USE OF DATA REPRESENTATION MAPPING IN AUTOMATIC
GENERATION OF DATA BASE ACCESS PROCEDURES

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ABSTRACT

Formal definition of data structure mappings are given and used to transform relational algebraic formula into data base access procedures which search a formally defined data structure. Especially, a hierarcical data structure is introduced to represent a set of relations which are inherently hiearchical. We show that a retrieval procedure using data accesses along a two level hierarchy can be obtained from an original formula on flat relations by applying equivalence transformation rules and data representation mappings.

1. INTRODUCTION

During the latest decade, data base theories and data structure theories have been developed independently. In data base area, much research has been concentrated on relational data base theories. As far as query processing is concerned, many logical query languages as well as deductive

query evaluation systems have been proposed. Query optimizations and processing based on relational algebla have also been investigated by many researchers.

On the other hand, theoretical researches on data structure have been developed in the field of program correctness studies [Hoare 72]. Recently, [Hansson 79] developed a framework to formally define data structures and prove correctness of programs manipulating them.

In this paper, we will combine the above two theoretical approaches and put them in a framework of data abstraction theory. Namely, we regard relations as objects of an abstract data type and relational algebra as its associated operations. Furthermore, we formally define data structures and represent relations by them. Our final goal is to transform relational algebraic query formulas into data base access procedures which search the defined data structures. Especially, we consider to represent a set of relations which are inherently hierarchical by a hierarchical data structure. We show that a retrieval procedure using data accesses along a two level hierarchy can be obtained from an original formula on flat relations by applying equivalence transformation rules and data representation mappings.

2. RELATION AND RELATIONAL ALGEBRA

First, we define the relational data base formally.

<u>Definition</u>. Let D1,...,Dn be n domains (not necessarily distinct) of elements and A1,...,An be n identifiers which are associated with D1,...,Dn respectively. A relation R on the set of identifiers {A1,...,An} is a subset of the cartesian product D1 ✗...✗ Dn. Ai is called an attribute of the relation R. The relation R on the attribute set {A1,...,An} is denoted by R(A1,...An).

We use A, B,... for a single attribute and X, Y,... for a set of attributes (including the null set).

<u>Definition</u>. A relation type ℝ(A1,...,An) on the attribute set {A1,...,An} is a set of all relations on the set {A1,...,An}.

A relation R(A1,...,An) is an element of the relation type ℝ(A1,...,An). A conceptual schema which determines the logical data structure is defined by a set of relation types. In this paper, we regard hierarchical data organizations to be matters of internal schema, which determine the physical data structure.

Figure 2-1 shows an example of a relational data base where the conceptual schema is the set, {EMP(emp, loc), ROOM(room#, sect)}.

Next, we define the relational algebraic operations ([Codd 72], [Reiter 78]).

<u>Definition</u>. Let R(A,X) be a relation. Then, the <u>projection</u> π of R with respect to A is a relation given by the equation

$$\prod_A (R) = \{ x : (\exists z) \langle z, x \rangle \in R \}$$

where x represents the sequence of values corresponding to the attribute set X and z the value corresponding to a chosen attribute A.

Definition. Let R(X) be a relation, and x be a variable which corresponds to X. Let f be a logical function X $\rightarrow$ {T,F}. Then, the <u>selection</u> of R with respect to f(x), denoted by $\Phi_{f(X)}$ (R), is a set of x values given by the equation

$$\Phi_{f(X)} (R) = \{ x : x \in R \wedge f(x) = T \}.$$

The selection operation $\Phi_{f(X)}(R)$ is an associative retrieval of the relation R under the condition that f(x)=T.

Definition. Let R1(A,X) and R2(B,Y) be relations, and f(A,B) a logical function. Then, the <u>join</u> of R1(A,X) and R2(B,Y) with respect to f(A,B) is denoted by R1(A,X) $*_{f(A,B)}$ R2(B,Y), and is given by

$$R1(A,X) *_{f(A,B)} R2(B,Y)$$

$$= \{ \langle a,x,b,y \rangle : \langle a,x \rangle \in R1 \wedge \langle b,y \rangle \in R2 \wedge f(a,b) = T \}.$$

If the join condition f(A,B) is given by an equality formula such as [loc = room#], then a = b always holds in the above set expression and the answer becomes redundant. In this case, we remove one of the two attributes from the result. This modified join is called the natural join.

3. QUERY TRANSFORMATION INTO HIERARCHICAL DATA ORGANIZATION

Figure 3-1 displays the logical structure of the data base given in Figure 2-1. This diagram is a Backman diagram and uses arrows to indicate one-to-many mappings (Backman [1969]). One-to-many mappings correspond to hierarchical relations and the arrows represent the access paths in hierarchical data bases; namely, subordinate data are retrieved by specifying the superordinate data and the access path between them.

Figure 3-2 shows an example of a concrete tree structure which has the logical structure given in Figure 3-1. Traversal from the root node in Figure 3-2 corresponds to retrieval along access path "a" in Figure 3-1. Given a piece of section information, we select an appropriate arc and follow it to reach the set of room#'s which belong to the section.

Let us assume that each set of room#'s forms a relation distinctively. We denote a set of room#'s under the selector "sect = ai" as ROOM.ai(room#). This relation can be defined by a selection followed by a projection as follows:

ROOM.ai(room#)

$$= \pi_{sect} \circ \Phi_{[sect=ai]} (ROOM(room\#,sect)). \tag{3.1}$$

If we regard ai to be a variable, then the above formula defines any relation under any selector.

Next, we define a hierarchical data organization for the EMP
relation similar to that which was defined for the ROOM
relation. Let us denote EMP.bj(emp) to be a relation which
comprises the employees who are located at bj. Then, the
formula corresponding to (3.1) is given by:

$$EMP.bj(emp) = \prod_{loc} \bullet \Phi_{[loc=bj]} (EMP(emp,loc)). \qquad (3.2)$$

Let us now consider the example query "Get all
employees who belong to INFO section":

$$Q = \prod_{loc} \bullet \prod_{sect} \bullet \Phi_{[sect=INFO]}$$
$$(EMP(emp,loc) \underset{[loc=room\#]}{*} (ROOM(room\#,sect))).$$
$$(3.3)$$

It has been shown in [Furukawa 79] that Q is equivalently
transformed to:

$$Q = \bigcup_{bj \in ROOM.INFO(room\#)} EMP.bj(emp). \qquad (3.4)$$

The evaluation of formula (3.4) proceeds as follows: first,
compute ROOM.INFO(room#) according to the definition (3.1);
then retrieve EMP.bj(emp) for all bj in ROOM.INFO(room#);
finally collect them together. This procedure is exactly the
structural retrieval along access path comprising "a" and
then "b" in Figure 4-2. However, this access method is
somewhat different from that of the hierarchical data bases.
In order to retrieve all of the employees who are working in
room bj, the above method relies upon the use of
associativity rather than a pointer to access EMP.bj

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relation. Although associative retrieval can be realized very efficiently with hashing techniques and/or specialized hardware, it is still less efficient than the pointer access method. We will show further transformation of expression (3.4) based on the data stucture mapping to realize the exact access method of the hierarchical model.

4. FORMAL DEFINITION OF DATA STRUCTURES AND PRIMITIVE FUNCTIONS

In this section, we formally define a hierarchical structure and its related primitive functions to manipulate the structure. In defining a data structure, we adopt a framework developed by [Hansson 79].

First, we define a list consisitng of arbitrary components.

Axiom 1 $Vw\{list(w) \leftrightarrow w=NIL \lor ExEy[w=x.y \,\&\, list(y)]\}$

Then, we define a selection list (slist) to represent a set of subrelations such as $\{EMP.r : r \in room\#\}$.

Axiom 2

$Vw\{slist(w) \leftrightarrow w=NIL \lor$

$EsExEtl[w=<s,x>.tl \,\&$

$Vs'Vx'(<s',x'> \in tl \rightarrow s'=s)) \,\&$

$list(x) \,\&\, slist(tl)]\}$,

where $\in$ is a membership predicate for slist and will be

defined later.

This axiom defines a list of pairs whose first components differs from each other. We introduce the following useful lemmas concerning to the slist.

Lemma 2.1   slist(NIL).

Lemma 2.2

VsVxVw{slist(<s,x>.w)

      ↔ Vs'Vx'[<s',x'> ← w → s'≠s] & list(x) &
      slist(w)}.

Next, we define a membership predicate   as follows:

Axiom 3.

VeVw{e ← w

      ⟷ slist(w) & EhdEtl[hd=e ∨ e ← tl]}

Like Axiom 2, we also introduce the following lemmas:

Lemma 3.1   Ve{~e ← NIL}.

Lemma 3.2   VeVw[e ← e.w ← slist(w)].

Lemma 3.3
  VeVhdVtl[e ← hd.tl ↔ slist(hd.tl) & e ← tl]

We are interested in representing hierarchy using slists. The following axiom defines a hierarchical data structure called multi-level slist structure or simply

mls-structure in terms of slist.

Axiom 4.

Vw{mls-structure(w)

↔ w=NIL v slist(w) & VsVx[<s,x> ∈ w → slist(x)]}.

Finally, we define a double list (dlist) whose components are all lists.

Axiom 5

Vw{dlist(w)

↔ w=NIL v EhdEtl[w=hd.tl &

list(hd) & dlist(tl)]}

Now, we introduce primitive predicates and their corresponding functions for manipulating slist and/or dlists.

(1) assoc(s,w,x)

assoc(s,w,x) corresponds to a LISP ASSOC function and is equal to "x = ASSOC(s,w)". A formal definition of assoc is given as follows:

Axiom 6

1. Vs[assoc(s,NIL,NIL)].

2. VsVxVw[assoc(s, s,x .w,x) ← slist(s,x .w)].

3. VsVs'VxVx'Vw[assoc(s, s',x' .w,x) ← slist(s',x' .w) &

s≠s' & assoc(s,w,x)].

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assoc(s,w,x) is similar to <s,x> $\in$ w. They differ only when w is an empty list, where assoc(s,NIL,x) is true for x=NIL, but <s,x> $\in$ w becomes false. Relations between assoc and $\in$ are stated by the following two theorems.

Theorem 1   VsVxVw{< s,x> $\in$ w $\rightarrow$ assoc(s,w,x)}.

Theorem 2   VsVxVw{~<s,x> $\in$ w $\rightarrow$ assoc(s,w,NIL)}.

(2) project-sublist(w,w')

project-sublist(w,w') corresponds to a projection π of a relational algebra. It projects out first components of each pair of w and obtains w' consisting of all of the second components of the pairs. Formally, it is stated as follows:

Axiom 7

1. project-sublist(NIL,NIL).
2. VsVxVwVw'[project-sublist(s,x .w,x.w')

                $\leftarrow$ slist(s,x .w) & project-sublist(w,w')].

We denote a function corresponding to project-sublist(w,w') as PRSUBL(w).

(3) project-selector(w,w')

project-selector(w,w') is similar to project-sublist(w,w'). The roles of first components and the second components are interchanged.

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Axiom $\underline{8}$

1. project-selector(NIL,NIL).

2. VsVxVwBw'[project-selector(s,x .w,s.w')

    ← slist(s,x .w) & project-selector(w,w')].

A corresponding function is denoted by PRSEL(w).

(4) dunion(w,w')

dunion(w,w') is a predicate which manipulates dlists and makes a list w' consisting of all elements of all components of w.

Axiom $\underline{9}$

1. dunion(NIL,NIL).

2. VxVwVw'[dunion(x.w,APPEND(x,w'))

    ← dlist(x.w) & dunion(w,w')].

A corresponding function is denoted by DUNION(w).

$\underline{5}$. DATA STRUCTURE MAPPING AND FURTHER QUERY TRANSFORMATION

In order to represent ROOM.s and EMP.r using an mls-structure, we need to associate each room# in ROOM.s relations with EMP.r. We introduce a pseudo relation called HROOM.s(room#, Emp) where Emp is a pointer to an EMP.r relation. The original ROOM.s(room#) relation is obtained from HROOM.s(room#, Emp) by

$$\text{ROOM.s(room\#)} = \prod_{\text{Emp}} (\text{HROOM.s(room\#, Emp)}) \qquad (5.1)$$

We organize HROOM.s and EMP.r in a ROOM-FILE as shown in Figure 5-1, where ROOM-FILE satisfies the following assertion:

Assertion 1   mls-structure(ROOM-FILE).

The mappings from ROOM-FILE to HROOM.s and EMP.r are given as follows:

Mapping 1   For any section s,

    if   Ex[<s,x> $\in$ ROOM-FILE]
    then Repr(ASSOC(s,ROOM-FILE)) = HROOM.s
    else HROOM.s = $\emptyset$.

From Assertion 1 and Mapping 1, it is shown that any non empty HROOM.s is represented by a selection list. Namely, the representation function Repr maps a selection list to a pseudo relation.

Each EMP.r is then represented by a second component of some element of the selection list corresponding to an HROOM.s such that r belongs to s.

Mapping 2   For any room# r,

1. Vw{EsEx[<s,w> $\in$ ROOM-FILE & <r,x> $\in$ w]
      $\rightarrow$ Repr(ASSOC(r,w)) = EMP.r}

2. ~EwEsEx[<s,w> $\in$ ROOM-FILE & <r,x> $\in$ w] $\rightarrow$ EMP.r = $\emptyset$.

From Mapping 1 and 2, it is shown that the following equations holds:

Mapping 1'   For any section s, HROOM.s is obtained from

HROOM.s = Repr(ASSOC(s,ROOM-FILE)).

Mapping 2'   For any room# r, EMP.r is obtained from

$$EMP.r = \bigcup_{w \in PRSUBL(ROOM-FILE)} (\text{if } r \in PRSEL(w) \text{ then } Repr(ASSOC(r,w)) \text{ else } \phi).$$

We are now able to construct a mapping from ROOM-FILE to each ROOM.s:

Mapping 1"   For any section s, ROOM.s is obtained from

ROOM.s = Repr(PRSEL(ASSOC(s, ROOM-FILE))).

Now, let us consider how to transform the query (3.4) into a program which searches the ROOM-FILE mls-structure.

By applying Mapping 1" and 2' to (3.4), we obtain

$$Q = \bigcup_{r \in Repr(PRSEL(ASSOC(INFO,ROOM-FILE)))}$$

$$\bigcup_{w \in PRSUBL(ROOM-FILE)}$$

$$(\text{if } r \in PRSEL(w) \text{ then } Repr(ASSOC(r,w)) \text{ else } \phi). \quad (5.2)$$

Since the functional dependency room# $\rightarrow$ section holds, r belongs to only one section (in this case, INFO), and "$r \in PRSEL(w)$" becomes false for any other sections. Therefore, (5.2) is reduced to

$$Q = \bigcup_{r \in \text{Repr(PRSEL(ASSOC(INFO,ROOM-FILE)))}} (\text{Repr(ASSOC(r,ASSOC(INFO,ROOM-FILE))))}$$

$$(5.3)$$

Generally, a reduction theorem from (5.2) to (5.3) is stated as follows:

Theorem 3   Let M be an mls-structure, S be a constant corresponding to a first level selector of M and f be an arbitrary function. Then,

$$\bigcup_{r \in \text{PRSEL(ASSOC(S,M))}} \circ \bigcup_{w \in \text{PRSUBL(M)}} (\text{if } r \in \text{PRSEL(w) then } f(r,w) \text{ else } \emptyset)$$

$$= \bigcup_{r \in \text{PRSEL(ASSOC(S,M))}} f(r,\text{ASSOC(S,M)}).$$

Since the range of r covers the entire set of all first components of pairs in HROOM.INFO, we do not need to search HROOM.INFO list for each r, but simply union all second elements of pairs in HROOM.INFO. The following theorem suggests us to use a DUNION function to compute the above union.

Theorem 4   Let W be an slist. Then,

$$\bigcup_{r \in \text{PRSEL(W)}} (\text{Repr(ASSOC(r,W)))}$$

$$= \text{Repr(DUNION(PRSUBL(W))).}$$

By applying Theorem 4 to (5.3), we obtain the following simple program to compute Q:

$$Q = \text{Repr(DUNION(PRSUBL(ASSOC(INFO,ROOM-FILE))))} \qquad (5.4)$$

Axiom 6, 7 and 9 define ASSOC, PRSUBL and DUNION respectively. These axioms can be considered to be PROLOG program segments and can be executed directly. It is also possible to generate LISP programs by adding a suitable set of transformation rules and applying the same transformation methods as above.

6. CONCLUDING REMARKS

In this paper, we have represented a set of subrelations by a formally defined list structure and generated a data retrieval procedure which manipulates the structure given a higher level relational algebraic query formula.

The idea that we represent relations instead of tuples or values in tuples is closely related to the data abstraction concept. We do not care the detailed structure of each relation, which is another lower level problem to be solved separately.

We also have shown that during the transformation of relational algebraic formulas into data structure manipulation procedures, simplifications due to the structure are performed. We think this simplification process is very important in data abstraction theory.

[Yonezawa 80] has attacked a similar problem to ours in the frame of first order logic. Yonezawa has achieved a very

neet method to generate recursive procedures for recursively defined queries.

Further consideration is needed to see whether our methods can be applied to the recursive case or not. We also plan to apply our methods to the network case.

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EMP(emp,      loc)

      SMITH    101
      JONES    102
      BROWN    101
      HENRY    301
      NELSON   102
      MURPHY   201
      LONG     202
      MORGAN   201
      LEE      202


ROOM(room#,  sect)

      101    INFO
      102    ARCH
      201    LANG
      202    INFO
      301    ARCH
```
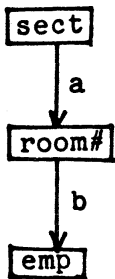
Figure 2-1. An Example
Relational Data Base.

Figure 3-1. A Backman
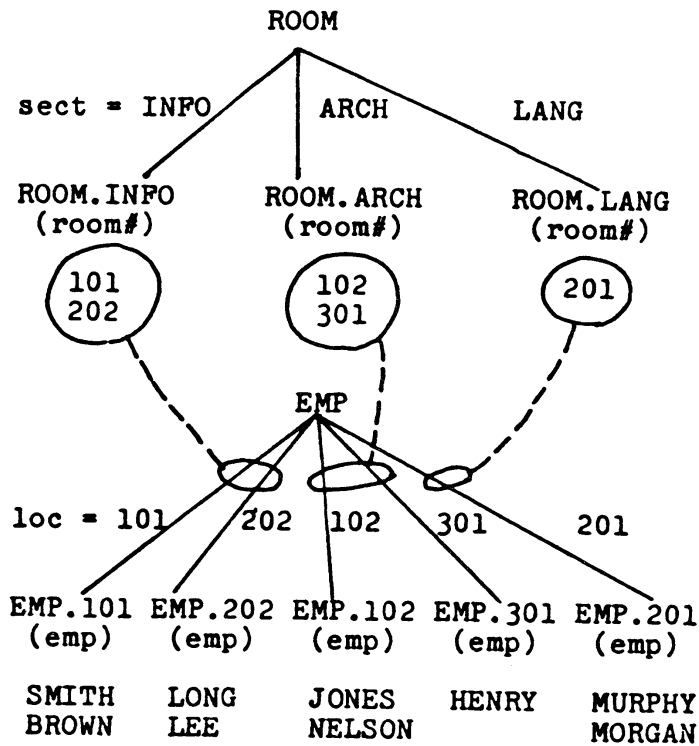diagram for EMP, ROOM
relational data base.

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ROOM

sect = INFO   ARCH   LANG

ROOM.INFO   ROOM.ARCH   ROOM.LANG
(room#)   (room#)   (room#)

    101   102   201
    202   301

                 EMP

loc = 101   202   102   301   201

EMP.101 EMP.202 EMP.102 EMP.301 EMP.201
(emp)   (emp)   (emp)   (emp)   (emp)

SMITH   LONG   JONES   HENRY   MURPHY
BROWN   LEE   NELSON   MORGAN

Figure 3-2. A structural organization for
the ROOM and EMP relations.

ROOM-FILE

        HROOM.INFO   HROOM.ARCH   HROOM.LANG
<INFO,<101, >.<202, >.NIL>.<ARCH,<102, >.<301, >.NIL>....NIL

    EMP.101   EMP.202   EMP.102   EMP.301

    SMITH.BROWN.NIL LONG.LEE.NIL JONES.NELSON.NIL HENRY.NIL
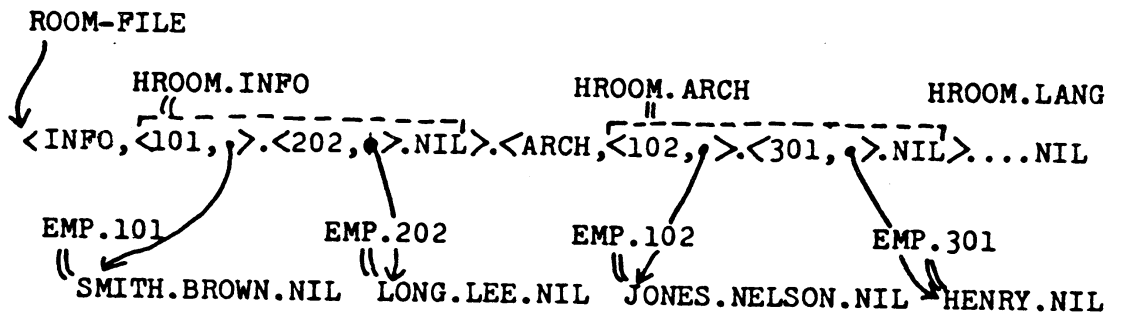
Figure 5-1. Organization of HROOM.s and EMP.r in
the ROOM-FILE mls-strucutre.