Numerical Analysis of Viscous Vortex Motion

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Head-on collision of two co-axial viscous vortex rings is simulated by numerical integration of the Navier-Stokes equations for viscous incompressible fluid. Numerical results are employed to observe the behaviors of the time dependent flow quantities and confirm the importance of viscosity. Sudden decrease of the total energy and circulation subsequent to the period of their conservation, is observed to be similar to the phenomenon of the energy catastrophe studied in the turbulence theory.

1.   Introduction

Recent experimental and theoretical investigation[1,2] upon acoustic wave generated by head-on collision of two vortex rings has stressed importance of viscosity as a dominant factor in determining the form of the acoustic signal emitted by the vortex motion. The results of their work also indicate the need to study the viscous motion of high Reynolds number.

In the inviscid fluid the head-on collision of two co-axial vortex rings possessing the same strength and the same size is

considered to be equivalent to the motion of one ring approaching a plane perpendicular to its axis. In this case the image inside the wall acts as the other ring.

However because of the symmetry of the vortex configuration, similar application can be made to the motion of viscous fluid. The principal difference from the inviscid case is that the total kinetic energy and the vortex strength are no longer conserved during the motion, due to the viscous diffusion of vorticity.

Thus the aim of the present study is to determine the behavior of time dependent flow quantities at high Reynolds numbers, by applying the Navier-Stokes equation to the motion of a viscous incompressible fluid. The Navier-Stokes equation and the continuity equation are transformed into the equations of vorticity and stream function. They are then solved by means of finite difference approximations for appropriate boundary conditions and given initial vorticity distributions. The numerical method used here is expected to give accurate results only for low and medium Reynolds numbers. However it is hoped that through these analyses one can get an idea about the behavior of viscous vortex motion at higher Reynolds numbers.

Recently collision of a single vortex ring with a solid wall was investigated experimentally by Yamada et al.[3], finding generation of a secondary vortex ring at the wall in addition to the main vortex. In comparison with the present study where stress-free condition is applied at the plane boundary, viscosity effect is enhanced in their experiment by the presence of the rigid wall where no-slip condition is imposed.

2.   Mathematical formulation

2.1   Governing equations

We consider an axisymmetric flow without swirl of an incompressible viscous flow in cylindrical polar co-ordinates (x, σ, ϕ) with the axis x coincident with the symmetry axis of the vortex. In view of the property that all the flow quantities are independent of the azimuthal angle ϕ, we have the velocity components ($u(x, \sigma)$, $v(x, \sigma), 0$) and the vorticity ($0, 0, \omega(x, \sigma)$). The governing equations of the unsteady incompressible viscous flow are the Navier-Stokes equation for the vorticity and the continuity equation :

$$\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial \sigma} = \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial \sigma^2} + \frac{1}{\sigma} \frac{\partial \omega}{\partial \sigma} - \frac{\omega}{\sigma^2} \right), \qquad (2.1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial \sigma} = 0, \qquad (2.2)$$

together with the relations

$$u = \frac{1}{\sigma} \frac{\partial \psi}{\partial \sigma}, \qquad v = -\frac{1}{\sigma} \frac{\partial \psi}{\partial x}, \qquad (2.3)$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial \sigma}, \qquad (2.4)$$

where ψ is the Stokes stream function, ν the kinematic viscosity and t the time.

To obtain the equivalent system of equations to be approximated by the finite difference equations, we transform the above eq.(2.1), using eq.(2.2), into the following conservation form:

$$\frac{\partial \omega}{\partial t} + \frac{\partial u \omega}{\partial x} + \frac{\partial v \omega}{\partial \sigma} = \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial \sigma^2} + \frac{1}{\sigma} \frac{\partial \omega}{\partial \sigma} - \frac{\omega}{\sigma^2} \right), \qquad (2.5)$$

in anticipation of the fact that the equation of motion in the

form of conservation law is more effective for convergence of the numerical iterations.

Using the eq.(2.3), the expression (2.4) can be transformed into the relation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial \sigma^2} - \frac{1}{\sigma}\frac{\partial \psi}{\partial \sigma} = -\sigma\omega \qquad \text{(2.6)}$$

Let us take the radius a of the vortex ring at an initial position as the reference length and a magnitude of vorticity in the vortex core as the reference vorticity and define

$$x = a\,x^* \quad,\quad \sigma = a\,\sigma^*,\quad \omega = \Omega_o\,\omega^*\ ,\quad u = a\Omega_o u^*\ ,\quad v = a\,\Omega_o v^*,$$
$$t = \Omega_o^{-1} t^* \quad,\quad \psi = a^3\Omega_o\,\psi^* \quad.$$

With these substitutions, eqs. (2.3), (2.5) and (2.6) retain the same form (for convenience the asterisk can be dropped) except all quantities are dimensionless and ν is replaced by 1/Re, where Re is the Reynolds number defined as Re = $a^2\Omega_o\,/\nu$.

2.2 Initial and boundary conditions

The plane wall against which the vortex collides is taken at x = 0 . In view of the axisymmetric nature of the problem, we consider the vortex motion in the region ($0\langle x\langle\infty,\ 0\leq\sigma\langle\infty$) .

The initial state of the vortex ring is assumed to have a compact vortex core, that is, the vorticity at t =0 takes a non-zero value C (with dimensional value C Ω_o) only in a compact domain D of the x-σ plane :

$$\omega\Big|_{t=0} = \begin{cases} C\ , & \text{inside D,} \\ 0\ , & \text{outside D .} \end{cases} \qquad \text{(2.7)}$$

A circular form is assumed for D: $\left\{ (x,\sigma) / \left[(x - x_c)^2 + (\sigma-1)^2 \right]^{1/2} \leqslant 0.3 \right\}$, where x_c (> 0) is the initial mean axial position of the vortex and the core radius is taken as 0.3 . The initial value of vorticity C $=-2.0$ is used throughout D, where the minus sign means that the vortex motion is to the wall at x = 0.

In order to improve the final results obtained from the computation, we employed another initial condition derived from the above condition, in which D is not necessarily circular and C is not constant.

The boundary condition on the symmetry axis $\sigma =0$ is that v = 0 and $\partial u/\partial\sigma= 0$ hold by the symmetry of the velocity field . The conditions at the wall are u = 0 and $\partial v/\partial x = 0$. Those conditions are transformed into the conditions

$$\psi = 0 \text{ and } \omega = 0 \text{ on } \sigma= 0 \text{ and } x = 0 , \qquad (2.8)$$

which manifest that the boundaries $\sigma = 0$ and x = 0 are coincident with a streamline and viscous stress vanishes there.

For the conditions at infinity we take

$$\omega = 0 \text{ and } \psi = 0 , \qquad (2.9)$$

since the velocity field induced by vortex rings is irrotational and decays as O(r^{-3}) where r $=\sqrt{x^2+ \sigma^2}$.

2.3 The analytical expressions

For later use, it is desirable to get definitions of some flow quantities as follows.

The circulation of the system can be defined as

$$\Gamma = \iint_A \omega \, dx \, d\sigma , \qquad (2.10)$$

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where A denote the domain considered on the axial plane (x, σ). The coordinates of the center of vortex core can be defined by the expressions ;

$$\bar{x} = \frac{\iint_A \omega x \, dx \, d\sigma}{\Gamma} \quad , \quad \bar{\sigma} = \frac{\iint_A \omega \sigma \, dx \, d\sigma}{\Gamma} \quad . \quad (2.11)$$

The total kinetic energy E of the system and its time derivatives can be expressed as follows ;

$$E = \frac{1}{2} \iint_A \{ u^2 + v^2 \} 2\pi \, \sigma \, dx \, d\sigma$$

$$= \pi \iint_A \omega \psi \, dx \, d\sigma \quad , \qquad (2.12)$$

$$\frac{dE}{dt} = -\frac{2\pi}{Re} \iint_A \omega^2 \, \sigma \, dx \, d\sigma \quad , \qquad (2.13)$$

$$\frac{d^2E}{dt^2} = -\frac{2}{Re} \iint_A \omega^2 v \, dx \, d\sigma + \frac{2}{Re^2} \iint_A \{ (\frac{\partial \omega}{\partial x})^2 + (\frac{\partial \omega}{\partial \sigma})^2 + (\frac{\omega}{\sigma})^2 \} \sigma \, dx \, d\sigma \quad . \quad (2.14)$$

3.   Finite difference equations

To solve the time dependent problem corresponding to the required physical situation, we use the finite difference equations approximating the eqs.(2.3), (2.5) and (2.6) (in dimensionless form) . To construct the finite difference equations we divide the region of consideration in the x- σ plane into rectangular meshes whoses intervals are taken as h and k in the x and directions respectively. The numbers of the mesh points in the corresponding directions are denoted by I and J respectively. The network of mesh points is shown in Fig.1. Here we consider the finite region OABC with such an extent that the boundary condition (2.9) at infinity is replaced with the suffi-

cient accuracy by $\psi = 0$ and $\omega = 0$ on the two sides AB and BC in Fig.1 . Therefore the two conditions (2.8) and (2.9) reduce to the boundary conditions :

$\psi = 0$ and $\omega = 0$ on all the boundaries OA, AB,

BC and CO , (3.1)

for the computation of the finite difference equations.

Time step is taken as τ and the values of ψ, ω ,u and v at the point (ih, jk) and at the time t= $n\tau$ are denoted by $\psi_{i,j}^{n}$, $\omega_{i,j}^{n}$, $u_{i,j}^{n}$ and $v_{i,j}^{n}$ respectively, where i and j are integers.

Using the above notations and substituting the forward difference formula for time derivative and the central difference formulae for space derivatives into the eqs. (2.5) and (2.6), we obtain the following finite difference equations:

$$\frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n}}{\tau} = -\left\{ \frac{(u\omega)_{i+1,j}^{n} - (u\omega)_{i-1,j}^{n}}{2h} + \frac{(v\omega)_{i,j+1}^{n} - (v\omega)_{i,j-1}^{n}}{2k} \right\} + \frac{1}{Re}\left\{ \frac{\omega_{i+1,j}^{n} + \omega_{i-1,j}^{n} - 2\omega_{i,j}^{n}}{h^2} \right.$$

$$\left. + \frac{\omega_{i,j+1}^{n} + \omega_{i,j-1}^{n} - 2\omega_{i,j}^{n}}{k^2} + \frac{1}{\sigma}\frac{\omega_{i,j+1}^{n} - \omega_{i,j-1}^{n}}{2k} - \frac{\omega_{i,j}^{n}}{\sigma^2} \right\}$$

$$= \mathcal{L}_{\sigma}^{n}(\omega) + \mathcal{L}_{x}^{n}(\omega) , \tag{3.2}$$

$$\mathcal{L}_{x}^{s}(\omega) = -\left\{ \frac{(u\omega)_{i+1,j}^{s} - (u\omega)_{i-1,j}^{s}}{2h} \right\} + \frac{1}{Re}\left\{ \frac{\omega_{i+1,j}^{s} + \omega_{i-1,j}^{s} - 2\omega_{i,j}^{s}}{h^2} \right\} , \tag{3.3}$$

$$\mathcal{L}_{\sigma}^{s}(\omega) = -\left\{ \frac{(v\omega)_{i,j+1}^{s} - (v\omega)_{i,j-1}^{s}}{2k} \right\} + \frac{1}{Re}\left\{ \frac{\omega_{i,j+1}^{s} + \omega_{i,j-1}^{s} - 2\omega_{i,j}^{s}}{k^2} + \right.$$

$$\left. \frac{1}{\sigma}\frac{\omega_{i,j+1}^{s} - \omega_{i,j-1}^{s}}{2k} - \frac{\omega_{i,j}^{s}}{\sigma^2} \right\} , \tag{3.4}$$

$$S = n ,$$

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$$\frac{\psi_{i-1,j}^{n} + \psi_{i+1,j}^{n} - 2\psi_{i,j}^{n}}{h^2} + \frac{\psi_{i,j-1}^{n} + \psi_{i,j+1}^{n} - 2\psi_{i,j}^{n}}{k^2} - \frac{1}{\sigma}\frac{\psi_{i,j+1}^{n} - \psi_{i,j-1}^{n}}{2k}$$

$$= -\sigma\,\omega_{i,j}^{n} \qquad . \qquad (3.5)$$

For eq.(2.3) finite difference approximations can be made as follows. Let us denote the values of a function f(x) and its derivatives at x = ih by f_i, f_i', f_i'' and so on. Then, using Taylor series expansions of f_{i-1}, f_{i+1}, f_{i-1}' and f_{i+1}', we can write

$$f_{i-1}' + 4\,f_i' + f_{i+1}' = \frac{3}{h}\,(f_{i+1} - f_{i-1}) + O(h^4) ,$$

which is in tridiagonal form and gives the first derivatives of f_i in terms of f_i to the accuracy of fourth order of h.

Employing the above formula to the calculation of the velocity components $u_{i,j}$ and $v_{i,j}$ from eq.(2.3), we obtain the finite difference equations

$$(\sigma u)_{i,j-1} + 4(\sigma u)_{i,j} + (\sigma u)_{i,j+1} = \frac{3}{k}(\psi_{i,j+1} - \psi_{i,j-1}) , \qquad (3.6)$$

$$(\sigma v)_{i-1,j} + 4(\sigma v)_{i,j} + (\sigma v)_{i+1,j} = -\frac{3}{h}(\psi_{i+1,j} - \psi_{i-1,j}) . \qquad (3.7)$$

To solve the difference equation (3.2), we apply the method based on that devised by Peaceman & Rachford[4] for diffusion problems, known as ADI (Alternating Direction Implicit) method. Thus each time step , nτ to (n+1)τ , is divided into two successive half steps, nτ to (n+1/2)τ and (n+1/2)τ to (n+1)τ , and hence eq.(3.2) becomes

$$\frac{\omega_{i,j}^{n+\frac{1}{2}} - \omega_{i,j}^{n}}{\tau/2} = L_\sigma(\omega) + L_x(\omega) , \qquad (3.8)$$

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$$\frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n+\frac{1}{2}}}{\tau/2} = \mathcal{L}_\sigma^{n+1}(\omega) + \mathcal{L}_x^{n+\frac{1}{2}}(\omega) \quad, \tag{3.9}$$

where $\mathcal{L}_\sigma^n(\omega)$, $\mathcal{L}_\sigma^{n+1}(\omega)$ and $\mathcal{L}_x^{n+\frac{1}{2}}(\omega)$ are as defined in eqs. (3.3) and (3.4).

Since eqs. (3.6)-(3.9) are the systems of equations in tridiagonal forms, we can apply Gauss elimination method to solve eqs.(3.6) and (3.7) for $u_{i,j}$ and $v_{i,j}$ from known values of $\psi_{i,j}$, eq.(3.8) for $\omega_{i,j}^{n+\frac{1}{2}}$ and eq.(3.9) for $\omega_{i,j}^{n+1}$.

To solve eq.(3.5), we use SOR (Successive Over-Relaxation) method after modifying the equation as

$$\psi_{i,j}^{s+1} = (1-\alpha)\,\psi_{i,j}^{s} + \frac{\alpha}{2(h^{-2}+k^{-2})} \left\{ \frac{\psi_{i+1,j}^{s} + \psi_{i-1,j}^{s+1}}{h^2} + \frac{\psi_{i,j+1}^{s} + \psi_{i,j-1}^{s+1}}{k^2} \right.$$
$$\left. - \frac{1}{\sigma}\,\frac{\psi_{i,j+1}^{s} - \psi_{i,j-1}^{s+1}}{2k} + \sigma\,\omega_{i,j}^{n} \right\}, \tag{3.10}$$

where α is the parameter known as the relaxation factor and the index s indicates the number of the iteration times. In this formula the latest possible values of $\psi_{i,j}$ are substituted in the right hand side for the rapid rate of convergence of the iterations and the optimum value of α has been chosen based on the formulae $\alpha = \dfrac{8 - 4\sqrt{4-\beta^2}}{\beta^2}$ and $\beta = \cos\dfrac{\pi}{I} + \cos\dfrac{\pi}{J}$ [5] and some trial test. Here I and J are the numbers of mesh points as stated above.

4. Computational procedure

4.1 Initial condition

Initially all dependent variables were set to zero except

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the vorticity for which we assigned $\omega_{i,j} = -2$ for the points which

lie in the core $\left\{ (x,\sigma) : ((x-X_c)^2 (\sigma-1)^2)^{1/2} \leq 0.3 \right\}$ and $\omega_{i,j} = 0$

for the remaining points. Here $(X_c,1)$ and 0.3 are the initial

coordinates of the core center and the core radius respectively.

Next, using the assigned values of $\omega_{i,j}$ and $\psi_{i,j}$ we improved

the values of $\psi_{i,j}$ by eq.(3.10). The iteration was continued until

the two successive values of $\psi_{i,j}^k$ and $\psi_{i,j}^{k+1}$ had met the convergence

condition $|\psi_{i,j}^{k+1} - \psi_{i,j}^k| < 10^{-6}$ (at $k = K$) . Taking the given values

of $\omega_{i,j}$ and the converged values of $\psi_{i,j}$ as the initial values we

proceeded to the next time $t = \tau, 2\tau, 3\tau, \ldots$. The procedure

from the time $t = n\tau$ to the next time $t = (n+1)\tau$ is described

in the next sub-section 4.2 .

Up to here we have assumed that the initial shape of vortex

core is exactly circle and approximated the initial vorticity

distribution by giving uniform values for mesh points which lie

in the circle and zero for outside points. However the actual

shape of the vortex core may be not necessarily circle and

vorticity distribution in the core may not be exactly uniform.

For this reason we devised the way to get the better approxi-

mation of the initial shape of vortex core and the vorticity

distribution in it. We recorded the vorticity distribution at

an appropriate time after proceeding the above computation with

the previous initial conditions, and the equi-vorticity lines

for that time were plotted.

From this procedure we obtained the more suitable set of

vorticity distribution which would give a reasonable shape of

vortex core and the vorticity distribution in the core region

in the state of translational motion. Thus we assigned the new

initial vorticity distribution by constructing the vortex core

which has the same shape and the same vorticity distribution in

it as those of the core of vortex given by the recorded data.

The size of the core was also chosen by specifying a minimum

value of the vorticity below which the value of $\omega_{i,j}$ was set equal

to zero. The center of the core was made to coincide with the

position of the previous initial core center. The other depen-

dent variables are all set to zero for all points as in the pre-

vious initial conditions. From those values, the values of $\psi_{i,j}$

were modified by the eq.(3.10) and the same procedure continued

as stated previously.

4.2   The steps required to proceed from time $n\tau$ to time $(n+1)\tau$

Let us suppose that the values of $\omega_{i,j}^m$, $\psi_{i,j}^m$, $u_{i,j}^m$ and $v_{i,j}^m$ are

known up to the time $n\tau$, where those of $u_{i,j}^m$ and $v_{i,j}^m$ are determined

from the relations (3.6) and (3.7), then the values of $\omega_{i,j}^{m+1}$, $\psi_{i,j}^{m+1}$,

$u_{i,j}^{m+1}$ and $v_{i,j}^{m+1}$ for the next time $(n+1)\tau$ can be calculated as

follows.

1. Using the formula $\psi_{i,j}^{m+1} = \psi_{i,j}^m + (\psi_{i,j}^m - \psi_{i,j}^{m-1})$, we approximate the

values of $\psi_{i,j}^{m+1}$.

2. Then the values of $u_{i,j}^{m+1}$ and $v_{i,j}^{m+1}$ can be calculated by using

eqs.(3.6) and (3.7) .

3. Systems of equations, eqs.(3.8) and (3.9), are solved to get

$\omega_{i,j}^{m+1}$ and $\omega_{i,j}^{m+1}$ by using ADI method.

4. Next, using the up-to-date values of $\omega_{i,j}^{m+1}$ we solve eq.(3.10)

to get the corrected values of $\psi_{i,j}^{m+1}$ by SOR method. The iteration

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was continued until the two successive values of $\psi_{i,j}^{s}$ and $\psi_{i,j}^{s+1}$

satisfy the condition $\left| \dfrac{\psi_{i,j}^{s+1} - \psi_{i,j}^{s}}{\psi_{h}} \right| < 10^{-5}$ where ψ_{h} denotes the

reference value of $\psi_{i,j}$, taken as the maximum value of $\psi_{i,j}$ at $t = 0$

in this case.

5. Using the latest revised values of $\psi_{i,j}^{n+1}$ we repeat step 2 to step 4

again and again until the values of $\omega_{i,j}^{n+1}$ no longer change. The

convergency of the whole iteration process was tested by the

condition $\left| (\omega_{i,j}^{n+1})^{k+1} - (\omega_{i,j}^{n+1})^{k} \right| < 10^{-5}$ where k and k +1 denote the numbers of

the successive iterations.

6. When the convergence condition in the step 5 is satisfied the

latest values of $\omega_{i,j}^{n+1}$ and $\psi_{i,j}^{n+1}$ together with $u_{i,j}^{n+1}$ and $v_{i,j}^{n+1}$ are

taken as the values for the time $(n+1)\tau$. Thus one time step

is completed and we can proceed to the next time step in the same

way.

5. Results and discussion

Computations were carried out by taking the grid points of

136x121 and the size of the space step 1/15 for both x and σ

directions. It was found that a further increment in the size

of the region did not produce any noticeable changes of the

results. It therefore confirms the chosen size of the region is

sufficiently large to approximate the infinite region of the

required flow field.

The error bounds for the values of $\psi_{i,j}$ and $\omega_{i,j}$ are that a

further reduction of them brought no improvement of the accuracy

of the solutions, at least for four significant figures. The

accuracy of the procedure is also checked by using the analytical

expressions (2.12)-(2.14) for E, dE/dt, d^2E/dt^2 as shown in Table(1).

Figures 2(a) to 2(d) show the equi-vorticity lines and streamlines at t = 0, 39.8, 49,8 and 59.8 for Re = 500,1000,2500 and 5000. . Figure 3 shows the paths of the centroid of vorticity defined in eq.(2.11), where the ordinate gives the radius of the vortex ring at the corresponding position. It can be seen from the figure that the radius of the vortex remains almost constant till the vortex reaches at a distance of the order of its core radius from the wall and after that it increases rapidly.

Figure 4 represents the variation of the circulation defined in eq.(2.10), and fig,5 the variation of energy of the system defined by eq.(2.12), both plotted against the dimensionless time t ᵼ for Re =500,2500 and 5000. Comparing with the curves of the vortex path, it is found that as the Reynolds number increases from 500 to 5000 the energy and the circulation curves tend to be flat for the vortex positions away from the wall, and decrease after the vortex has reached very near to the wall. This indicates that the effect of viscosity becomes dominant as the vortex comes close to the wall at distances of the order radius of the core. It appears that, as the Reynolds number is increased indefinitely, the circulation and the energy of the system tend to be conserved in an initial period and then suddenly damped by the viscous action when the vortex comes near the wall. This sort of behavior of total energy, that is, initial conservation and subsequent sudden increase, is called as energy catastrophe in the theory of homogeneous turbulence by Andre and Lesieur[6].

In Fig.6 , the distribution of vorticity across the diameter parallel to the symmetry axis is plotted for t =49.8 and 59.8 for Re =2500.. From the figure it can be seen that the vorticity distributions compare fairly well with the Gaussian distribution. Figure 7 shows the variations of maximum vorticity with time t and it can be seen that near t =50 the maximum vorticity increases abruptly and subsequently decreases. This increase is probably due to the stretching of vortex when it collides against the wall, and the subsequent decrease is due to viscous dissipation. Considering the fact that this case of a single vortex colliding against the wall is equivalent to that of head-on collision of two vortices, the viscous decay of the total circulation of the single vortex implies pair annihilation of positive and negative vorticities.

6.   Conclusion

Although the Reynolds numbers used in the present study were not strictly comparable to that of ref.1 and 2) , it can be said that our results confirm the important role played by viscosity when the cores of two vortex rings come into collision. Therefore this analysis suggests that the acoustic emission studied in ref.2) must have been affected largely by the action of viscosity.

It is interesting to note that the viscous decay of the total circulation of the single vortex is interpreted as pair annihilation of the vorticities of opposite senses, and that the sudden decrease of the energy and circulation is subsequent to the period of their conservation is observed to be similar to the phenomenon of the energy catastrophe studied in the turbulence theory.

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t \ Re		500	1000	2500	5000
10.	NE	-1.074×10^{-3}	-8.875×10^{-4}	-5.420×10^{-4}	-2.670×10^{-4}
	AE	-1.074×10^{-3}	-8.875×10^{-4}	-5.388×10^{-4}	-2.689×10^{-4}
30.	NE	-5.490×10^{-4}	-5.335×10^{-4}	-3.950×10^{-4}	-2.235×10^{-4}
	AE	-5.481×10^{-4}	-5.323×10^{-4}	-3.928×10^{-4}	-2.221×10^{-4}
50.	NE	-3.555×10^{-4}	-4.305×10^{-4}	-5.410×10^{-4}	-3.265×10^{-4}
	AE	-3.555×10^{-4}	-4.220×10^{-4}	-4.870×10^{-4}	-2.671×10^{-4}
10.	NEE	4.635×10^{-5}	2.790×10^{-5}	1.040×10^{-5}	4.208×10^{-6}
	AEE	4.480×10^{-5}	2.646×10^{-5}	0.942×10^{-5}	3.979×10^{-6}
30.	NEE	1.465×10^{-5}	1.093×10^{-5}	4.908×10^{-6}	2.043×10^{-6}
	AEE	1.423×10^{-5}	1.038×10^{-5}	4.526×10^{-6}	1.875×10^{-6}
50.	NEE	5.208×10^{-6}	-4.370×10^{-6}	-3.980×10^{-5}	-1.952×10^{-5}
	AEE	5.017×10^{-6}	-4.652×10^{-6}	-4.077×10^{-5}	-2.004×10^{-5}

NE =Numerically obtained value of dE/dt

AE =Analytically obtained value of dE/dt

NEE =Numerically obtained value of d^2E/dt^2

AEE =Analytically obtained value of d^2E/dt^2

Table 1 .

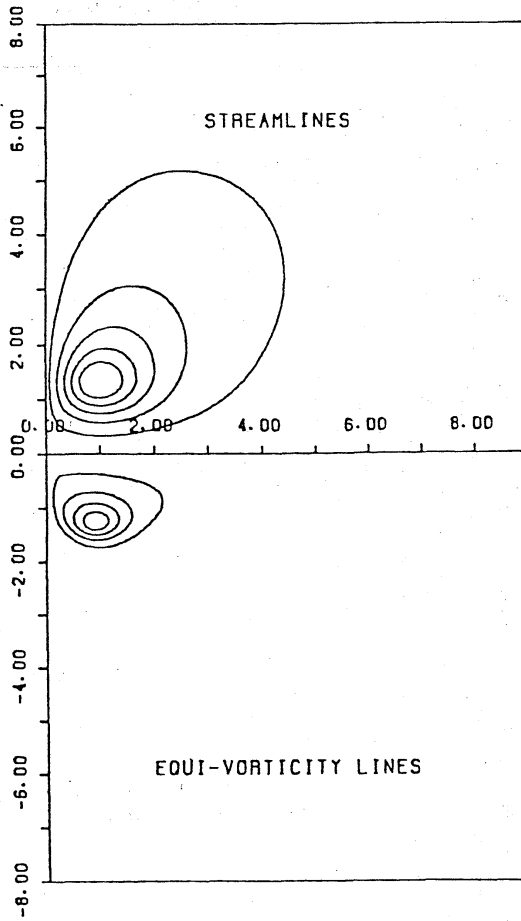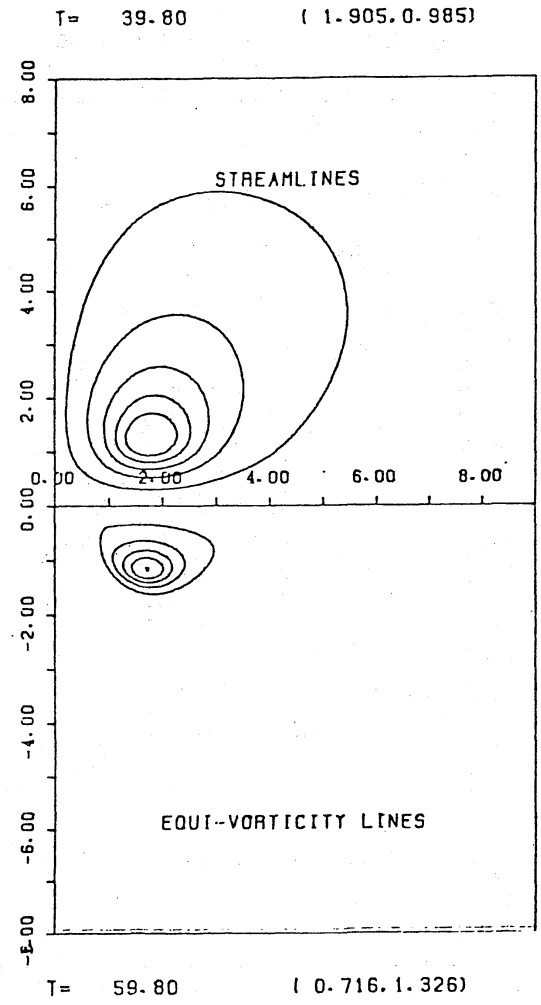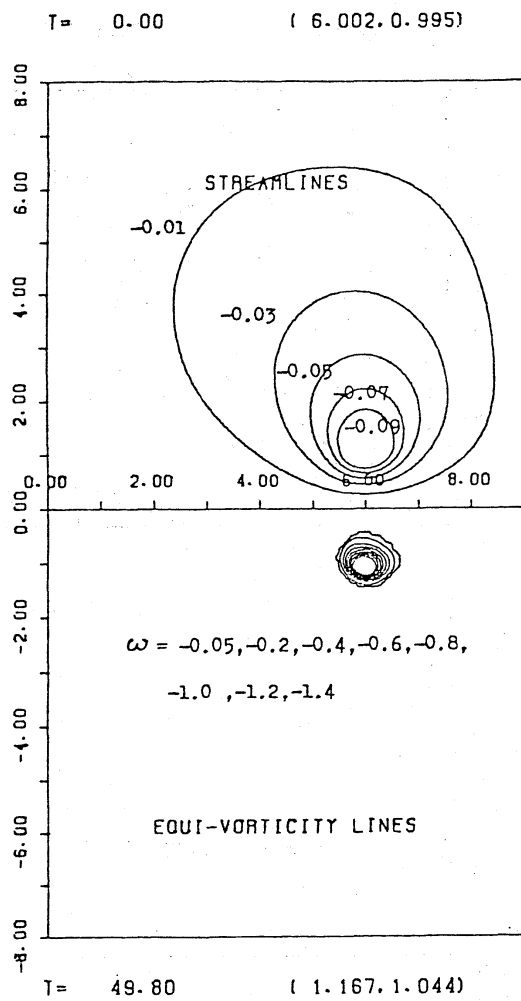$$D : \left\{ (x,\sigma) / \left[(x-x_\ell)^2 + (\sigma-1)^2 \right]^{\frac{1}{2}} \leq \varepsilon \right\}$$
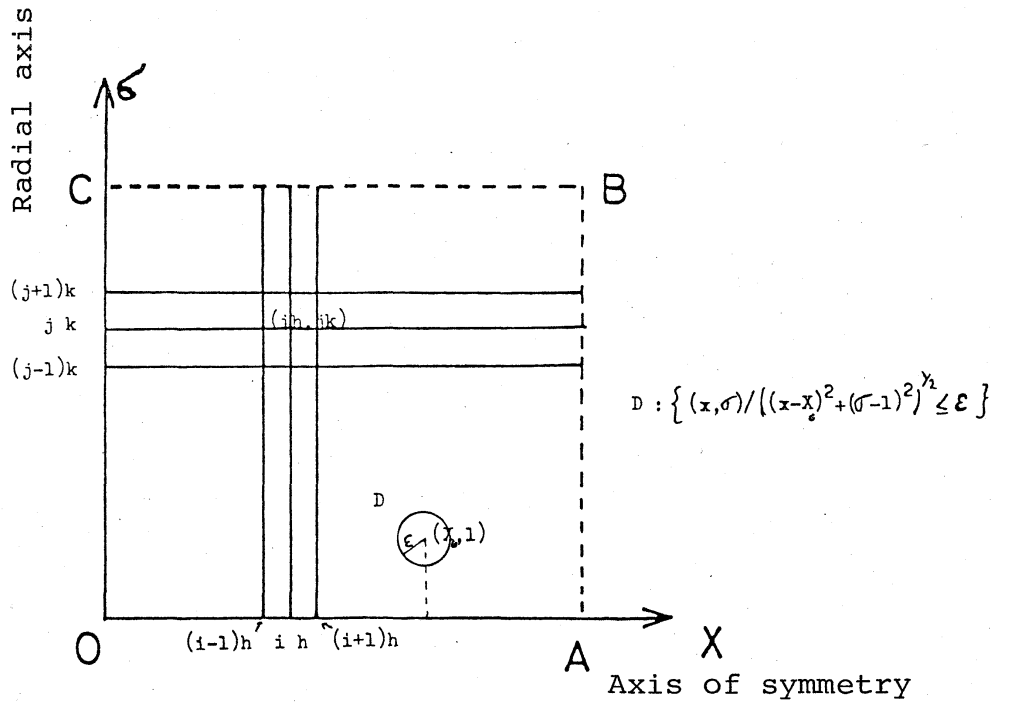
Fig.1

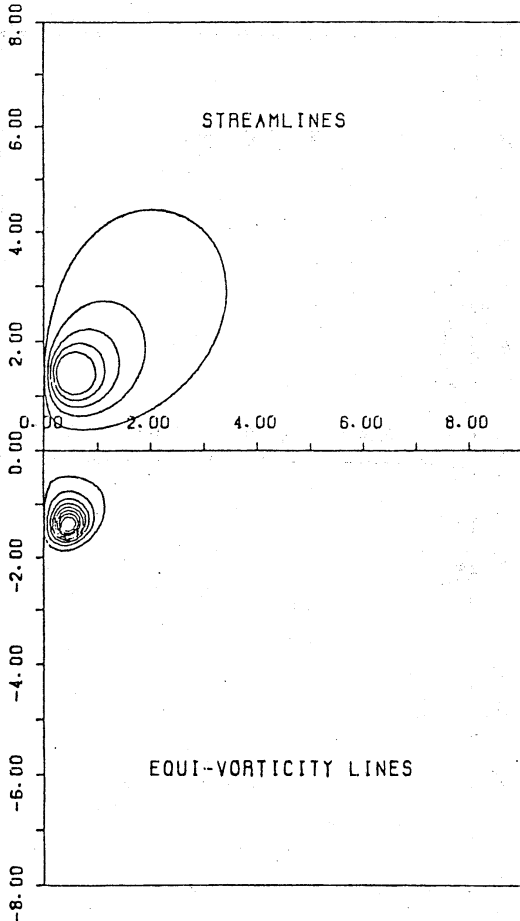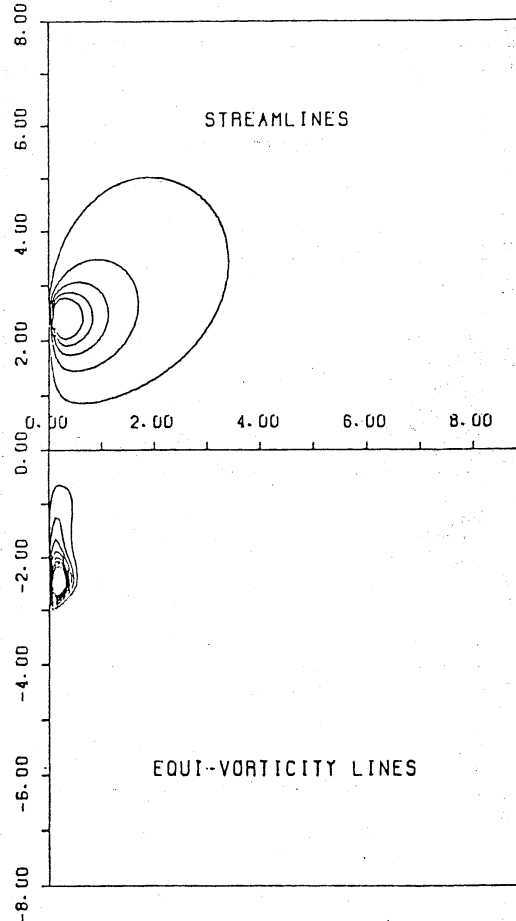REYNOLDS NUMBER= 500.0 Fig.2(a)

T= 0.00 (5.993,0.988)

STREAMLINES

-0.01

-0.03

-0.05

-0.07

-0.09

ω = -0.05,-0.2,-0.4,-0.6,-0.8,

-1.0,-1.2,-1.4

EQUI-VORTICITY LINES

T= 39.80 (2.527,0.987)

STREAMLINES

EQUI-VORTICITY LINES

T= 49.80 (1.931,1.013)

STREAMLINES

EQUI-VORTICITY LINES

T= 59.80 (1.462,1.086)

STREAMLINES

EQUI-VORTICITY LINES

REYNOLDS NUMBER= 1000.0   Fig.2(b)

T=   0.00   (6.002.0.995)

STREAMLINES

-0.01

-0.03

-0.05

-0.07

-0.09

$\omega = -0.05, -0.2, -0.4, -0.6, -0.8,$

$-1.0, -1.2, -1.4$

EQUI-VORTICITY LINES

T= 39.80   (1.905.0.985)

STREAMLINES

EQUI-VORTICITY LINES

T= 49.80   (1.167.1.044)

STREAMLINES

EQUI-VORTICITY LINES

T= 59.80   (0.716.1.326)

STREAMLINES

EQUI-VORTICITY LINES

T= 0.00   (6.035.0.996)   T= 39.80   (1.335.1.004)

STREAMLINES

-0.01

-0.03

-0.05

-0.07

-0.09

STREAMLINES

$\omega = -0.05, -0.2, -0.4, -0.6, -0.8,$

$-1.0, -1.2, -1.4$

EQUI-VORTICITY LINES

EQUI-VORTICITY LINES

T= 49.80   (0.537.1.237)   T= 59.80   (0.258.2.141)

STREAMLINES

STREAMLINES

EQUI-VORTICITY LINES

EQUI-VORTICITY LINES

RETNOLDS NUMBER= 5000.0   Fig.2(d)

T=   0.00   (6.050.0.986)   T=   39.80   (1.431.0.996)

STREAMLINES

-0.01

-0.03

-0.05

-0.07

-0.09

STREAMLINES

$\omega = -0.05, -0.2, -0.4, -0.6, -0.8,$
$-1.0, -1.2, -1.4$

EQUI-VORTICITY LINES

EQUI-VORTICITY LINES

T=   49.80   (0.548.1.169)   T=   59.80   (0.200.2.145)

STREAMLINES

STREAMLINES

EQUI-VORTICITY LINES

EQUI-VORTICITY LINES

Fig.3 Paths of centroid of vorticity
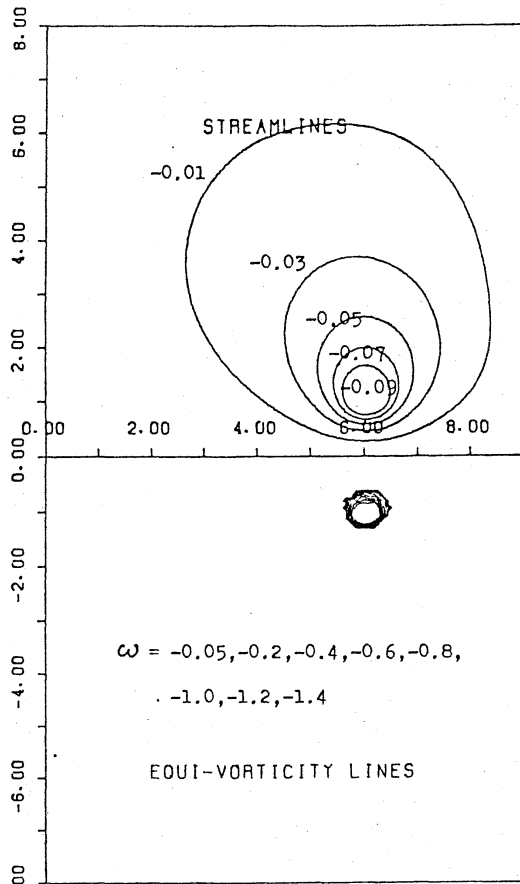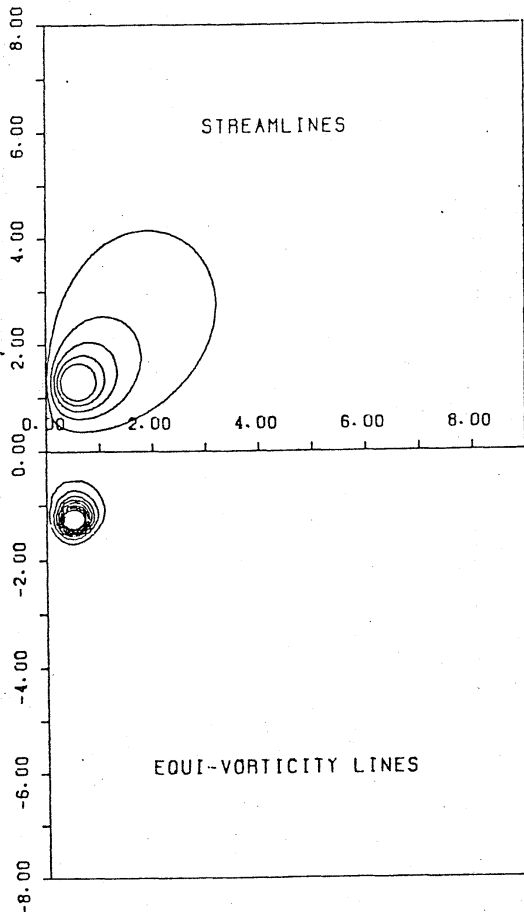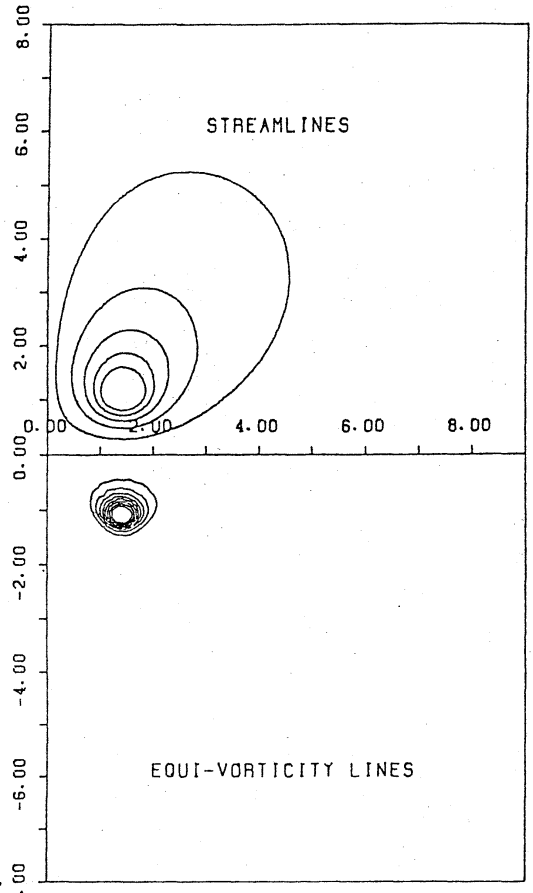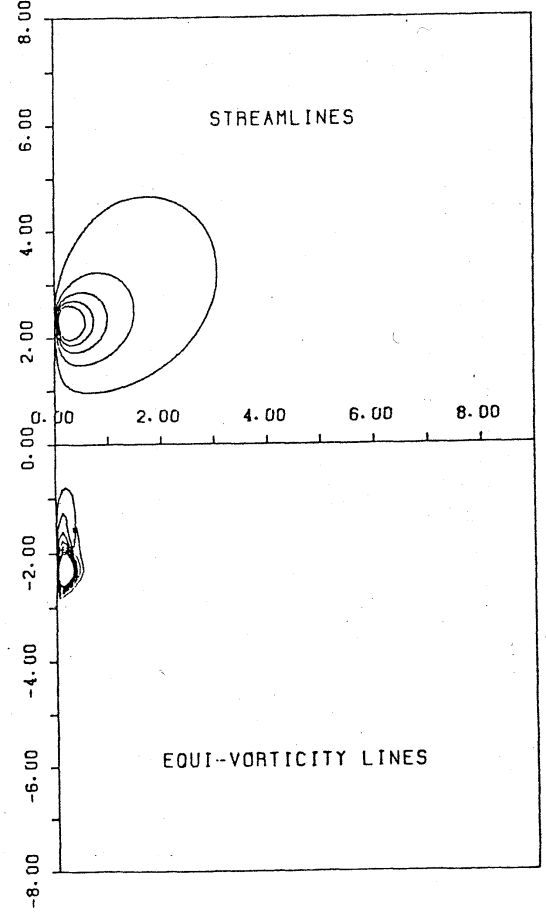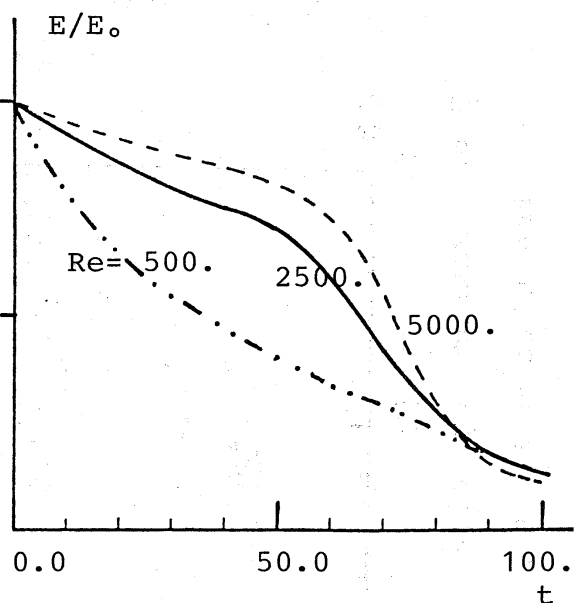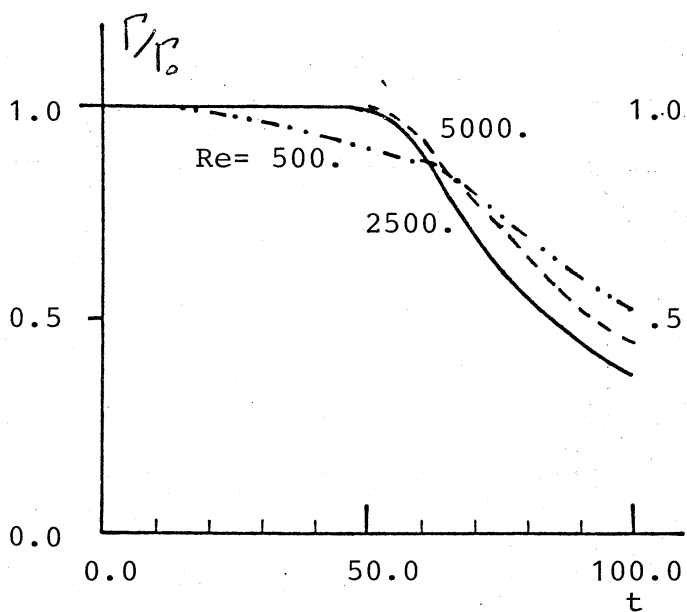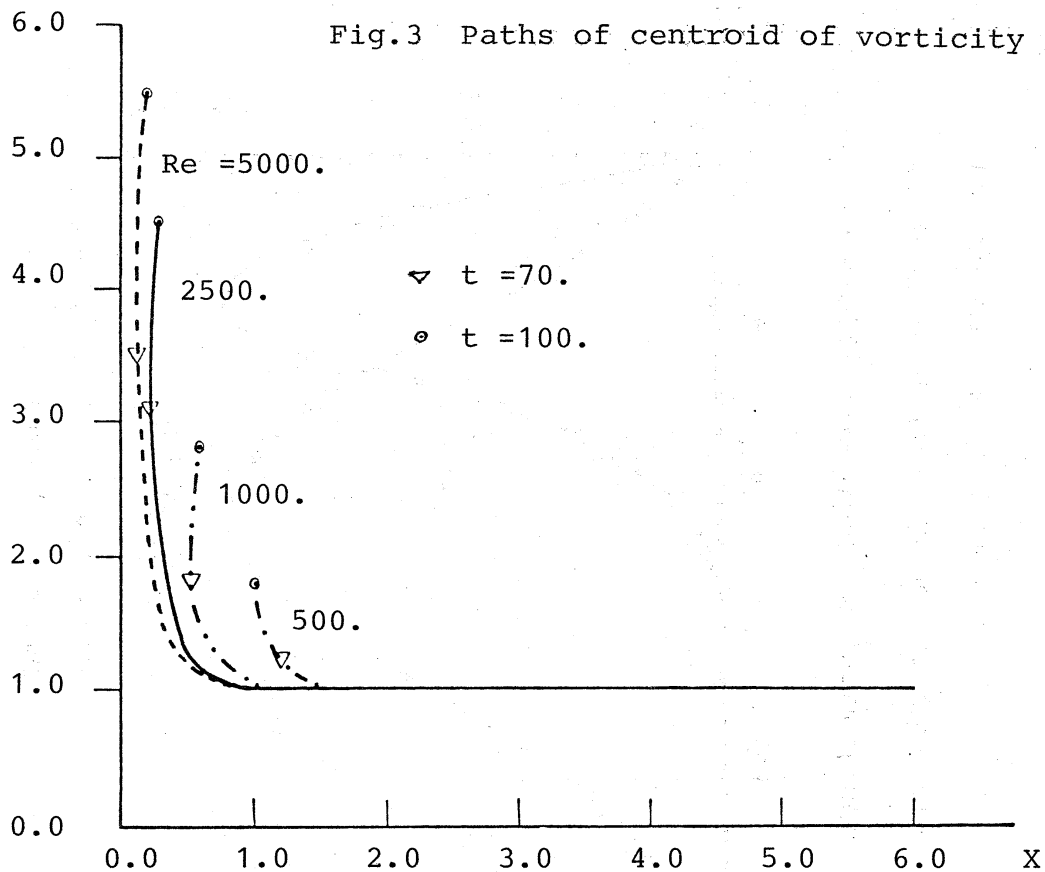
Fig.4 Circulation normalized
by the value at t=0.

Fig.5 Total energy normal-
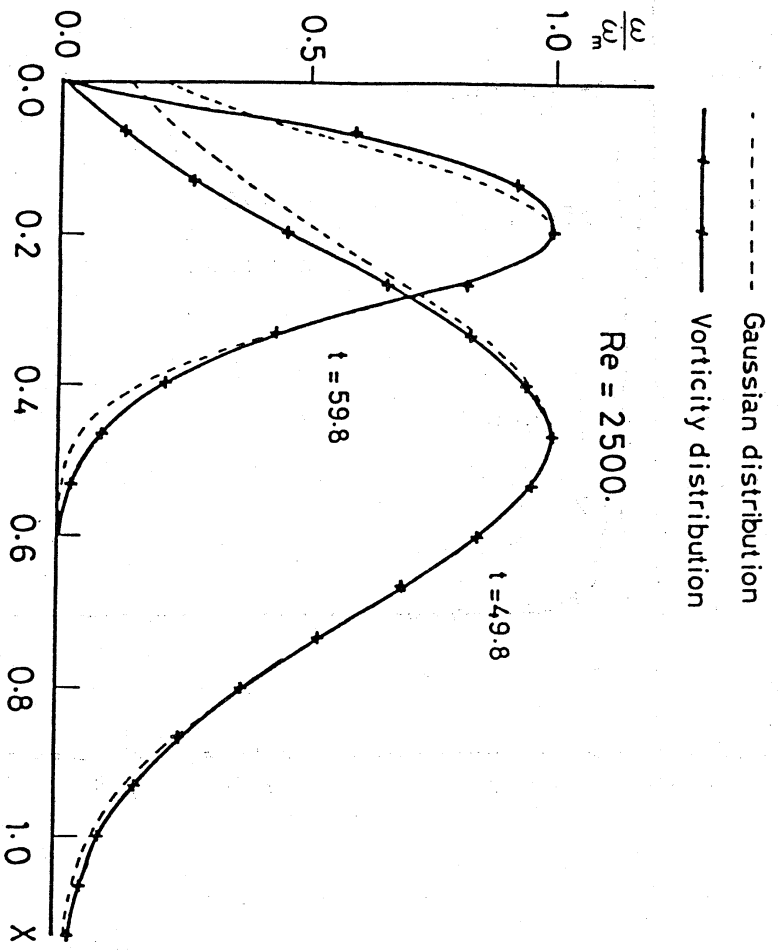ized by the value at t

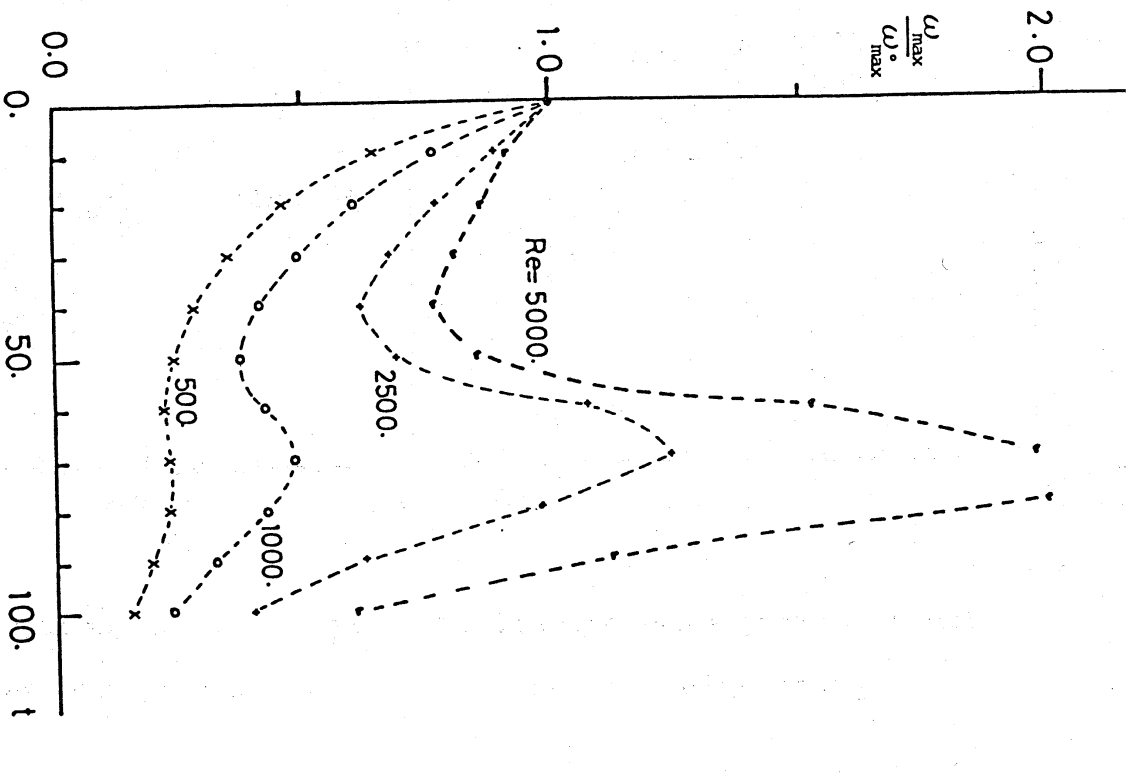Fig.6 Vorticity distribution (normalized by the maximum value ω_m) along the core axis parallel to the symmetric axis X .

Fig.7 Maximum vorticity, normalized by the value at t = 0 .