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A REMARK ON PLANARITY

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A classical theorem of Hassler Whitney states that a 3-connected planar graph G can be drawn in the plane in essentially one way, meaning that the circuits that bound faces are uniquely determined by the structure of G. [3]. The simplest way to prove this is to characterize those circuits combinatorially. In [2] they are identified with the peripheral circuits of G, that is circuits having only one bridge. In [1] such circuits are called non-separating: if a peripheral circuit is contracted into a single vertex the graph G remains non-separable.

A convex representation of G is a drawing of G in the plane in which each edge is represented by a straight segment and each face is the interior or exterior of a convex polygon. Moreover the vertices of the bounding circuit of the face are to be drawn as the vertices of the convex polygon. Of course only one face, the outer or infinite face, is the exterior of a convex polygon.

In the theory of convex representations of G we first suppose that a circuit J of G, necessarily peripheral, is chosen to the boundary of the outer face. It is then supposed that J

is drawn in the plane as the boundary of an arbitrary convex
polygon Q, with appropriate number of vertices. The drawing of
J is defined by a 1 - 1 mapping of the vertices of J onto Q,
preserving cyclic order.

The mapping of f is now supposed to be extended as a map-
ping of the vertex-set V(G) of G onto some set of points of the
plane. Initially the extended mapping f need not be one to one.
We call J the <u>frame</u> of the extended mapping. We now ask such
questions as "Can the extended mapping f be chosen so as to
determine a convex representation of G?" and "What restrictions
on f will ensure that it defines a convex representation?"
The answer to the first of these questions is "Yes". See for
example the existence proof in [1]. A partial answer to the
second question is found in [2]: if each vertex not in J is
drawn at the centroid of the representative points of its
neighbours in G, then a convex representation results. Moreover
such a "barycentric" representation can indeed be found as the
solution of a set of linear equations.

It seems convenient to point out here one obvious property
of convex representations. Let us say that the mapping f is
<u>enclosing</u> if it satisfies the following condition: for each
vertex v of G not in J the point f(v) lies in the interior of
some convex polygon whose vertices are the representative points
of some of the neighbours of v. Clearly if f defines a convex
representation of G, then f is an enclosing mapping.

Let us make another definition. We call f a <u>balanced</u>
mapping if for each vertex v of G not in J the following

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assertion is true: if a line through f(v) has the representative point of a neighbour of v on one side, then it has the representative point of another neighbour of v on the other side. Clearly every enclosing mapping is balanced and every barycentric mapping is balanced.

Consider the proof in [2] that a barycentric mapping determines a convex representation. The only property of a barycentric mapping used in this proof is that of being balanced. Accordingly, after a few verbal changes, the proof shows that if f is balanced it determines a convex representation of G. Incidentally we see that for the 3-connected graph G a balanced mapping is necessarily an enclosing one.

The purpose of this Note is to draw attention to the following theorem.

THEOREM. The mapping of f, with frame J, of G into the plane defines a convex representation of G if and only if it is enclosing(or, equivalently, balanced).

It seems to the author that this theorem is worthy of explicit statement, but he is not aware of any such explicit statement in the literature.

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