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本書を

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に捧ぐ

SYMPOSIUM ON COMBINATORIAL STRUCTURES OF CONFIGURATIONS

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Organizer : Sanpei Kageyama, Department of Mathematics, Faculty of School Education, Hiroshima University

This book is dedicated to the memory of Professor Goro Ishii.

PROGRAM AND ABSTRACT

1. T. Atsumi (Kagoshima University)

Johnson scheme and the fundamental relations of t-designs.

The following results are given: Theorem 1. $V_h = W_{h,h}M_h$ is an eigenspace for $M_\ell^T M_\ell$ ($\ell = 1,\ldots,t-h$). Theorem 2. Let (P, $\mathbb{B}$) be a t-(v,k,1) design (i.e., Steiner system) such that for any block B of $\mathbb{B}$, the set of points outside B, with blocks of the form $B' - (B \cap B')$ where B' is a block with $|B \cap B'| = t-1$ is a 2-(v-k,k-t+1,c) design for some integer c. Then $W_{2,2}M_2$ is an eigenspace for the adjacency matrices of the Steiner system (P, $\mathbb{B}$).

2. N. Ito (University of Illinois at Chicago Circle)

Classifications of Hadamard 2-designs I, II.

There exist 1102 non-isomorphic 2-(23,11,5) designs (see [4]). Based on this classification we consider a few ideas on the classification of Hadamard 2-designs.

3. M. Jimbo (Science University of Tokyo)
Cyclic neofields and cyclic 2-designs.

There is the close relation between the concept of the neofields and that of the 2-designs with block size 3. In this paper we shall show that the existence of the cyclic 2-designs of block size 4 is equivalent to that of some cyclic neofields.

4. N. Hamada (Hiroshima University)
The geometric structure and the p-rank of an affine triple system derived from a Moufang loop.

H.P. Young showed that there is a one to one correspondence between affine triple systems and exp. 3-Moufang loops (ML). Recently, L. Beneteau showed that (i) $3 \leqq |Z(E)| \leqq 3^{n-3}$ for any non-associative exp. 3-ML $(E,\cdot)$ with $|E| = 3^n$ where $n \geqq 4$ and $Z(E)$ is an associative center of $(E,\cdot)$ and (ii) there exists exactly one exp. 3-ML, denoted by $(E_n,\cdot)$, such that $|E_n| = 3^n$ and $|Z(E_n)| = 3^{n-3}$ for any integer $n \geqq 4$. The purpose of this paper is to investigate the geometric structure of the affine triple system derived from the exp. 3-ML $(E_n,\cdot)$ using the transitivity of the parallelism and subsystems and to compare with the structure of an affine geometry AG(n,3).

5. J. Ogawa (University of Calgary)

Non-existence of certain block designs.

In the non-existence proofs of symmetrical BIBD and PBIBD, a more or less systematic method is the use of the Hasse-Minkowski p-invariant under the rational congruence of quadratic forms. In this article an exposition of the Hasse-Minkowski p-invariant and its use for deriving necessary conditions for existence of symmetrical BIBD and symmetrical and regular PBIBD are presented.

6. K. Ushio (Niihama Technical College)

On bipartite decomposition of a complete bipartite graph.

A complete bipartite graph K_{n_1,n_2} is said to have a bipartite decomposition if it can be decomposed into a union of line-disjoint subgraphs each isomorphic to a complete bipartite graph K_{k_1,k_2}. In this paper, a theorem which states a necessary and sufficient condition for a complete bipartite graph K_{n_1,n_2} to have a bipartite decomposition is given. And several corollaries are also given.

7. S. Kageyama (Hiroshima University) and T. Tanaka (Hatsukaichi High School)

On group divisible designs.

Generalizing methods of constructions of Hadamard group divisible designs due to Bush (1979), some new families of semi-regular or regular group divisible designs are produced. Furthermore, new nonisomorphic solutions for some known group divisible designs are given, together with useful group divisible designs not listed in Clatworthy (1973).

8. K. Sawada (Nagoya Institute of Technology)
T-matrices of a special form.

It seems that recent advances in the construction of Hadamard matrices owe much to Baumert-Hall array and Williamson matrices. In order to construct a new Baumert-Hall array, Cooper and Wallis considered T-matrices. A T-matrix is, by definition, a set of four circulant (1,0,-1)-matrices X_1, X_2, X_3, X_4 of order n, such that only one of a_k, b_k, c_k and d_k is non-zero, where a_k, b_k, c_k, d_k $(k=0,1,\ldots,n-1)$ denote the first rows of X_1, X_2, X_3, X_4 respectively, and satisfies $X_1X_1^t+X_2X_2^t+X_3X_3^t+X_4X_4^t=nI_n$. Cooper and Wallis showed that if there exist T-matrices of order n, then a Baumert-Hall array of the same order can be constructed by using them.

In this paper we consider T-matrices of prime order $p \equiv 1 \pmod 3$, of a special form. Such a p can be uniquely written as $\alpha^2+3\beta^2$, $\alpha \equiv 1 \pmod 3$, $\beta > 0$. Let ζ be a primitive p-th root of unity, γ a primitive root of p, w an element satisfying $w \not\equiv 1$, $w^3 \equiv 1 \pmod p$, and $F_1(x)$, $F_2(x)$, $F_3(x)$, $F_4(x)$ generating functions of X_1, X_2, X_3, X_4 respectively. Then $H=\langle\tau\rangle$ for the automorphism $\tau:\zeta \longrightarrow \zeta^w$ is a subgroup of order 3 of Galois group of $Q(\zeta)/Q$. We consider T-matrices satisfying the following conditions: (i) $F_1(\zeta)$ is invariant under H; (ii) $F_2(\zeta)$, $F_3(\zeta)$, $F_4(\zeta)$ are transformed cyclically by τ, in this order; (iii) $F_1(1)=\alpha$ and $F_2(1)=F_3(1)=$ $=F_4(1)=\pm\beta$; (iv) $F_1(\zeta)F_1(\zeta^{-1})+F_2(\zeta)F_2(\zeta^{-1})+F_3(\zeta)F_3(\zeta^{-1})+F_4(\zeta)F_4(\zeta^{-1})$ $=p$.

We conjectured the existence of T-matrices of the above form and have searched for them for p=13, 19, 31 and 37 by an electronic computer. Our result is that there exist much more T-matrices of the above form, compared to the usual Williamson matrices in our form, appreciably more than we have expected.

9. M. Yamada (Tokyo Woman's Christian University)
On the Goethals-Seidel matrices of a special type.

Whiteman proved the following theorem. If p is a prime and $q=2p-1$ is a prime power, then there exists an Hadamard matrix of order $4(2p+1)$. I.e. Let A,B,C,D be circulant matrices of order 2n with elements ± 1, let the number of 1's in any row or column of A be n-1, and the number of 1's in any row or column of B,C,D be n. If

$$A^tA + B^tB + C^tC + D^tD = 4(2n+1)I - 4J, \qquad (1)$$

then

$$H = \begin{pmatrix} A & BR & CR & DR & X \\ -BR & A & -{}^tDR & {}^tCR & Y \\ -CR & {}^tDR & A & -{}^tBR & Z \\ -DR & -{}^tCR & {}^tBR & A & W \\ -{}^tX & {}^tY & {}^tZ & {}^tW & K \end{pmatrix}$$

is an Hadamard matrix of order 4(2n+1), where J is the square matrix of order 2n with every element 1, R a fundamental back circulant matrix of order 2n, X,Y,Z,W 2n×4 matrices defined by $X=(w,w,w,w)$, $Y=(w,w,-w,-w)$, $Z=(w,-w,w,-w)$, $W=(-w,w,w,-w)$ with the column vector w consisting of 1's, and K the circulant matrix of order 4 whose top row is (1,-1,-1,-1).

We interprete his construction by using the theory of a finite field. We let: I_2 the unit matrix of order 2, I_p the unit matrix of order p, T a fundamental circulant matrix of order 2, T_p a fundamental circulant matrix of order p, ξ a primitive element of $GF(q^2)$, ζ_p a p-th root of unity, χ the Legendre character of GF(q), S the trace from $GF(q^2)$ to GF(q), and ψ the quadratic chracter modulo p. We define $a_r=\chi(2)\chi(S\xi^{r-p})$, $b_r=\chi(2)\chi(S\xi^r)$, and A,B,C,D by $A=-(I_2+T)\otimes I_p+(I_2-T)\otimes\sum_{r=1}^{p-1}a_{4r}T_p^r$, $B=(I_2-T)\otimes\sum_{r=0}^{p-1}b_{4r}T_p^r$, $C=D=(I_2+T)\otimes\sum_{r=1}^{p-1}\psi(r)T_p^r+(I_2-T)\otimes I_p$. Furthermore we define $Q=B+i^{-p}A$, $U=(1+i)C$, so that the left-hand side member of the equation (1) can be expressed as QQ^*+UU^*. Diagonalizing Q, we have $\theta=\sum_{r=0}^{p-1}(-1)^r\{\chi(S\xi^{2r})+i^p\chi(S\xi^{2r+p})\}\zeta_p^r$ for the eigenvalue 1 of T and diagonalizing U, we have $\eta=\sum_{r=1}^{p-1}\psi(r)\zeta_p^r$ for the eigenvalue -1. We know that θ is the ratio of a Gauss sum over $GF(q^2)$ to a Gauss sum over GF(q), and η is a Gauss sum over GF(p). The diagonal elements of QQ^* and UU^* when diagonalized, are 4q and 8 for the eigenvalue 1 of T, and 4 and 8p for the eigenvalue -1, but the sum of these is 4(2p+1) in both cases.

10. K. Yamamoto (Tokyo Woman's Christian University)

On the Williamson equation generalized.

If A,B,C,D are circulant (1,-1)-matrices of order n satisfying $AA^*+BB^*+CC^*+DD^*=4nI$, then they will give rise to an Hadamard matrix of order 4n of the Goethals-Seidel type. Assume n be odd and denote the first row of A by $(a_0,a_1,\dots,a_{n-1})$ and similarly for B,C and D. If $a_0=b_0=c_0=d_0=1$ and $a_ib_ic_id_i=-1$ for $i\geq 1$, then the above equation is written as

$$(1+2\sum_{m\in A} e_m x^m)(1+2\sum_{m\in A} e_m x^{-m})+(1+2\sum_{m\in B} e_m x^m)(1+2\sum_{m\in B} e_m x^{-m})+$$
$$+(1+2\sum_{m\in C} e_m x^m)(1+2\sum_{m\in C} e_m x^{-m})+(1+2\sum_{m\in D} e_m x^m)(1+2\sum_{m\in D} e_m x^{-m}) \equiv 4n \pmod{x^n-1},$$

a generalization of the Williamson equation, where A,B,C,D is now a partition of the set $\{1,2,\dots,n-1\}$. The above equation seems to have much more solutions compared to the usual Williamson equation.