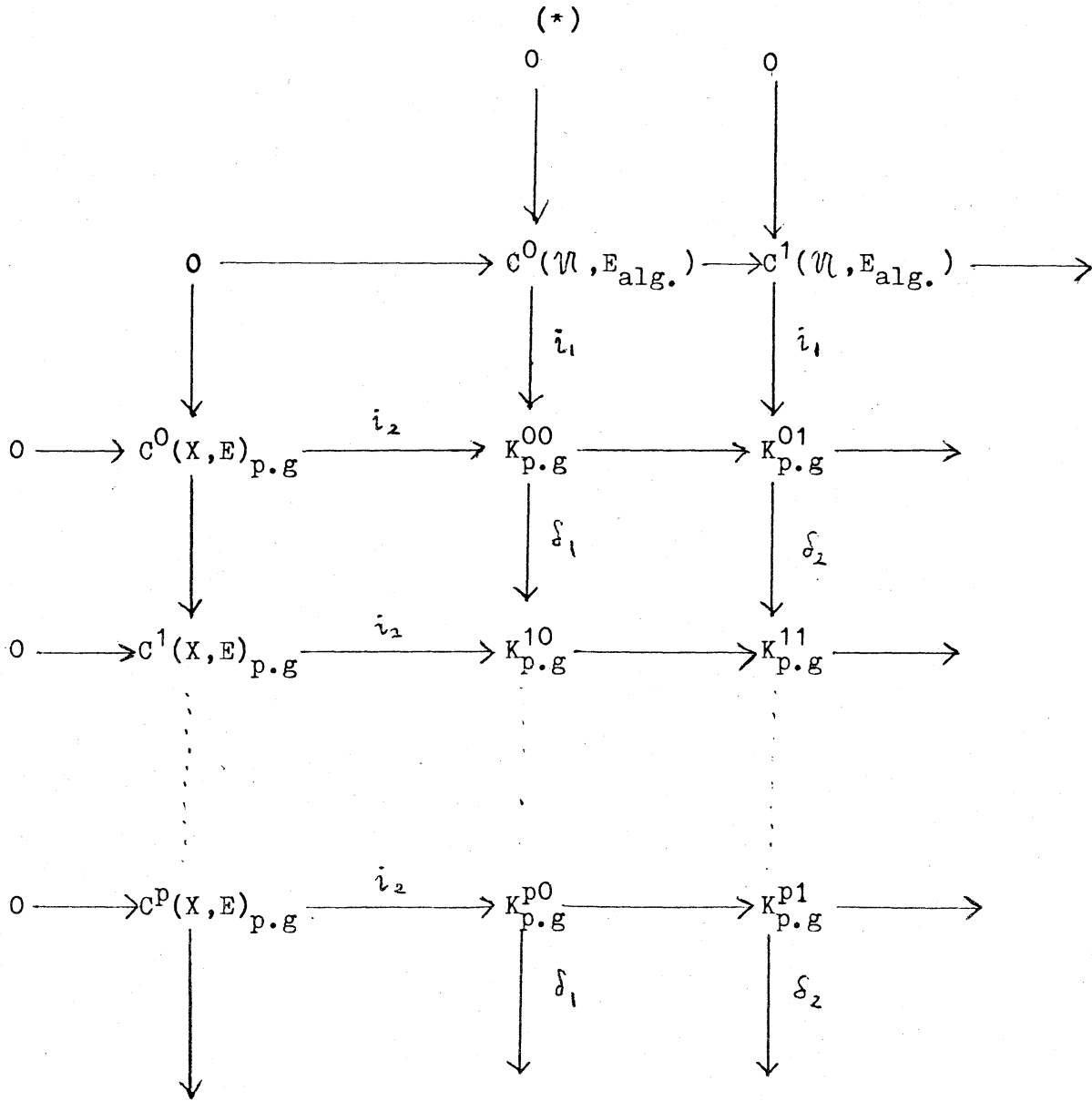
Theorem(3.0)

$$H^q(C^*(X,E)_{p.g}) = 0 \quad \text{for} \quad q \geqq 1,$$

$$H^0(C^*(X,E)_{p.g}) \cong \Gamma(X_{alg.}, E_{alg.}).$$

When we regard E as an algebraic vector bundle, we write $E_{alg.}$.

In order to prove the Th.(3.0) we think the following diagram;

(*)

, where $\mathcal{U} = \{X_i\}_{i \in I}$ is an affine open covering and

$$K^{pq}_{p \cdot g} = \varinjlim_{\sigma} K^{pq}_{\sigma, p \cdot g} .$$

By the simple observation we have the following:

(a) $K^{0q}_{p \cdot g} = \varinjlim_{\sigma} \overset{alt.}{\underset{i_0 \cdots i_q}{\bigoplus}} C^0(\hat{\mathcal{A}}_{\sigma}(X_{i_0 \cdots i_q}), E)_{p \cdot g} \longleftrightarrow \overset{alt.}{\underset{i_0 \cdots i_q}{\bigoplus}} \Gamma(X_{i_0 \cdots i_q}, E)_{p \cdot g}$

(b) $\overset{alt.}{\underset{i_0 \cdots i_q}{\bigoplus}} \Gamma(X_{i_0 \cdots i_q}, E)_{p \cdot g} = \overset{alt.}{\underset{i_0 \cdots i_q}{\bigoplus}} \Gamma(X_{i_0 \cdots i_q}, E_{alg.})$

$$= C^q(\mathcal{U}, E_{alg.}) .$$

We derive the first equality in (b) from the Sasakura's th.(1.2) and the second equality in (b) from the Serre's vanishing theorem of affine varieties. Therefore i_1 is injective from (a) and (b). Also i_2 is injective by Prop.(2.3). On the other hand

$$H^p(K^{*q}_{p \cdot g}) = \overset{alt.}{\underset{i_0 \cdots i_q}{\bigoplus}} \varinjlim_{\sigma} H^p(C^*(\hat{\mathcal{A}}_{\sigma}(X_{i_0 \cdots i_q}), E)_{p \cdot g} = 0 \quad \text{for} \quad p \geq 1 .$$

by the Sasakura's vanishing theorem(1.2). This means that (1) in the following proposition is true.

Proposition(3.1)

The complexes,

(1) $K^{0q}_{p \cdot g} \longrightarrow K^{1q}_{p \cdot g} \longrightarrow \cdots \cdots \longrightarrow K^{pq}_{p \cdot g} \longrightarrow \cdots$ for $q \geq 0$

(2) $K^{p0}_{p \cdot g} \longrightarrow K^{p1}_{p \cdot g} \longrightarrow \cdots \cdots \longrightarrow K^{pq}_{p \cdot g} \longrightarrow \cdots$ for $p \geq 0$,
are exact.

We must prove (2).

Lemma(3.2)

Let $\mathcal{U} = \{X_i\}_{i \in I}$ be as above. Then $H^1(K^{p*}_{p \cdot g}) = 0$ for $p \geq 0$.

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proof of Lemma(3.2)

We shall prove it by the induction on the number of sheets of the covering $\mathcal{U}$.

(i) When #I = 2, $\mathcal{U} = \{X_1, X_2\}$.

We define a neighborhood N_σ of $\{f_1 = 0\}$ in the following manner:

$N_\sigma = \bigcup_{Q \in V_1 - V_{12}} U_\sigma(Q; d(Q, V_{12}))$, where $V_1 = \{f_1 = 0\}$ and $V_{12} = \{f_1 = 0\} \cap \{f_2 = 0\}$

and $d(Q, V_{12})$ is a distance from Q to V_{12} and $U_\sigma(Q; d(Q, V_{12}))$ is the disc whose center is Q and radius $\frac{1}{\sigma_1} d(Q, V_{12})^{+\sigma_2}$. Then we have the following figure:
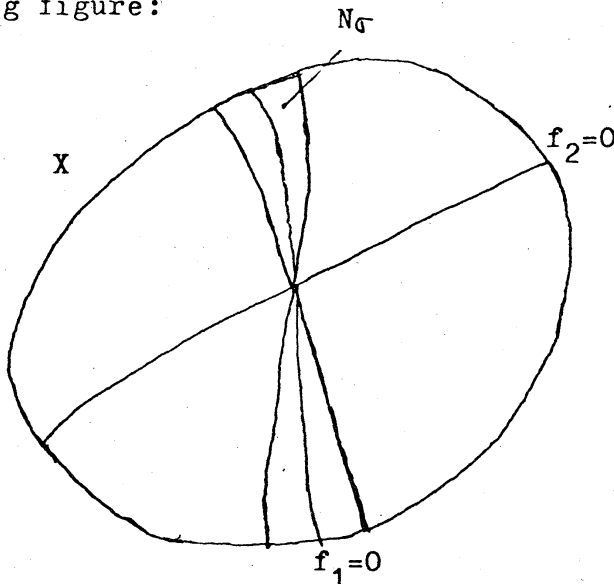
We take an element $\varphi_{12} \in C^p(\hat{\mathcal{R}}_\sigma(X_{12}), E)_{p \cdot g}$, then we give φ_1 and φ_2 in the following:

$\varphi_1 \begin{cases} = -\varphi_{12} & \text{on } N_\sigma \\ = 0 & \text{on } X - N_\sigma \end{cases}$

$\varphi_2 \begin{cases} = 0 & \text{on } N_\sigma \\ = \varphi_{12} & \text{on } X - N_\sigma . \end{cases}$

Then we have $\varphi_{12} = \varphi_2 - \varphi_1$. When we take sufficiently large $\sigma'(\geq \sigma)$, $\varphi_i \in C^p(\hat{\mathcal{R}}_{\sigma'}(X_i), E)_{p \cdot g}(i = 1, 2)$ by the Lojasiewicz inequality and the triangle inequality. Therefore $H^1(K^{p*}_{p \cdot g}) = 0$ for $\forall p \geq 0$.

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(ii) When $\#I=3$, $\mathcal{U} = \{X_1, X_2, X_3\}$ and we take

$$\varphi = (\varphi_{12}, \varphi_{13}, \varphi_{23}) \in \overset{alt.}{\underset{i,j}{\bigoplus}} \; C^p(\hat{\mathcal{A}}_\sigma(X_{ij}), E)_{p.g}.$$

Applying the assumption of the induction to $\{X_1, X_2\}$, there exist

$\varphi_1 \in C^p(\hat{\mathcal{A}}_\sigma(X_1), E)_{p.g}$ and $\varphi_2 \in C^p(\hat{\mathcal{A}}_\sigma(X_2), E)_{p.g}$ such that

$\varphi_{12} = \varphi_2 - \varphi_1$. By the cocycle condition, $\varphi_{23} - \varphi_{13} + \varphi_{12} = 0$, we

have $(\varphi_{23} + \varphi_2) - (\varphi_{13} + \varphi_1) = 0$. We define newly as follows:

$$\tilde{\varphi}_{23} = \varphi_{23} - \varphi_2 \;, \quad \tilde{\varphi}_{13} = \varphi_{13} - \varphi_1 \;, \quad \tilde{\varphi}_{12} = 0 \;.$$

Since $(\tilde{\varphi}_{13}, \tilde{\varphi}_{23})$ is a cocycle for (X_{13}, X_{23}), we can define $\tilde{\varphi}_3$ as follows:

$$\tilde{\varphi}_3 \begin{cases} = \varphi_{13} & \text{on } X_{13} \\ = \varphi_{23} & \text{on } X_{23} \\ = 0 & \text{on } X_3 - (X_{13} \cup X_{23}). \end{cases}$$

This cochain $\tilde{\varphi}_3$ is not contained in $C^p(\hat{\mathcal{A}}_\sigma(X_3), E)_{p.g}$. Now we define

$\varphi_3 \in C^p(\hat{\mathcal{A}}_\sigma(X_3), E)_{p.g}$ in the following manner:

$$\varphi_3 \begin{cases} = \tilde{\varphi}_3 & \text{on } X_3 - N \\ = 0 & \text{on } N \end{cases}$$

$$\varphi_1 \begin{cases} = \tilde{\varphi}_3 & \text{on } N - X_1^c \\ = 0 & \text{on otherwise} \end{cases}$$

$$\varphi_2 \begin{cases} = \tilde{\varphi}_3 & \text{on } N - X_3^c \\ = 0 & \text{otherwise} \end{cases}$$

, where X_i^c is a complement of X_i. Then $\varphi_3 \in C^p(\hat{\mathcal{A}}_\sigma(X_3), E)_{p.g}$ and

$H^1(K_{p.g}^{p*}) = 0$ for $p \geq 0$.

(iii) When $\#I=n$, we can show that $H^1(K_{p.g}^{p*}) = 0$ for $p \geq 0$ in the

same manner as (ii).

Q.E.D.

Let $\mathcal{U}^n = \{X_1, \ldots, X_n\}$ be a covering of X and $K_{p.g}^{p*}(\mathcal{U}^n)$ as

$K_{p.g}^{p*}$ for $\mathcal{U}^n$.

Lemma(3.3)

We assume that $H^i(K^{p*}_{p.g}(\mathcal{N}^n)) = 0$ for $i = q-1$, q $(q-1 \geqq 1)$.

Then $H^q(K^{p*}_{p.g}(\mathcal{N}^{n+1})) = 0$.

proof

We take an element $\varphi_{n+1} \in Z^q(K^{p*}_{p.g}(\mathcal{N}^{n+1})$. We restrict φ_{n+1}

$\mathcal{N}^n$ and $\varphi_n \in Z^q(K^{p*}_{p.g}(\mathcal{N}^n))$ denotes it. By the assumption of the

induction there is $\Psi_n \in K^{pq-1}_{p.g}(\mathcal{N}^n)$ such that $\delta_2 \Psi_n = \varphi_n$. Now

we define $\Psi_{n+1} \in K^{pq-1}_{p.g}(\mathcal{N}^{n+1})$ as follows:

$$\begin{cases} \Psi_{n+1}(X_{i_1} \wedge \cdots \wedge X_{i_q}) = \Psi_n(X_{i_1} \wedge \cdots \wedge X_{i_q})(1_= i_1 < \cdots < i_q = n) \\ \Psi_{n+1}(X_{i_1} \wedge \cdots \wedge X_{i_q}) = 0 \qquad\qquad (i_q = n+1). \end{cases}$$

We define an element $\varphi'_{n+1} \in Z^q(K^{p*}_{p.g}(\mathcal{N}^{n+1}))$ by

(*) $\varphi'_{n+1} = \varphi_{n+1} - \delta_2 \Psi_{n+1} \begin{cases} = 0 \quad\text{on } X_{i_1} \wedge \cdots \wedge X_{i_{q+1}} (i_{q+1} \leqq n) \\ = \varphi_{n+1}(X_{i_1} \wedge \cdots \wedge X_{i_q} \wedge X_{n+1})(i_q \leqq n). \end{cases}$

The number of the sheets of the covering $\mathcal{B}^n = \{X_i \cap X_{n+1}; i = 1 \cdots n\}$

is n. From (*) we can regard φ'_{n+1} as an element φ'_n of

$Z^{q-1}(K^{p*}_{p.g}(\mathcal{B}^n))$ by using the covering $\mathcal{B}^n$. By the assumption of the

induction there exists $\widetilde{\varphi}_n \in K^{pq-2}_{p.g}(\mathcal{B}^n)$ such that $\delta_2 \widetilde{\varphi}_n = \varphi'_n$. We

pull back $\widetilde{\varphi}_n$ and we have $\widetilde{\varphi}_{n+1} \in K^{pq-1}_{p.g}(\mathcal{N}^{n+1})$.

<div align="right">Q.E.D.</div>

proof of (2) in the Proposition(3.1)

By $H^2(K^{p*}_{p.g}(\mathcal{N}^2)) = 0$, Lemma(3.2) and Lemma(3.3) we have

$H^2(K^{p*}_{p.g}(\mathcal{N}^3)) = 0$. By $H^2(K^{p*}_{p.g}(\mathcal{N}^3)) = 0$, Lemma(3.2) and Lemma(3.3)

we have $H^2(K^{p*}_{p.g}(\mathcal{N}^4)) = 0$. By using Lemma(3.2),(3.3) repeatedly

we have (2).

<div align="right">Q.E.D.</div>

<u>proof of Th.(3.0)</u>

By using Prop.(3.1) and the commutative diagram (*) we have

(a) $\quad H^p(C^*(X,E)_{p.g}) \cong H^p(C^*(\mathcal{Vl},E_{alg.}))$ for $p \geqq 0$.

By the Serre's vanishing theorem we get

(b) $\quad H^p(C^*(\mathcal{Vl},E_{alg.})) = 0$ for $p \geqq 1$.

Therefore this completes the proof by (a) and (b).

$$Q.E.D.$$

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