

Geometric Invariant Theory

(\*) " The story starts with nineteenth century invariant theory, which was concerned with the determination of the invariants of "quantics" (in our language, invariants for the action of  $SL(m, \mathbf{C})$  given by the symmetric power representations). The first step of relevance to us was the proof by Gordon [5] of the finite generation of such invariants in the case  $m = 2$ , using the methods that had been developed for computational purposes. This result was subsequently extended to arbitrary  $m$  by Hilbert [6]; Hilbert's proof depended on his basis theorem (proved in the same paper) and the use of Cayley's  $\Omega$ -process for constructing a projection from the ring  $\mathbf{C}[X_1, \dots, X_n]$  onto the subring of elements invariant under the action of  $SL(m, \mathbf{C})$ .

Various attempts were made in succeeding years to extend Hilbert's result to the natural action of an arbitrary subgroup of  $GL(n, \mathbf{C})$  on  $\mathbf{C}[X_1, \dots, X_n]$ . (See, for example, Hurwitz [9], Fischer [3] for some interesting special cases, and Noether [12,13] for a complete solution in the case of a finite subgroup; this last is valid for any  $k$ . Unsuccessful attempts on the general problem may be found in Maurer [10,11] and Weitzenböck [14], although the latter contains a correct proof for a subgroup isomorphic to  $\mathbf{C}$  or  $\mathbf{C}^*$ .) The most significant contribution was that of H. Weyl, whose main results (cf. Weyl [15]) can be stated in our language as follows:

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(\*) The following is quoted from Newstead [135; § 6 of Chap. 3]. We shall make this rather long quotation because this is a good and brief account on the development of the invariant theory.

- (i) any connected semi-simple group over  $\mathbf{C}$  is linearly reductive;
- (ii) the ring of invariants for a linear action of such a group is finitely generated.

Weyl's original methods combined the integral procedure developed by Hurwitz and Schur and the infinitesimal methods of E. Cartan; subsequently he obtained similar results by algebraic methods, valid for any algebraically closed field of characteristic 0 (cf. Weyl [16]). In 1933, part of Weyl's argument was simplified and generalized by M. Schiffer, who gave a simple algebraic argument (replacing the use of Cayley's  $\mathcal{Q}$ -process) by which (ii) can be deduced from (i) (cf. Weyl [16; 2nd ed. Supplement C]). Schiffer stated his result in a form adapted to the classical case, but his argument is valid generally and indeed in any characteristic; it extends without difficulty to give the following more general result.

THEOREM. For any rational action of a linearly reductive group  $G$  on a finitely generated  $k$ -algebra  $R$ ,  $R^G$  is finitely generated.

(A detailed proof of this theorem on the lines of Schiffer's argument may be found in Fogarty [27] or Mumford-Suominen [120] and also, in a slightly disguised form, in Mumford [116]. An alternative proof, due to Nagata, appears in Nagata [64] and Dieudonné-Carrell [25]).

The next advance concerned the construction of counter-examples. In 1900 Hilbert had proposed as his fourteenth problem the following question (cf. Hilbert [8]):

Is  $L \cap k[X_1, \dots, X_n]$  a finitely generated  $k$ -algebra for every subfield  $L$  of  $k(X_1, \dots, X_n)$  ?

(Clearly this contains the problem of the finite generation of  $k[X_1, \dots, X_n]^G$  for a subgroup  $G$  of  $GL(n)$ , though it seems that at the time Hilbert believed that Maurer had solved this problem, at least in the case  $k = \mathbf{C}$ .) In 1953 Zariski proposed a generalization of Hilbert's question and answered it in the affirmative in a special case (cf. Zariski [95,96]). However, in 1957, Rees [74] found a counter-example to Zariski's problem; and a year later Nagata [60] gave a negative answer to Hilbert's fourteenth problem itself. In fact, in Nagata's counter-example,  $L \cap k[X_1, \dots, X_n]$  is a ring of invariants; so this finally disposed of attempts to prove that such rings are always finitely generated. Over the next few years, Nagata produced a simpler counter-example (cf. Nagata [61]), and also gave a complete identification of linearly reductive groups in arbitrary characteristics (cf. Nagata [62]). (All these results are contained in Nagata's notes on lectures given at the Tata Institute in 1961-62 (cf. Nagata [64]).)"

A modern treatment of the geometric invariant theory perhaps started with Mumford's monograph [116], in which he prefaced that the theory comprises essentially the following two problems:

- (i) When does an orbit space of an algebraic scheme acted on by an algebraic group exist ?
- (ii) Construct moduli schemes for various types of algebraic objects.

Concerning the first problem, he established the concept of orbit

spaces (or quotient spaces) in explicit and rigorous ways, which had not been sufficiently understood (but only intuitively), and sorted it to give various definitions of quotient spaces, among which the most significant is the concept of geometric quotient. If an algebraic group  $G$  acts on an algebraic scheme  $X$ , there does not exist in general the geometric quotient of  $X$  by  $G$  and there arises a question of finding in a natural way a  $G$ -invariant open set  $U$  of  $X$ , for which the geometric quotient by  $G$  exists; of course, the choice of  $U$  has to meet the demands from various moduli problems. For this purpose, he introduced the notion of stability (and semi-stability as well) and proved that the set  $X^S$  consisting of all stable points is a  $G$ -invariant open set for which the geometric quotient by  $G$  exists. Also, he gave an effective criterion to test the stability (also the semi-stability as well) in the case where  $G$  is a reductive algebraic group, which is stated in terms of the behaviour of the action of  $G$  on  $X$  when it is restricted to one-parameter subgroups of  $G$ . In fact, the notion of semi-stability is important to compactify in a natural fashion the moduli scheme obtained as  $X^S/G$ . This theory is tested in various moduli problems, e.g.,  $n$ -dimensional vector spaces endowed with endomorphisms, binary forms, elliptic curves with neutral points, vector bundles over a curve with given rank and degree, etc.; for the excellent, elementary accounts on these examples, the readers may refer to Mumford-Suominen [121] and Newstead [135]. Mumford applied successfully his theory to the following moduli problems:

- (i)  $\underline{A}_{d,g,n}$  :  $g$ -dimensional abelian varieties with level  $n$  structures and polarizations of degree  $d$ ; the fine moduli space

exists if  $n$  is sufficiently large to  $g$  and  $d$ , and the coarse moduli space exists for all possible values of  $g, d, n$ .

(ii)  $\underline{M}_g$  : nonsingular curves of genus  $g$ ; the coarse moduli space exists.

Mumford's arguments depended greatly on the theory of Hilbert schemes, due to Grothendieck [105], and were restricted to the case of characteristic 0 for the lack of results on the finite generation of the ring of invariants under reductive group actions in the case of positive characteristic. The desired result were conjectured in the preface of [116] as:

MUMFORD'S CONJECTURE. (\*) Any reductive algebraic group is geometrically reductive.

The subsequent development of the theory took place surrounding this conjecture. Nagata [65] proved that if a geometrically reductive group  $G$  acts rationally on a finitely generated  $k$ -algebra  $R$  then  $R^G$  is finitely generated, and Nagata and Miyata [58] proved that a geometrically reductive group is reductive. Oda [68] proved Mumford's conjecture for  $SL(2)$  (and hence  $GL(2)$ ) over a field of characteristic 2 and Seshadri [83] for the same group in general. There is also a contribution by Sumihiro [91]. Finally, in 1974, Haboush [33] proved the conjecture in general, one of the cores in his proof is reduced to the work of Steinberg [88,89]. Soon after, Seshadri [86] generalized the results over arbitrary base rings. There are some fundamental works by Seshadri [84,85] on the construction of geometric quotients.

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(\*) Different in the form from the original one.

Some positive results have also been obtained for certain actions of non-reductive groups; see, for example, Seshadri [81], Grosshans [30,31] and Hochschild-Mostow [34].

Finally, as introductory books and references, the reporter recommends Seshadri [143], Mumford-Suominen [121], Newstead [135], Fogarty [27], and Dieudonné-Carrell [25] and Springer [\*] for classical flavors.

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