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Rational singularities with C*-action.

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Introduction.

Let k be a field of characteristic 0, Y be a normal variety of finite type over k and Φ: Y' $\longrightarrow$ Y be a resolution of Y. A point y of Y is called a rational singularity ([T.E], Chap.I, §3) if $(R^q\Phi_*(O_{Y'}))_y = 0$ for $q > 0$. It is known that $R^q\Phi_*(O_{Y'})$ is independent of the resolution Φ and that the higher direct images reflect the ring-theoretic properties of the local ring $O_{Y,y}$. For example, if y is an isolated singularity, $(R^q\Phi_*(O_{Y'}))_y$ is isomorphic to the local cohomology module $H_y^{q+1}(O_Y)$ $(0 < q < \dim Y - 1)$ and, consequently, $O_{Y,y}$ is a Cohen-Macaulay ring if and only if $(R^q\Phi_*(O_{Y'}))_y = 0$ for $0 < q < \dim Y - 1$. From these facts, we are led to consider the following question; "Is it possible to describe the modules $(R^q\Phi_*(O_{Y'}))_y$ by purely ring-theoretic data of $O_{Y,y}$?" The first aim of this paper is to show that that is indeed the case if y is an isolated singularity with C*-action.

If (Y,y) has a C*-action, we can attach a normal graded ring R to (Y,y). Hereafter, we will use the langauge of graded rings and graded modules. To get the result, we consider the space $\mathrm{Proj}(R^\natural)$ defined in E.G.A. Chap.II,§ 8. As this space is described directly from R, we can describe the geometric properties of this space from the ring-theoretic properties of R and there is a proper birational map from $\mathrm{Proj}(R^\natural)$ to Spec(R). If we know that $\mathrm{Proj}(R^\natural)$ has only rational singularities, then we can express $R^q\Phi_*(O_{Y'})$ by the cohomology groups of $\mathrm{Proj}(R^{\natural})$ and hence by the local cohomology groups of R. This idea was carried out by Pinkham [P] when dim R = 2 and by the author [W_1] in some special cases. The essential part of the proof is to show $\mathrm{Proj}(R^{\natural})$ has only rational singularities under the assumption that $\mathrm{Spec}(O_{Y,y}) - \{y\}$ has only rational singularities.

It was shown by Demazure [D] that the normal graded ring R is characterized by X = Proj(R) and a "rational coefficient Weil divisor" D on X (we will write R = R(X,D) in this case). This expression is very helpful to construct examples of graded rings. In §3, we will give a condition for R(X,D) to be a canonical singularity in terms of (X,D).

By our theorem, R(X,D) is a rational singularity if and only if $H^q(X,nD) = 0$ for $q > 0$ and $n \geqq 0$, once we know that $\mathrm{Spec}(R) - \{y\}$ has only rational singularities. But the condition "$\mathrm{Spec}(R) - \{y\}$ has only rational singularities" is a rather delicate one if the fractional part of D has singular points even if X is smooth. We will give some partial answers to this problem in §4.

§ 1. Notations and preliminaries.

In this paper, we will use the following notations.

(1.1) Notation. k is a field of characteristic 0.

$R = \bigoplus_{n \geqq 0} R_n$ is a Noetherian normal graded ring with $R_0 = k$.

$\underline{m} = R_+ = \bigoplus_{n>0} R_n$, the unique graded maximal ideal of R.

If $M = \bigoplus_{n \in Z} M_n$ is a graded R-module or $S = \bigoplus_{n \in Z} S_n$ is a graded ring, we put $M|_t = \bigoplus_{n \geqq t} M_n$ or $S|_0 = \bigoplus_{n \geqq 0} S_n$. Note that $S|_0$ is again a graded ring.

$Y = \mathrm{Spec}(R)$ and $y = \{m\} \in Y$.

$U = Y - \{y\}$.

$\Phi: Y' \longrightarrow Y$ is a resolution of the singularities of Y. (In this paper, we don't treat Y' itself. We only treat the higher direct images $R^p\Phi_*(O_{Y'})$ $(p>0)$, which are independent of the resolution chosen.)

$X = \mathrm{Proj}(R)$, $O_X(n) = \tilde{R}(n)$.

Note that X is a normal projective variety over k, since R is a normal graded ring finitely generated over k.

$H^q_{\underline{m}}(R)$ is the q-th local cohomology group of R with respect to $\underline{m}$. Note that $H^q_{\underline{m}}(R)$ has the natural structure of a graded R-module.

$K_R = (H^d_{\underline{m}}(R))^*$ $(d = \dim(R))$ is the canonical module of R. (See [G.W], p.184, for the definition of the functor $(.)^*$.)

$a(R) = \max\{n \mid (H^d_{\underline{m}}(R))_n \neq 0\} = -\min\{n \mid (K_R)_n \neq 0\}$.

Proposition (1.2) ([H.R], §5). There is a canonical isomorphism of graded R-modules

$$H^q_{\underline{m}}(R) \cong \bigoplus_{n \in Z} H^{q-1}(X, O_X(n)) \quad (q \geqq 2).$$

As is shown in [D], the graded ring R is determined by X and a rational coefficient Weil divisor D. Namely,

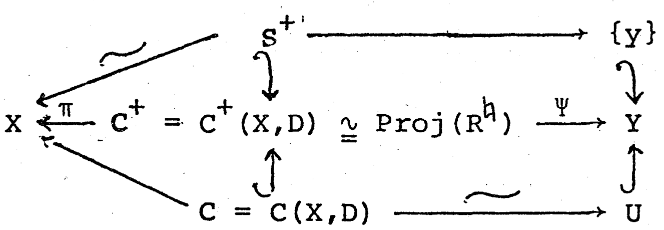
Theorem (1.3) ([D], 3.5). If T is a homogeneous element of degree 1 in the quotient field of R (we may assume the existence of $T \neq 0$ by adjusting the grading of R, if necessary), then there exists unique divisor $D \in \mathrm{Div}(X,Q) = \mathrm{Div}(X) \otimes Q$, such that ND is an ample Cartier divisor for some positive integer N and

$$R_n = H^0(X, O_X(nD)).T^n$$

for every $n \geqq 0$. (For the definition of $O_X(nD)$, see [D], §1 or $[W_1]$.)

We will fix this D and we will write R = R(X,D) if we want to specify D. If R = R(X,D), then $O_X(n) \cong O_X(nD)$ for every integer n ($[W_1]$, (2.1)).

(1.4) We have the following commutative diagram of morphisms;



where $C^+(X,D) = \mathrm{Spec}_X(\bigoplus_{n \geqq 0} O_X(n))$, $C(X,D) = \mathrm{Spec}_X(\bigoplus_{n \in Z} O_X(n))$ is an open subset of $C^+(X,D)$ and $S^+ = C^+ - C$. The morphism Ψ is a projective morphism, $\Psi(S^+) = \{y\}$ and the restriction of Ψ on C is an isomorphism. The graded ring $R^\natural$ is defined in E.G.A. Chap. II, §8. (It is easy to see that C^+ is naturally isomorphic to $\mathrm{Proj}(R^\natural)$.) The morphism π is the structure morphism and, so, is an affine morphism. The restriction of π to S^+ is an isomorphism. Watching this diagram, we have the isomorphisms

$$R^q\Psi_*(O_{C^+}) \cong H^q(C^+, O_{C^+}) \cong \bigoplus_{n \geqq 0} H^q(X, O_X(n)) \cong H^{q+1}_{\underline{m}}(R)|_0.$$

§ 2. A criterion for rational singularities.

In this section, we will give a necessary and sufficient condition for R to be a rational singularity.

Definition (2.1). Let Z be a normal scheme essentially of finite type over k and let f: Z' $\longrightarrow$ Z be a resolution of singularities of Z. We say Z has rational singularities if $R^i f_*(O_{Z'}) = 0$ for i>0. Note that, in our terminology, Z has rational sigularities if Z is smooth.

We say a ring A is a rational singularity if Spec(A) has rational singularities.

Theorem (2.2). R is a rational singularity if and only if the following conditions hold; (i) U = Spec(R) - $\{\underline{m}\}$ has rational singularities. (ii) R is a Cohen-Macaulay ring. (iii) a(R) < 0.

(Proof) Let Ψ: $C^+ \longrightarrow Y$ be as in (1.4). The conditions (ii) and (iii) implies that $R^q\Psi_*(O_{C^+}) = 0$ for q>0. Now, we may assume that the resolution Φ factors through Ψ. Put $\Phi = \Theta\circ\Psi$. If C^+ has rational singularities, then by Leray spectral sequence, we have $R^q\Phi_*(O_{Y'}) \cong R^q\Psi_*(O_{C^+})$ for every q and R is a rational singularity.

Conversely, if R is a rational singularity, it is known that R is Cohen-Macaulay and the condition (i) is trivial. The condition (iii) follows from (1.4) and the above argument if we can prove that C^+ has rational singularities. So, it suffices to prove

Lemma (2.3). If U has rational singularities, so does C^+.

To prove this lemma, we need the following

Theorem (2.4) (Boutot). Let B be a k-algebra essentially of finite type over k and let A be a subalgebra of B which is a direct summand of B as an A-module. If B is a rational singularity, so is A. (The author heard the proof of this theorem orally from M. Hochster and does not know where the proof is to be published.)

(Proof of (2.3)) As the condition is local, it suffices to prove that $\pi^{-1}(W)$ has rational singularities for every affine open set W of X. As $C^+ = \mathrm{Spec}_X(\oplus_{n\geqq 0} O_X(n))$ and $U \cong C = \mathrm{Spec}_X(\oplus_{n\in Z} O_X(n))$, it suffices to prove the following

Lemma (2.5). If $S = \oplus_{n\in Z} S_n$ is a graded k-algebra essentially of finite type over k and if S has rational singularities, then so does S_0 and $S|_0 = \oplus_{n\geqq 0} S_n$.

(Proof) As S_0 is a direct summand of S as an S_0-module, S_0 has rational singularities by (2.4). If we take a variable T over S and if we put deg(T) = -1 in the graded ring S[T], it is easy to see that $S|_0$ is isomorphic to $(S[T])_0$ as graded k-algebras and the latter has rational singularities since so does S[T]. This concludes the proof of Theorem (2.2).

Corollary (2.6). If U has rational singularities, then $R^q\Phi_*(O_{Y'}) \cong (H_{\mathfrak{m}}^{q+1}(R))|_0$ for $q\geqq 1$.

(Proof) If U has rational singularities, so does C^+ by (2.3) and we have $R^q\Phi_*(O_{Y'}) \cong R^q\Psi_*(O_{C^+})$. On the other hand, we have seen in (1.4) that the latter is isomorphic to $(H_{\mathfrak{m}}^{q+1}(R))|_0$ for $q\geqq 1$.

Remark. If U has rational singularities, then $H_{\mathfrak{m}}^q(R)$ (q<d) has finite length and $H_{\mathfrak{m}}^q(R) \cong R^{q-1}\Phi_*(O_{Y'})$ by the duality theorem and Grauert-Riemenschneider vanishing theorem. As $H_{\mathfrak{m}}^q(R) \cong \oplus_{n\in Z} H^{q-1}(X, O_X(nD))$, we have

"If X is a normal projective variety over k and if D is a rational coefficient Weil divisor satisfying the conditions (a) ND is an ample Cartier divisor for some positive integer N (b) C(X,D) has rational singularities, then $H^q(X,O_X(nD)) = 0$ for $q < \dim X$ and $n<0$."

Definition (2.7). If U has rational singularities, we define

$$p_g(R) = \dim_k R^{d-1}\Phi_*(O_{Y'})$$

and we will call it the geometric genus of R. Note that if U has rational singularities, R is a rational singularity if and only if R is Cohen-Macaulay and $p_g(R) = 0$.

Corollary (2.8). If R is a Gorenstein ring and if U has rational singularities, then $p_g(R) = \sum_{n=0}^{a(R)} \dim_k R_n$.

(Proof) In this case, $R^{d-1}\Phi_*(O_{Y'}) \cong H^d_{\underline{m}}(R)|_0 \cong R^*(-a(R))|_0$ by (2.6) and [G.W], (3.1.4).

Remark (2.9). If $S = \bigoplus_{n\geq 0} S_n$ is a Cohen-Macaulay graded ring with $S_0 = k$ and if $x \in S_m$ is an S-regular element, then $a(S) = a(S/xS) - m$. So the invariant a(S) can be computed from that of an Artinian ring and if S is Artinian $a(S) = \max\{n | S_n \neq 0\}$. Also, if $R = k[X_1,\ldots,X_d]$ is a polynomial ring, then $a(R) = -\sum_{i=1}^{d} \deg(X_i)$ ([G.W], (2.2.8) and (2.2.10)).

Corollary (2.10). Let $R = k[X_1,\ldots,X_{d+s}]/(f_1,\ldots,f_s)$ be a complete intersection and assume that U has rational singularities. Then, R is a rational singularity if and only if $\sum_{j=1}^{s}\deg(f_j) < \sum_{i=1}^{d+s}\deg(X_i)$.

(The geometric genus of R is computed in $[W_2]$ if R is a hypersurface with isolated singularity.)

Remark (2.11). If U has rational singularities, then so does X by (2.5). But U may have non-rational singularities even if X is smooth. We will treat this problem in §4.

Remark (2.12). To prove that R is a rational singularity, it suffices to show that $R_{\underline{m}}$ is a rational singularity, since "rational singularity" is an open condition and Y has a k*-action.

§ 3. A criterion for canonical singularities.

In this section, we put $R = R(X,D)$, where $D = \sum p_V/q_V \varepsilon \mathrm{Div}(X)\otimes Q$. (We assume that p_V and q_V are relatively prime integers for every V, $q_V>0$ and ND is an ample Cartier divisor for some positive integer N. We will fix p_V, q_V and N for this meaning throughout this paper.)

For a normal variety Z, we denote by ω_Z the dualizing sheaf of Z defined by $\omega_Z = i_*(\Omega^n_{\mathrm{Reg}(Z)})$, where $i: \mathrm{Reg}(Z) \longrightarrow Z$ is the inclusion map and $n = \dim Z$. As ω_Z is a divisorial O_Z-Module, we can write $\omega_Z = O_Z(K_Z)$ for some $K_Z \varepsilon \mathrm{Div}(Z)$. We will call K_Z the canonical divisor of Z. Also, we put $\omega_Z^{[r]} = i_*((\Omega^n_{\mathrm{Reg}(Z)})^{\otimes r}) \cong O_Z(rK_Z)$.

Definition (3.1)(M.Reid). (Y,y) is a canonical singularity if it satisfies the following conditions:

(i) $\omega^{[r]}_{Y,y}$ is an invertible $O_{Y,y}$-module for some integer r.

(ii) $\Phi_*(\omega^{\otimes r}_{Y'})_y = \omega^{[r]}_{Y,y}$.

Recently, R.Elkik proved that a canonical singularity is a rational singularity ([E_2]) and a rational Gorenstein singularity is a canonical singularity by definition.

Proposition (3.2). If $R = R(X,D)$ is a canonical singularity, then $r(K_X+D') = a'D$ for some positive integer r and an integer a' with $a' \leqq -r$, where we put $D' = \sum (q_V-1)/q_V$.

(Proof) As $\Phi_*(\omega^{\otimes r}_{Y'}) \subset \Psi_*(\omega^{[r]}_{C^+}) \subset \omega^{[r]}_Y$, it suffices to prove the following two lemmas.

Lemma (3.3). $\omega^{[r]}_Y$ is invertible if and only if $r(K_X+D')$ is linearly equivalent to $a'D$ for some integer a'.

(Proof) This follows from the calculation of Cl(R) and $cl(\omega_R)$. (cf. (1.6) and (2.9) of $[W_1]$.)

Lemma (3.4). (i) $\omega_{C^+} \cong \bigoplus_{n>0} O_X(K_X+D'+nD)$. In other words, $K_{C^+} \equiv -S^+ + \pi^*(K_X+D')$.

(ii) If $r.(K_X+D') \equiv a'D$, then $\Psi^*(\omega_R^{[r]}) \cong O_{C^+}(r.\pi^*(K_X+D')+a'S^+)$ $= \omega_{C^+}((a'+r)S^+)$.

(Proof) (i) If D is a Cartier divisor, then C^+ is an A^1-bundle over X and this formula is well known. In general, if we put $C^+_{(N)} = \mathrm{Spec}_X(\bigoplus_{n\geqq 0} O_X(nND))$, then $C^+_{(N)}$ is an A^1-bundle over X, since ND is a Cartier divisor by the assumption and we can get ω_{C^+} by $\omega_{C^+} \cong \mathrm{Hom}_{O_{C^+_{(N)}}}(O_{C^+}, \omega_{C^+_{(N)}})$.

(ii) As $\omega_R^{[r]}$ is equal to $\omega_{C^+}^{[r]}$ on $U = C^+ - S^+$, $\Psi^*(\omega_R^{[r]})$ $= \omega_{C^+}^{[r]}(bS^+) \cong O_{C^+}(r.\pi^*(K_X+D')+(b-r)S^+)$ for some integer b. On the other hand, the restriction of $\Psi^*(\omega_R^{[r]})$ to S^+ is trivial, since S^+ contracts to a point by Ψ. As S^+ is linearly equivalent to $-\pi^*(D)$ ([D],2.9), and as $r.(K_X+D') = a'D$, we get $\Psi^*(\omega_R^{[r]})$ $\cong O_{C^+}((b-a'-r)S^+) \cong O_{C^+}(\pi^*((a'+r-b)D)$. This implies $b = a'+r$. (In fact, since $\Psi^*(\omega_R^{[r]})$ is invertible, $(a'+r-b)D$ is an integral divisor and if nD is an integral divisor, $O_X(nD) \cong O_X$ implies $n=0$, since ND is ample.)

Corollary (3.5). (i) $\Psi_*(\omega_{C^+}) = \omega_R$ if and only if $a(R) < 0$.

(ii) If $r.(K_X+D') \equiv a'D$ for some integer a', then $\Psi_*(\omega_{C^+}^{[r]})$ $= \omega_R^{[r]}$ if and only if $a' \leqq -r$.

(Proof) (i) By $[W_1]$, (2.8), $\omega_R \cong \bigoplus_{n\in Z} H^0(X, O_X(K_X+D'+nD))$

and $\Psi_*(\omega_{C^+}) \cong \underset{n>0}{\oplus} H^0(X, O_X(K_X + D' + nD)) \cong \omega_R|_1$ by (3.4). Thus $\Psi_*(\omega_{C^+}) = \omega_R$ if and only if $a(R)<0$ by the definition of $a(R)$.

(ii) If $r.(K_X + D') \equiv a'D$, then $\omega_R^{[r]} \cong R(a')$ as graded R-modules and $\Psi_*(\omega_{C^+}^{[r]}) \cong \underset{n\geq r}{\oplus} H^0(X, O_X(r(K_X+D')+nD)) \cong \underset{n\geq r}{\oplus} H^0(X, O_X((a'+n)D))$. Thus $\Psi_*(\omega_{C^+}^{[r]}) = \omega_R^{[r]}$ if and only if $a'+r \leq 0$.

Corollary (3.6). If $C^+(X,D)$ has canonical singularities and and if $r.(K_X+D') \equiv a'D$ for some positive integer r and for some integer a' with $a' \leq -r$, then $R = R(X,D)$ is a canonical singularity.

Corollary (3.7). If X and D satisfies one of the following conditions and if $r.(K_X+D') \equiv a'D$ for some positive integer r and an integer $a' \leq -r$, then $R = R(X,D)$ is a canonical singularity.

(i) X has canonical singularities and D is an ample Cartier divisor (in this case, $D' = 0$).

(ii) U has rational singularities, X is a factorial variety and $p_V \equiv 1$ (mod. q_V) for every V.

(Proof) By (3.6), it suffices to prove that $C^+(X,D)$ has canonical singularities in both cases. In case (i), C^+ is an A^1-bundle over X. In case (ii), C^+ has rational singularities by (2.3) and C^+ is a Gorenstein variety since ω_{C^+} is invertible by our conditions and by (3.4). So, in both cases, C^+ has canonical singularities.

Remark. The conditions of (3.2) are not the sufficient condition for R to be a canonical singularity, even if we assume that U has canonical singularities. For example, if $X = P^1$ and if $D = 3/4.P$ for some point $P \in P^1$, $K_X \equiv -2P$ and we have $3(K_X+D') \equiv -5D$. In this case, U is smooth and (X,D) satisfies the conditions of (3.2). But R(X,D) is not a canonical singularity since R(X,D) is not a

Gorenstein ring and two-dimensional canonical singularities are Gorenstein rings (cf. [R]).

§ 4. Some criteria for U to have rational singularities.

If we want to construct rational singularities with C*-action, it is important to know the condition for $U = C(X,D)$ to have rational singularities by (2.2). So we consider the problem; "Find the conditions on (X,D) for C(X,D) to have rational singularities (or to be smooth). Although we cannot give the complete answer, we can solve the problem in some useful cases.

(4.1) ($[W_1]$, (3.1)). If (X,D) satisfies the following conditions, then C(X,D) has rational singularities.

(i) X has rational singularities.

(ii) If $x \in X$ is a singular point of X, D is a Cartier divisor on some neighborhood of x.

(iii) If $x \in X$ is a smooth point of X, $\mathrm{Supp}(D - \llcorner D \lrcorner)$ is normal crossing at x.

Proposition (4.2). Take a point $x \in X$ and put $A = O_{X,x}$. If D is linearly equivalent to $\frac{p}{q}.V$, where V is a Cartier divisor near x defined by $f \in A$ (we assume p and q are relatively prime and $0<p<q$), there is an etale covering from $\mathrm{Spec}(A[Z]/(Z^q - f)[T,T^{-1}])$ onto $C(X,D) \times_X \mathrm{Spec}(A)$. (Where Z and T are variables over A.)

(Proof) Put $C(X,D) \times_X \mathrm{Spec}(A) = \mathrm{Spec}(B)$, where $B = \bigoplus_{n \in Z} O_X(nD)_x = A[f^a T^n \mid qa+np \geq 0]$. If we take p' $(0<p'<q)$ so that $pp'+bq=1$ for some integer b, it is easy to see that $B = A[f^b T^{p'}, f^p T^q, (f^p T^q)^{-1}] \cong A[u,v,v^{-1}]/(u^q - fv^{p'})$. If we define the A-homomorphism

ϕ: B $\longrightarrow$ $A[Z,T,T^{-1}]/(Z^q-f)$ by putting $\phi(u) = ZT^{p'}$ and $\phi(v) = T^q$, ϕ is an etale homomorphism and we get the desired result.

Corollary (4.3). Under the assumptions of (4.2),

(i) $C(X,D)\times_X \mathrm{Spec}(A)$ has rational singularities if and only if $\mathrm{Spec}(A[Z]/(Z^q-f))$ has rational singularities.

(ii) $C(X,D)\times_X \mathrm{Spec}(A)$ is smooth if and only if X and V are smooth at x or X is smooth at x and q=1.

Corollary (4.4). Let us assume that k is algebraically closed and (X,D) satisfies the conditions of (4.2) at x. If X is smooth at x, dim X = 2, q>1 and if V is singular at x, then $C(X,D)\times_X \mathrm{Spec}(A)$ has rational singularities if and only if the singularity of V at x is analytically isomorphic to one of the following ones.

(i) If $q \geq 6$, $xy=0$.

(ii) If q=5,4, $xy=0$ or $x^2=y^3$.

(iii) If q=3, $x^2=y^n$ (n=2,3,4,5).

(iv) If q=2, $x^2=y^n$ $(n\geq 2)$, $x(y^2-x^n)=0$ $(n\geq 2)$, $x^3=y^n$ (n=3,4,5), or $y(y^2-x^3)=0$.

(Proof) Straightforward from (4.3) and the classification of rational Gorenstein surface singularities. (cf.[A])

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