

MANIFOLDS WHICH DO NOT ADMIT EXPANSIVE HOMEOMORPHISMS
WITH PSEUDO ORBIT TRACING PROPERTY

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Abstract

In this paper we study some necessary conditions for a homeomorphism of compact manifold to be an expansive homeomorphism with pseudo orbit tracing property and give theorems which are extensions of K. Shiraiwa's result [3]. As their applications, we give some examples of compact manifolds which do not admit expansive homeomorphisms with pseudo orbit tracing property.

The proofs will be treated in the future paper.

Let M be a compact connected topological manifold without boundary with a metric d and f a homeomorphism from M onto itself. We say that f is expansive if there exists $\epsilon > 0$ such that $d(f^n(x), f^n(y)) \leq \epsilon$ for all $n \in \mathbb{Z}$ implies $x = y$. We say that f has the pseudo orbit tracing property (abbrev. P.O.T.P.) if for any $\epsilon > 0$ there exists $\delta > 0$ such that for every sequence $\{x_i\}_{i \in \mathbb{Z}}$ of M with $d(f(x_i), x_{i+1}) < \delta$ for all $i \in \mathbb{Z}$ there exists $x \in M$ such that $d(f^i(x), x_i) < \epsilon$ for all $i \in \mathbb{Z}$. The author [1] defined the notion of orientability of local unstable sets of an expansive homeomorphism f with P.O.T.P..

Theorem 1. Let M be a compact connected topological manifold

without boundary. If $f: M \rightarrow M$ is an expansive homeomorphism with P.O.T.P. and if local unstable sets of f are orientable, then for any $n > 0$ the induced homomorphism $f_*^n: H_*(M, \mathbb{R}) \rightarrow H_*(M, \mathbb{R})$ is not the identity map where $H_*(M, \mathbb{R}) = \bigoplus_{q \geq 0} H_q(M, \mathbb{R})$ and $H_q(M, \mathbb{R})$ denotes the q -dimensional homology group of M with coefficients in the field \mathbb{R} of real numbers.

Corollary 2. Any simply connected rational homology sphere does not admit expansive homeomorphisms with P.O.T.P.. Therefore any manifold whose universal covering space is a rational homology sphere does not admit expansive homeomorphisms with P.O.T.P..

We say that a manifold M satisfies condition (T) if the cohomology algebra $H^*(M, \mathbb{R})$ of M is a graded exterior algebra on generators of odd degree. An element of $H^*(M, \mathbb{R})$ is called decomposable if it is a sum of products of two elements of positive degree. The set D of all decomposable elements of $H^*(M, \mathbb{R})$ is an ideal in the algebra $H^*(M, \mathbb{R})$. Put $P = \bigoplus_{q \geq 1} H^q(M, \mathbb{R}) / D$. Then P has naturally a graded \mathbb{R} -module structure. Let $f: M \rightarrow M$ be a continuous map. Then f induces homomorphisms on $H^*(M, \mathbb{R})$ and on D . Therefore it naturally induces a homomorphism $f_P: P \rightarrow P$.

Theorem 3. Let M be as in Theorem 1 and satisfy condition (T). If $f: M \rightarrow M$ be an expansive homeomorphism with P.O.T.P. and if local unstable sets of f are orientable, then $f_P: P \rightarrow P$ is hyperbolic.

Corollary 4. Let G be a compact connected Lie group. If $f:$

$G \rightarrow G$ is an expansive homeomorphism with P.O.T.P., then $f_p: P \rightarrow P$ is hyperbolic.

Corollary 5. $SO(n)$, $Spin(n)$, $SU(n)$, $SP(n)$ and the exceptional Lie groups G_2 , F_4 , E_6 , E_7 , E_8 do not admit expansive homeomorphisms with P.O.T.P..

Remark. It was proved in [2] that compact surfaces except a torus do not admit expansive homeomorphisms with P.O.T.P..

References

- [1] K.Hiraide, Fixed point indices of expansive homeomorphisms with the pseudo orbit tracing property, preprint.
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- [3] K.Shiraiwa, Some conditions on Anosov diffeomorphisms, Manifolds-Tokyo 1973, Univ. of Tokyo Press, 1975, 205-209.

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