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Intertwining operatorsの間のspectrumの関係について

東北大 教養　吉野 崇 (Takashi Yoshino)

XおよびYを、2つの複素バナッハ空間とする。XからYへの有界線形作用素の全体をB(X, Y)で表わす。特にX = Yの場合は、B(X)で表わす。A∈B(X)及びB∈B(Y)に対して、C∈B(X,Y)が、CA = BCをみたす時、C intertwines A and Bと云う。特にCが、injectiveで、dense rangeを持つようにとれる時は、Aは、a quasi-affine transform of Bであると云い、A≺Bで表わす。A≺B且つB≺Aである時は、AとBとは、quasi-similarであると云う。Cがinvertibleにとれる時は、AとBとは、similarであると云う。similarな作用素の間のspectrumの関係及びquasi-similarな作用素の間のspectrumの関係については、多くの研究結果が知られているが、ここでは、intertwiningな作用素の間のspectral manifoldsの関係を調べ、その応用として、spectrumの関係について、報告する。

§1. 準備

$A\in B(X)$が、a spectral operator であるとは、Aと可換で、$\sigma(A|E(\sigma)X)\subset\bar{\sigma}$ for all Borel sets σ なる性質を持つ a spectral measure $E(\cdot)$が存在する場合である。ここで、$\sigma(T)$は作用素Tの spectrum を表わす。この spectral operators の概念は、Foiaş [3]によって、§4に述べる decomposable operators へと一般化される。$A\in B(X)$に対して、

$$\sigma_p^0(A)=\{\lambda\in\mathbb{C};\ (z-A)f(z)\equiv 0 \text{ for some non-zero analytic function } f: D_r(\lambda)\to X\}$$

とする。但し、$D_r(\lambda)=\{z\in\mathbb{C};\ |z-\lambda|<r\}$である。$\sigma_p^0(A)=\emptyset$なる場合に、$A$は、the single-valued extension property を持つと云う。$A\in B(X)$及び、a closed set $\sigma\subset\mathbb{C}$に対して、

$$X_A(\sigma)=\{x\in X;\ (z-A)f(z)\equiv x \text{ for some analytic function } f: \mathbb{C}\setminus\sigma\to X\}$$

とし、任意の$\sigma\subset\mathbb{C}$に対しては、$X_A(\sigma)=\cup\{X_A(\tau);\ \tau\subset\sigma \text{ and } \tau \text{ is closed}\}$とする。集合$X_A(\sigma)$を the spectral manifold of Aと云う。$X_A(\sigma)$がAの an invariant linear manifold になることや、$\sigma_1\subset\sigma_2$ならば、$X_A(\sigma_1)\subset X_A(\sigma_2)$なることが知られている。又、$X_A(\sigma)=X_A(\sigma\cap\sigma(A))$, $X_A(\sigma(A))=X$及び$X_A(\emptyset)=\{0\}$なることも容易にわかる。

<u>Proposition</u>. ([9]) If $X_A(\sigma)$ is closed, then $\sigma(A|X_A(\sigma))\subset\sigma\cup\overline{\sigma_p^0(A)}$.

$A \in B(X)$ が "Dunford's closure condition" をみたすとは，任意の closed set $\sigma \subset \mathbb{C}$ に対して，$X_A(\sigma)$ が closed なる時である。この時には，A は，the single-valued extension property を持つことが知られている。$A \in B(X)$ の不変部分空間 Z が，a spectral maximal space であるとは，A の，$\sigma(A|Z') \subset \sigma(A|Z)$ なる任意の不変部分空間 Z' に対して，$Z' \subset Z$ なる時である。$A \in B(X)$ が decomposable であるとは，$\sigma(A)$ の任意の open covering $\{G_1, G_2, \cdots, G_n\}$ に対して，A の spectral maximal spaces の系 $\{Z_1, Z_2, \cdots, Z_n\}$ が存在して，$X = Z_1 + Z_2 + \cdots + Z_n$ 且つ $\sigma(A|Z_j) \subset G_j$ for every $j = 1, 2, \cdots, n$ なる時である。

§2. 主な結果.

<u>Lemma.</u> *If $C \in B(X, Y)$ intertwines $A \in B(X)$ and $B \in B(Y)$, then $CX_A(\sigma) \subset X_B(\sigma)$ for an arbitrary set $\sigma \subset \mathbb{C}$. In particular $CX \subset X_B(\sigma(A))$.*

<u>Proof.</u> If $x \in X_A(\sigma)$, then $x \in X_A(\tau)$ for some closed set $\tau \subset \sigma$ and $(z-A)f(z) \equiv x$ for some analytic function $f: \mathbb{C} \setminus \tau \to X$. Since $Cx \equiv C(z-A)f(z) = (z-B)Cf(z)$ and since $Cf(z): \mathbb{C} \setminus \tau \to Y$ is analytic, $Cx \in X_B(\tau) \subset X_B(\sigma)$ and hence $CX_A(\sigma) \subset X_B(\sigma)$. And the rest follows by $X_A(\sigma(A)) = X$.

<u>Theorem 1.</u> (Radjavi-Rosenthal [5]) If $C \in B(X,Y)$ intertwines spectral operators $A \in B(X)$ and $B \in B(Y)$ with spectral measures $E(\cdot)$ and $F(\cdot)$ respectively, then C intertwines $E(\sigma)$ and $F(\sigma)$ for every Borel set σ.

<u>Proof.</u> Since $X_A(\sigma) = E(\sigma)X$ and $X_B(\sigma) = F(\sigma)Y$ for every closed set $\sigma \subset \mathbb{C}$ by [1; Theorem 4], $CE(\sigma)X \subset F(\sigma)Y$ by Lemma and hence $F(\sigma)CE(\sigma) = CE(\sigma)$ and $F(\mathbb{C}\setminus\sigma)CE(\sigma) = 0$ because $F(\sigma)CE(\sigma)x = CE(\sigma)x$ for all $x \in X$ and $F(\mathbb{C}\setminus\sigma)CE(\sigma)X \subset F(\mathbb{C}\setminus\sigma)F(\sigma)Y = F([\mathbb{C}\setminus\sigma]\cap\sigma)Y = F(\phi)Y = \{0\}$. By regularity of $E(\cdot)$ and $F(\cdot)$, $F(\sigma)CE(\sigma) = CE(\sigma)$ and $F(\mathbb{C}\setminus\sigma)CE(\sigma) = 0$ for every Borel set σ. And hence $F(\sigma)C = F(\sigma)C[E(\sigma) + E(\mathbb{C}\setminus\sigma)]$
$= F(\sigma)CE(\sigma) + F(\sigma)CE(\mathbb{C}\setminus\sigma) = F(\sigma)CE(\sigma) = CE(\sigma)$.

<u>Theorem 2.</u> (Rosenblum [6]) If $C \in B(X,Y)$ intertwines $A \in B(X)$ and $B \in B(Y)$ and if $\sigma(A) \cap \sigma(B) = \phi$, then $C = 0$.

<u>Proof.</u> Since $CX \subset X_B(\sigma(A)) = X_B(\sigma(A)\cap\sigma(B)) = X_B(\phi) = \{0\}$ by Lemma, $CX = \{0\}$ and hence $C = 0$.

<u>Theorem 3.</u> (Fong [4], Stampfli [7], Williams [8]) If $C \in B(X,Y)$ intertwines $A \in B(X)$ and $B \in B(Y)$ and has dense range

and if B satisfies the Dunford's closure condition, then $\sigma(B) \subset \sigma(A)$.

<u>Proof.</u> Since $\sigma_p^o(B) = \emptyset$ and since $X_B(\sigma(A))$ is closed by the assumption, $Y = \overline{CX} \subset X_B(\sigma(A)) \subset Y$ by Lemma and hence $Y = X_B(\sigma(A))$. Therefore $\sigma(B) = \sigma(B/Y) = \sigma(B/X_B(\sigma(A))) \subset \sigma(A) \cup \overline{\sigma_p^o(B)}$ $= \sigma(A)$ by Proposition.

<u>Theorem 4.</u> (Fialkow [2]) If $C \in B(X, Y)$ intertwines $A \in B(X)$ and $B \in B(Y)$ and is injective and if A is decomposable, then $\sigma(A) \subset \sigma(B)$.

<u>Proof.</u> If $\lambda \notin \sigma(B)$, then there exists an open ball $D_r(\lambda)$ such that $D_r(\lambda) \cap \sigma(B) = \emptyset$, and then $CX_A(D_r(\lambda)) \subset X_B(D_r(\lambda))$ $= X_B(D_r(\lambda) \cap \sigma(B)) = X_B(\emptyset) = \{0\}$ by Lemma and hence $X_A(D_r(\lambda))$ $= X_A(D_r(\lambda) \cap \sigma(A)) = \{0\}$ because C is injective. If $D_r(\lambda) \cap \sigma(A) \neq \emptyset$, then there exists r' such that $0 < r' < r$ and $D_{r'}(\lambda) \cap \sigma(A) \neq \emptyset$. Since $D_r(\lambda)$ and $\mathbb{C} \setminus \overline{D_{r'}(\lambda)}$ are open covering of $\sigma(A)$ and since A is decomposable by the assumption, A satisfies the Dunford's closure condition and $X = X_A(D_r(\lambda)) + X_A(\mathbb{C} \setminus \overline{D_{r'}(\lambda)})$ $\subset X_A(D_r(\lambda)) + X_A(\mathbb{C} \setminus D_{r'}(\lambda)) \subset X$ (see [9]). And since $\sigma(A | X_A(\mathbb{C} \setminus D_{r'}(\lambda)))$ $\subset [\mathbb{C} \setminus D_{r'}(\lambda)] \cap \sigma(A) = \sigma(A) \setminus D_{r'}(\lambda) \subsetneq \sigma(A)$ by Proposition,

$X_A(\mathbb{C}\setminus D_{r'}(\lambda)) \neq X$ and hence $X_A(D_r(\lambda)) \neq \{0\}$. This contradicts with $X_A(D_r(\lambda)) = \{0\}$. Therefore $D_r(\lambda) \cap \sigma(A) = \emptyset$ and $\lambda \notin \sigma(A)$.

Remark. In the above proof, the decomposability of A is only used to show that $X_A(\sigma) = \{0\}$ implies $\sigma \cap \sigma(A) = \emptyset$. If $C \in B(X,Y)$ intertwines $A \in B(X)$ and $B \in B(Y)$ and is injective, then $\sigma_p(A) \subset \sigma_p(B)$ and $\sigma_p^\circ(A) \subset \sigma_p^\circ(B)$, where $\sigma_p(A)$ denotes the point spectrum of A. Because if $\lambda \notin \sigma_p(B)$ and if $(\lambda - A)x = 0$, then $0 = C(\lambda - A)x = (\lambda - B)Cx$ and $Cx = 0$ and hence $x = 0$ because C is injective. Therefore $\lambda \notin \sigma_p(A)$. If $\lambda \notin \sigma_p^\circ(B)$ and if $(z-A)f(z) \equiv 0$ for some analytic function $f : D_r(\lambda) \to X$, then $0 \equiv C(z-A)f(z) = (z-B)Cf(z)$ and $Cf(z) ; D_r(\lambda) \to Y$ is analytic. Hence $Cf(z) = 0$ by the assumption and $f(z) = 0$ because C is injective. Therefore $\lambda \notin \sigma_p^\circ(A)$.

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