数理解析研究所講究録
第548巻 1985年 201-222

Conjugate Gradient and Chebyshev Accelerations on SAOR Method.

Ichi-ann Ohsaki, Masatoshi Ikeuchi and Hiroshi Niki,
大崎 一安　池内 雅紀　仁木 滉

Okayama University of Science.

Abstract :

The paper is concerned with an improvement over the symmetric accelerated overrelaxation (SAOR) method for an iterative solution of large linear systems. At first, the Chebyshev acceleration (or semi-iteration) is introduced on the SAOR method, and then the Non-Adaptive SAOR-SI algorithm is constructed and moreover by introducing the adaptive procedure which estimates the spectral radius $S(H(\gamma,\omega))$ of the SAOR iteration matrix $H(\gamma,\omega)$, the Partial-Adaptive SAOR-SI algorithm is developed. Next, the conjugate gradient acceleration on the SAOR method is considered, and the Non-Adaptive SAOR-CG algorithm is presented. Moreover by introducing the adaptive procedure which determines the SAOR parameters (γ,ω) automatically, the Adaptive SAOR-CG algorithm is developed. Numerical results including the comparison with the other algorithms are also given. It is finally proved that the proposed algorithms are considerably efficient and guarantee the feasibility for the iterative solution.

. *Introduction.*

We study on the iterative solution of large and sparse linear systems

$$A\,u = b \qquad (1),$$

where A is the $N \times N$ real and nonsingular matrix, b is the $N \times 1$ column vector and u is the $N \times 1$ column vector to be determined. Recently the accelerated overrelaxation (AOR) method was introduced by Hadjidimos[1], which was an iterative method accelerated with two parameters (γ,ω). By an analogy with the symmetric successive overrelaxation (SSOR) method the authors have developed the symmetric AOR (SAOR) method[5,6]. It is natural to consider, the acceleration of convergence on the SAOR method, as well as the SSOR method[4], because the SAOR iteration matrix $H(\gamma,\omega)$ has real and positive eigenvalues [2,6].

In this paper we consider two acceleration procedures on the SAOR method ; one is the Chebyshev acceleration or semi-iteration (Non-Adaptive SAOR-SI algorithm) and the other is the conjugate gradient (CG) acceleration (Non-Adaptive SAOR-CG algorithm). Moreover, we try to apply the adaptive procedure [4,9] to the above Non-Adaptive SAOR-SI and SAOR-CG algorithms. For the Chebyshev acceleration we construct the Partial-Adaptive SAOR-SI algorithm, which employs the adaptively estimated maximum eigenvalue $S(H(\gamma,\omega))$ of the SAOR iteration matrix $H(\gamma,\omega)$. For the CG acceleration we construct the Adaptive SAOR-CG algorithm which includes the procedure to determine the SAOR parameters (γ,ω) automatically. Numerical test is carried out for simple model problems. In the test we give the characteristics of new algorithms and some comparison results. In addition, we will demonstrate the efficiency and feasibility of the present algorithms by further applications for more general problems.

2. *SAOR Method.*

Assume that the coefficient matrix A of (1) is symmetric and positive definite. Without loss of generality, A may be splitted into

$$A = I - L - U \tag{2},$$

where I is the identity, and L and U are respectively the lower and upper triangular parts of A. For the nth iterated vector $u^{(n)}$, the SAOR method is defined [2,5,6] as

$$u^{(n+1/2)} = \tilde{L}(\gamma,\omega)u^{(n)} + k_F \tag{3}$$

and

$$u^{(n+1)} = \tilde{U}(\gamma,\omega)u^{(n+1/2)} + k_B \tag{4},$$

where γ and ω are respectively called the acceleration and overrelaxation parameters. Also $\tilde{L}(\gamma,\omega)$ and $\tilde{U}(\gamma,\omega)$ are respectively the corresponding iteration matrices to the forward AOR and backward AOR methods [2,6] expressed as

$$\tilde{L}(\gamma,\omega) = (I-\gamma L)^{-1}[(1-\omega)I + (\omega-\gamma)L + \omega U] \tag{5}$$

$$= I - \omega(I-\gamma L)^{-1}A$$

and

$$\tilde{U}(\gamma,\omega) = (I-\gamma U)^{-1}[(1-\omega)I + (\omega-\gamma)U + \omega L] \quad (6).$$

$$= I - \omega(I-\gamma U)^{-1}A$$

Eliminating $u^{(n+1/2)}$ from (3) and (4), we obtain

$$u^{(n+1)} = H(\gamma,\omega)u^{(n)} + k(\gamma,\omega) \; ; \quad (7)$$

$$H(\gamma,\omega) = \tilde{U}(\gamma,\omega)\tilde{L}(\gamma,\omega) \quad (8)$$

$$= I - \omega^2(I-\gamma U)^{-1}M(I-\gamma L)^{-1}A$$

and

$$k(\gamma,\omega) = \tilde{U}(\gamma,\omega)k_F + k_B \quad (9),$$

$$= \tilde{U}(\gamma,\omega)(I-\gamma L)^{-1}b + (I-\gamma U)^{-1}b$$

where $H(\gamma,\omega)$ is the SAOR iteration matrix, and M is defined by

$$M = \frac{1}{\omega}[(2-\omega)I + (\omega-\gamma)B] \quad (10),$$

in which $B(=L+U)$ is the Jacobi iteration matrix. Notice that $\gamma=\omega$ $H(\gamma,\omega)$ is equivalent to the iteration matrix of the SSOR method[6].

Now, let $A^{1/2}$ be the square root satisfying $(A^{1/2})^2=A$. Then we can define the matrices $H'(\gamma,\omega)$, $\tilde{L}'(\gamma,\omega)$ and $\tilde{U}'(\gamma,\omega)$ being similar to $H(\gamma,\omega)$, $\tilde{L}(\gamma,\omega)$ and $\tilde{U}(\gamma,\omega)$, respectively, as follows :

$$H'(\gamma,\omega) = A^{1/2}H(\gamma,\omega)A^{-1/2} = \tilde{U}'(\gamma,\omega)\tilde{L}'(\gamma,\omega) \quad (11),$$

where

$$\tilde{L}'(\gamma,\omega) = A^{1/2}\tilde{L}(\gamma,\omega)A^{-1/2} \quad (12)$$

$$= I - \omega A^{1/2}(I-\gamma L)^{-1}A^{1/2}$$

and

$$\tilde{U}'(\gamma,\omega) = A^{1/2}\tilde{U}(\gamma,\omega)A^{-1/2} \quad (13).$$

$$= I - \omega A^{1/2}(I-\gamma U)^{-1}A^{1/2}$$

Since A is symmetric, we can readily see

$$\tilde{U}'(\gamma,\omega) = (\tilde{L}'(\gamma,\omega))^{\mathrm{T}} \quad (14),$$

which in view of (8) gives rise to

$$H'(\gamma,\omega) = (\tilde{L}'(\gamma,\omega))^{\mathrm{T}}(\tilde{L}'(\gamma,\omega)) \quad (15).$$

If we choose γ and ω such that

$$0 < \gamma < 2 \text{ and } \omega + \frac{2-\omega}{m(B)} < \gamma < \omega + \frac{2-\omega}{M(B)} \quad (16),$$

in which $m(B)$ and $M(B)$ are respectively the minimum and maximum eigenvalues of B, then the real symmetric matrix M defined by (10) is proved to be positive definite (see [2,9]). From the relation in (8), we obtain

$$I - H'(\gamma,\omega) = A^{1/2}(I-H(\gamma,\omega))A^{-1/2} \quad (17),$$

$$= [\omega A^{1/2}(I-\gamma L)^{-1}A^{1/2}]^{\mathrm{T}}[\omega A^{1/2}(I-\gamma L)^{-1}A^{1/2}]$$

which is symmetric and positive definite. Hence we can use the $A^{1/2}$ as a symmetrization matrix [3] required in applying the acceleration procedure to the SAOR method.

3. Chebyshev Acceleration.

In the application of the Chebyshev acceleration procedure, it is necessary to assume three values of the SAOR parameters (γ,ω) and the spectral radius $S(H(\gamma,\omega))$ of the SAOR iteration matrix $H(\gamma,\omega)$. In this paper, we will present two versions of the Chebyshev acceleration : one is the Non-Adaptive SAOR-SI algorithm which estimates neither (γ,ω) nor $S(H(\gamma,\omega))$, and the other is the Partial-Adaptive SAOR-SI algorithm which estimates only the $S(H(\gamma,\omega))$.

(1) Non-Adaptive SAOR-SI Algorithm.

Let us define the nth iterated vector $u^{(n)}$ during the Non-Adaptive SAOR-SI algorithm as

$$u^{(n+1)} = \rho_{n+1}(\nu_{n+1}\delta^{(n)} + u^{(n)}) + (1 - \rho_{n+1})u^{(n-1)} \quad (18),$$

where $\delta^{(n)}$ is the pseudo-residual vector represented by

$$\delta^{(n)} = H(\gamma,\omega)u^{(n)} + k(\gamma,\omega) - u^{(n)} \quad (19),$$

also ν_n and ρ_n are the Chebyshev parameters defined by

$$\nu_{n+1} = \frac{S(H(\gamma,\omega))}{1 - S(H(\gamma,\omega))} \quad (20)$$

and

$$\begin{cases} \rho_1 = 1 \\ \rho_2 = (1 - \frac{1}{2}\sigma^2)^{-1} \\ \rho_{n+1} = (1 - \frac{1}{4}\sigma^2 \rho_n)^{-2} \; ; \quad n > 2 \end{cases} \qquad (21),$$

in which σ is given by

$$\sigma = \frac{S(H(\gamma,\omega))}{2 - S(H(\gamma,\omega))} \qquad (22).$$

In the Non-Adaptive algorithm the formulae (18)-(22) are simply iterates until a suitable criterion for convergence is achieved. The algorithm is shown in the flowchart of Figure 1.

(2) Partial-Adaptive SAOR-SI Algorithm.

We try to apply the partially adaptive procedure to the Non-Adaptive SAOR-SI algorithm for estimating the spectral radius $S(H(\gamma,\omega))$ of the SAOR iteration matrix $H(\gamma,\omega)$. The Partial-Adaptive SAOR-SI algorithm is expressed as follows : for the nth iterated vector $u^{(n)}$

$$u^{(n+1)} = \rho_{n+1}(\nu_{n+1}\delta^{(n)} + u^{(n)}) + (1 - \rho_{n+1})u^{(n-1)} \qquad (23),$$

where $\delta^{(n)}$ is the pseudo-residual vector which is the same form with the one given by (19), and ν_n and ρ_n are the Chebyshev parameters defined by

$$\nu_{n+1} = \frac{2}{2 - S_E(H(\gamma,\omega))} \qquad (24)$$

and

$$\rho_{n+1} = \begin{cases} 1 & ; n = s \\ (1 - \frac{1}{2}\sigma_E^2)^{-1} & ; n = s+1 \\ (1 - \frac{1}{4}\sigma_E^2\rho_n)^{-1} & ; n \geqslant s+2 \end{cases} \quad (25),$$

in which

$$\sigma_E = \frac{S_E(H(\gamma,\omega))}{2 - S_E(H(\gamma,\omega))} \quad (26).$$

The Partial-Adaptive version involves the parameter change test and parameter estimation procedures.

(1) Parameter Change Test Procedure.

We change $S(H(\gamma,\omega))$ whenever

$$\frac{\|\delta^{(n)}\|_{A^{1/2}}}{\|\delta^{(s)}\|_{A^{1/2}}} < \left(\frac{2r^{p/2}}{1 + r^p}\right)^F \quad (27),$$

where

$$p = n - s \quad (28)$$

and

$$r = \frac{1 - \sqrt{1 - \sigma_E^2}}{1 + \sqrt{1 - \sigma_E^2}} \quad (29).$$

Here F is the damping factor to be selected in the interval $[0,1]$.

(2) Parameter Estimation Procedure.

Once we have decided to change $S(H(\gamma,\omega))$, we take new value of

$$[S'_E(H(\gamma,\omega))]_{\mathrm{NEW}} = max(\ S_E(H(\gamma,\omega)),\ S'_E(H(\gamma,\omega))\ ) \qquad (30),$$

where $S'_E(H(\gamma,\omega))$ are computed by the following Rayleigh quotient

$$S'_E(H(\gamma,\omega)) = \frac{(\ W\delta^{(n)},\ WH(\gamma,\omega)\delta^{(n)})}{(\ W\delta^{(n)},\ W\delta^{(n)}\ )} \qquad (31).$$

If the new value is determined, we set $s=n$. This partial adaptive procedure is shown in the flowchart of Figure 2.

4. *Conjugate Gradient Acceleration.*

In the application of the CG acceleration procedure, it requires the SAOR parameters (γ,ω) only, so we can consider the two versions of the CG accelerations : one is the Non-Adaptive SAOR-CG algorithm which iterates with the fixed (γ,ω), and the other is the Adaptive SAOR-CG algorithm which includes the procedure to determine (γ,ω) adaptively.

(1) Non-Adaptive SAOR-CG Algorithm.

Let us define the nth iterated vector during the Non-Adaptive SAOR-CG algorithm as

$$u^{(n+1)} = \rho_{n+1}(\ \nu_{n+1}\delta^{(n)} + u^{(n)}\ ) + (\ 1 - \rho_{n+1}\ )u^{(n-1)} \qquad (32),$$

where $\delta^{(n)}$ is the pseudo-residual vector represented by

$$\delta^{(n)} = H(\gamma,\omega)u^{(n)} + k(\gamma,\omega) - u^{(n)} \tag{33}$$

also ν_n and ρ_n are the CG parameters defined by

$$\nu_{n+1} = \left(1 - \frac{(W\delta^{(n)}, WH(\gamma,\omega)\delta^{(n)})}{(W\delta^{(n)}, W\delta^{(n)})} \right)^{-1} \tag{34}$$

and

$$\begin{cases} \rho_1 = 1 \\ \rho_{n+1} = \left(1 - \dfrac{\nu_{n+1}}{\nu_n} \dfrac{(W\delta^{(n)}, W\delta^{(n)})}{(W\delta^{(n-1)}, W\delta^{(n-1)})} \dfrac{1}{\rho_n} \right)^{-1} \end{cases} \tag{35}$$

Instead of W, we employ the $A^{1/2}$ in (34) and (35). The Non-Adaptive version iterates simply the formulae (32)-(35) and are shown in the flowchart of Figure 1.

(2) Adaptive SAOR-CG Algorithm.

Let us introduce the adaptive procedure which determine automatically the SAOR parameters (γ,ω) to the Non-Adaptive SAOR-CG algorithm. Our adaptive procedure involves two procedures : one is the stopping procedure which tests whether the convergence has been achieved or not ; the other is the parameter estimation procedure which determines the SAOR parameters (γ,ω) adaptively.

(1) Stopping Procedure.

A stopping test is expressed as

$$\frac{\| \varepsilon^{(n)} \|}{\| \bar{u} \|} < \frac{1}{1 - M_E} \frac{\| \delta^{(n)} \|}{\| u^{(n)} \|} < \xi \qquad (36),$$

where $\varepsilon^{(n)}$ is the nth error vector defined by $\varepsilon^{(n)}=u^{(n)}-\bar{u}$ and M_E is an estimate of the maximum eigenvalue of $H(\gamma,\omega)$ computed from

$$M_E = M(T_{n,s}) \qquad (37).$$

Here $T_{n,s}$ is the symmetric and tridiagonal matrix given by [9] in which the maximum eigenvalue $M(T_{n,s})$ is computed by the method of bisection.

(2) Parameter Estimation Procedure.

We assume that

$$\begin{cases} m(B) \geqslant m \geqslant -2\sqrt{\beta} \\ M(B) \leqslant M \leqslant 2\sqrt{\beta} \\ M < 1 \\ \rho(LU) \leqslant \beta \end{cases} \qquad (38).$$

If we choose γ as

$$\gamma_1 = \begin{cases} \dfrac{2}{1 + \sqrt{1 - 2M + 4\beta}} & ; \quad M \leqslant 4\beta \\ \\ \dfrac{2}{1 + \sqrt{1 - 4\beta}} & ; \quad M \geqslant 4\beta \end{cases} \qquad (39),$$

then the spectral radius of the SSOR iteration matrix $H(\gamma,\gamma)$ is minimized and given by

$$\rho(H(\gamma_1,\gamma_1)) \leqslant \begin{cases} \dfrac{1 - \dfrac{1-M}{\sqrt{1-2M+4\beta}}}{1 + \dfrac{1-M}{\sqrt{1-2M+4\beta}}} & ; M \leqslant 4\beta \\[2ex] \dfrac{1-\sqrt{1-4\beta}}{1+\sqrt{1-4\beta}} & ; M \geqslant 4\beta \end{cases} \qquad (40).$$

Thus we can surely obtain the minimized spectral radius of the SSOR iteration matrix $H(\gamma,\gamma)$. Furthermore, by use of the parameter s $(=\omega/\gamma)$, it is possible to determine the overrelaxation parameter ω so that

$$\rho(H(\gamma,\omega)) < \rho(H(\gamma_1,\gamma_1)) \qquad (41).$$

The parameter s may be chosen in the interval $[0.95, 1.10]$. If $s=1.0$, then our algorithm is of course reduced to an SSOR-CG algorithm which differs from the Young's version [4]. After each iteration, we compute $M_E=M(T_{n,s})$, and then we change $M_E(B)$ if

$$\Phi(M_E) > \Phi(\rho(H(\gamma_1,\gamma_1)))^F \qquad (42),$$

where $\Phi(X)$ is defined for $[0,1]$ by

$$\Phi(X) = \frac{1-\sqrt{1-X}}{1+\sqrt{1-X}} \qquad (43)$$

and F is called the damping factor. Decided to change the parameter (γ,ω), we compute new $M_E(B)$ from

$$M_E(B) = max(\ M_E(B),\ M'_E(B)\ ) \qquad (44),$$

where

$$M'_E(B) = \frac{\|B\delta^{(n)}\|_A{}^{1/2}}{\|\delta^{(n)}\|_A{}^{1/2}} \qquad (45).$$

Once a new value of γ has been determined, ω and $\rho(H(\gamma,\gamma))$ are readily computed by setting $M{=}M_E(B)$. The iterative formulae for the Adaptive SAOR-CG algorithm are the same forms with the ones as for the Non-Adaptive SAOR-CG algorithm, except

$$\rho_{n+1} = \begin{cases} 1 & ;\ n = s \\ \left(1 - \dfrac{\nu_{n+1}}{\nu_n} \dfrac{(W\delta^{(n)},\ W\delta^{(n)})}{(W\delta^{(n-1)},\ W\delta^{(n-1)})} \dfrac{1}{\rho_n}\right)^{-1} & ;\ n \geqslant s+1 \end{cases} \qquad (46).$$

All the iterative procedures of the Adaptive SAOR-CG algorithm are shown in the flowchart of Figure 3.

5. *Numerical Experiments.*

In order to examine our new algorithms we work out two types of model problems which involve the generalized Dirichlet problem with respect to the elliptic partial differential equation

$$\frac{\partial}{\partial x}\left(A\frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial y}\left(C\frac{\partial U}{\partial y}\right) = 0 \qquad (47)$$

in the unit square domain ($0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1$), where $U=0$ is imposed on the whole boundary. Various choices of the coefficients $A(x,y)$ and $C(x,y)$ [8,9] are considered. We now deal with the first type (model 1) that $A(x,y)=1$ and $C(x,y)=1$, i.e., the Laplace's equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \qquad (48).$$

Here the five-points difference formula is adopted for the discretization of the model problem. All the algorithms to be treated in the numerical experiments are terminated when the iterated vector $u^{(n)}$ is satisfied by the following criterion :

$$\| \varepsilon^{(n)} \| < \xi = 10^{-6} \qquad (49),$$

where $\varepsilon^{(n)}$ is the nth error vector for the exact solution $\bar{u}$. Also the initial vector $u^{(0)}$ is chosen such as all its elements are equal to be $1/(1/h-1)$, in which h is the square mesh size.

(1) Characteristics of Chebyshev and CG Accelerations.

At first, we shall expose the characteristics of the Chebyshev and CG accelerations on the SAOR method. Figure 4 and Figure 5 show the iteration numbers required for convergence in connection with the damping factor F in the Partial-

Adaptive SAOR-SI and Adaptive SAOR-CG algorithms. If we work with F being very close to the unity, we can see that the parameters (γ,ω) are changing much frequently. With too small values of F, they are not changing enough. However, as seen from the results in Figure 4,5, the effectiveness of the adaptive procedure is relatively insensitive to F only in view of the iteration numbers required for convergence. Thus it seems that the selection of F is not very important, but if F is very close to the unity, it causes the loss of computational time and the waste of arithmetic works for the adaptive procedure (see [9]). Taking account of the above facts, we take the damping factor $F = 0.75$ and 0.85 as their typical values for the Partial-Adaptive SAOR-SI and Adaptive SAOR-CG algorithms, respectively. Figure 6 and Figure 7 show the convergence domain with respect to the SAOR parameter γ and s $(=\omega/\gamma)$ in the SAOR algorithm and the Non-Adaptive SAOR-SI and Adaptive SAOR-CG algorithms, respectively. In the Partial-Adaptive SAOR-SI algorithm the input data $S(H(\gamma,\omega))=0.99$ is employed. By combining with the Chebyshev or CG acceleration procedure, the convergence domain in the SAOR method is extended, and thus we can expect a fast convergence for a rough selection of (γ,ω) in each accelerated SAOR algorithms.

(2) Comparisons.

Here we will give two comparisons. One is the comparison with the accelerated SAOR algorithms and the optimum SOR algorithm. The other is the comparison with the Chebyshev accelerations and the CG accelerations on the SAOR method. Table 1 gives the iteration numbers required for convergence in all the algorithms, i.e., the Non-Adaptive SAOR-SI algorithm, Partial-Adaptive SAOR-SI algorithm, the Non-Adaptive SAOR-CG algorithm, the Adaptive SAOR-CG algorithm and the optimum SOR algorithm. In three of the Non-Adaptive SAOR-SI algorithm, Partial-Adaptive SAOR-SI algorithm and Non-Adaptive SAOR-CG algorithm, the SAOR parameters (γ,ω) are taken as $(\gamma,\omega) = (1.40, 1.54)$ and (γ_b,ω_b), where (γ_b,ω_b) are the optimum parameters determined experimentally. In the Adaptive SAOR-CG algorithm, the SAOR parameters (γ,ω) are determined automatically during the iteration process. Also the input data for the estimates of $S(H(\gamma,\omega))$ in the Non-Adaptive SAOR-SI algorithm are *0.99*. The SOR parameter ω is taken as $\omega = 2(1 + \sqrt{1-M(B)^2}\ )^{-1}$,which $M(B)$ is the maximum eigenvalue of the Jacobi iteration matrix B. As expected, the SAOR method accelerated with the Chebyshev procedure or CG procedure has achieved the considerably faster convergence than the optimum SOR algorithm. Next we consider a comparison with the Chebyshev acceleration and CG acceleration on the SAOR method. In the case of the non-adaptive versions, the effectiveness of both the algorithms with the optimum parameters are almost comparable. However, taking account that the Non-Adaptive SAOR-SI algorithm requires not only the SAOR parameters (γ,ω) but also $S(H(\gamma,\omega))$, it is advantageous for the CG acceleration to require only the SAOR parameters (γ,ω) and to be at least comparable in the iteration numbers required for convergence to the Non-Adaptive SAOR-SI algorithm. In the case of the adaptive versions, we can suggest that the Partial-Adaptive SAOR-SI algorithm is inferior to the Adaptive SAOR-CG algorithm because of the same facts with the above.

(3) Further Applications.

We try to test the feasibility and efficiency of our new algorithms for more general problems, i.e., we choose the coefficients $A(x,y)$ and $C(x,y)$ in (47) as in Table 2. Table 2 gives the iteration numbers required for convergence in the Non-Adaptive SAOR-SI algorithm, the Partial-Adaptive SAOR-SI algorithm and the Adaptive SAOR-CG algorithm. It is clear that the new algorithms are superior to the SOR algorithm, even in comparison with the Non-Adaptive SAOR-SI algorithm.

6. *Concluding Remarks.*

In this paper, we have proposed the new algorithms, the Non-Adaptive and Partial-Adaptive SAOR-SI algorithms based on the Chebyshev acceleration and the Non-Adaptive and Adaptive SAOR-CG algorithms based on the CG acceleration. In the numerical test, the following observations are made :

(1) In the test of the non-adaptive versions, i.e., the Non-

Adaptive SAOR-SI and Non-Adaptive SAOR-CG algorithms, they have enough fast convergence in spite of the haphazard selection of the parameters (γ,ω). Moreover, when the optimum parameters are employed, they converges fast considerably.

(2) From the fact that fewer iteration numbers and no parameter requirement, it can be suggested that the Adaptive SAOR-CG algorithm are far superior to the other algorithms including even the Partial-Adaptive SAOR-SI algorithm.

(3) The introduction of the adaptive procedures make the application to more general problems possible. Thus it proves that the Adaptive SAOR-CG algorithm is feasible for iterative solution.

In the future, we will attempt to develop the Full-Adaptive SAOR-SI algorithm which estimates both the SAOR parameters (γ,ω) and the spectral radius $S(H(\gamma,\omega))$.

References :

[1] A.Hadjidimos, Accelerated overrelaxation method, Math.Comp. 32(1978)147-157.

[2] A.Hadjidimos, Symmetric accelerated overrelaxation (SAOR) method, MACS.(1982)72-76.

[3] L.A.Hageman and D.M.Young, Applied Iterative Methods (Academic Press, New-York, 1981).

[4] L.A.Hayes and D.M.Young, The accelerated SSOR method for solving large linear systems : preliminary report, CNA123 (1977).

[5] S.Yamada, M.Ikeuchi, H.Sawami and H.Niki, Convergence rate of symmetric accelerated overrelaxation method, in : Proceedings of the 24th National Congress (IPSJ, Tokyo, 1982)897-898.

[6] S.Yamada, Adaptive SAOR-CG Algorithm for Large Linear Systems (master thesis) (Okayama University of Science , Okayama, 1983).

[7] D.M.Young, Iterative Solution of Large Linear Systems (Academic Press, New York, 1971).

[8] D.M.Young, On the accelerated SSOR method for solving large linear systems, CNA92(1974).

[9] S.Yamada, I.Ohsaki, M.Ikeuchi and H.Niki, Non-Adaptive and adaptive SAOR-CG algorithms, (submitted).

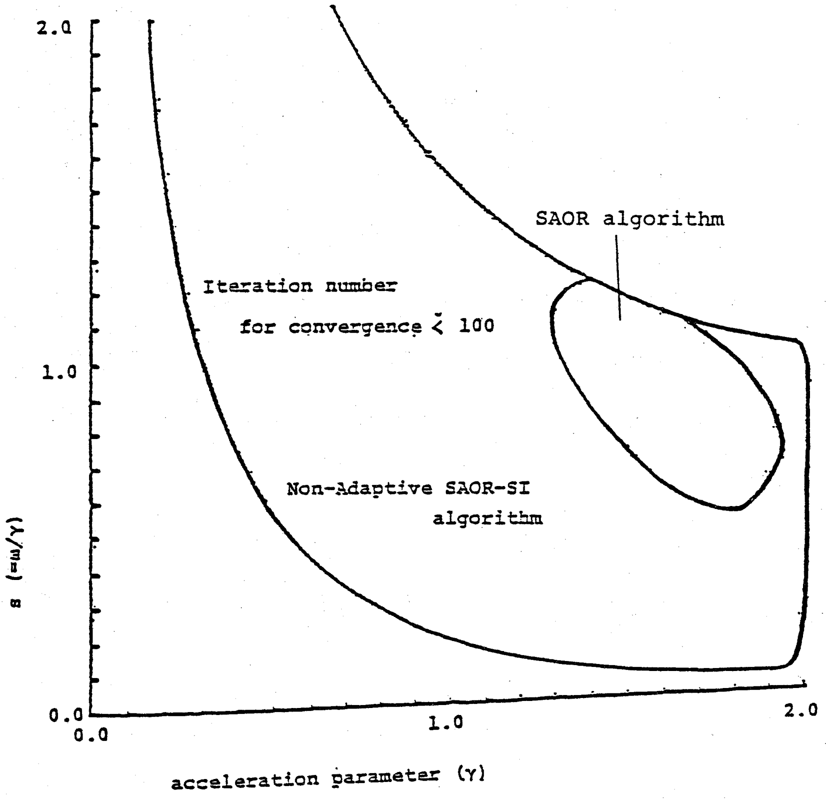

Figure 6. Convergence domain of Non-Adaptive SAOR-SI algorithm.

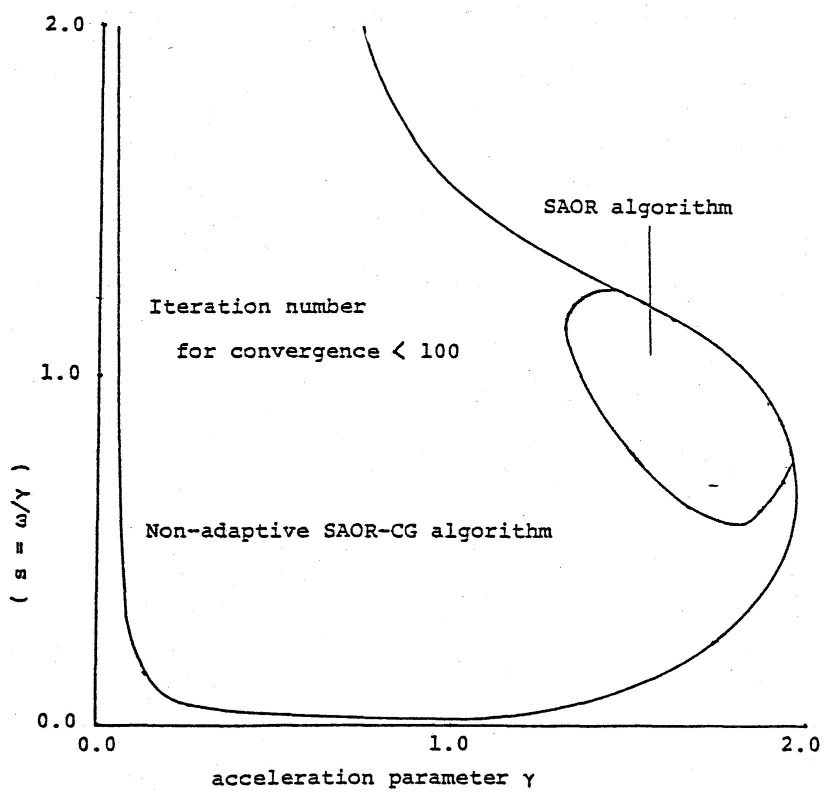

Figure 7. Convergence domain of Non-Adaptive SAOR-CG algorithm.

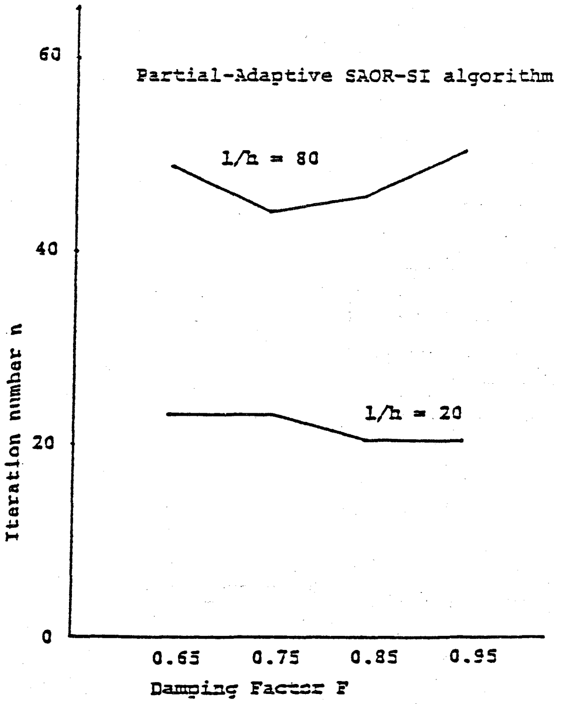

Figure 4. Effectiveness of damping factor in the Partial-Adaptive SAOR-SI algorithm.

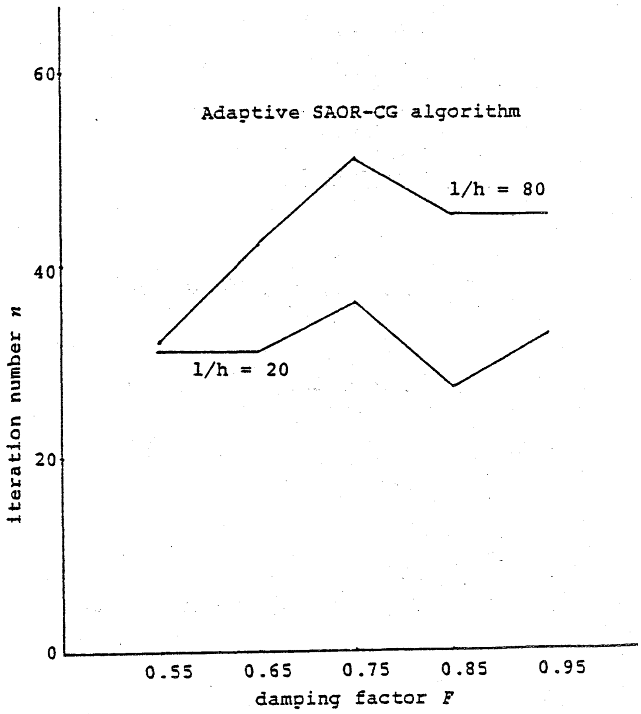

Figure 5. Effectiveness of damping factor in the Adaptive SAOR-CG algorithm.

Table 2. Further application (model 2).

$A = C = e^{10(x+y)}$

1/h	20	40	60	80	100
SOR algorithm	72	161	241	321	401
Non-Adaptive SAOR-SI algorithm					
(r,w) = (1.40,1.54) ME = 0.99	88	90	90	89	89
Partial-Adaptive SAOR-SI algorithm					
(r,w) = (ropt,wopt) F = 0.65	22	32	42	50	53
Adaptive SAOR-CG algorithm F = 0.85	26	61	87	102	95

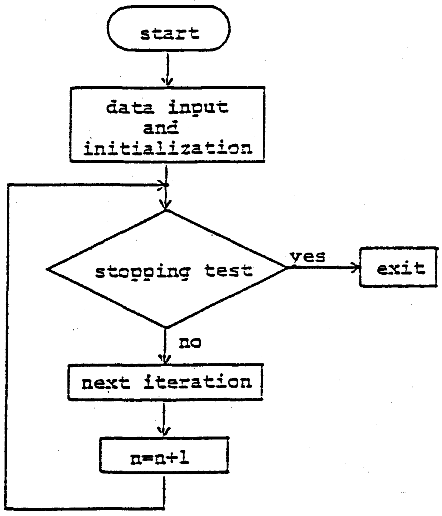

Figure 1. Flowchart of Non-Adaptive SAOR-SI and SAOR-CG algorithms.

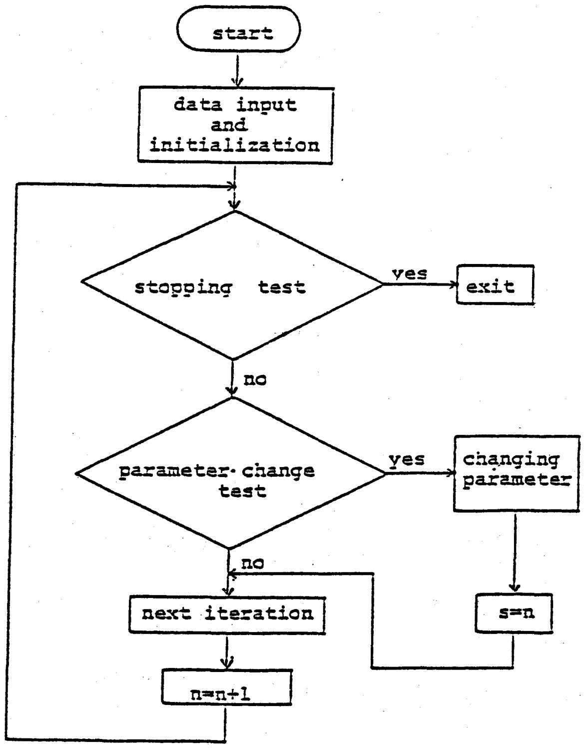

Figure 2. Flowchart of Partial-Adaptive SAOR-SI algorithm.

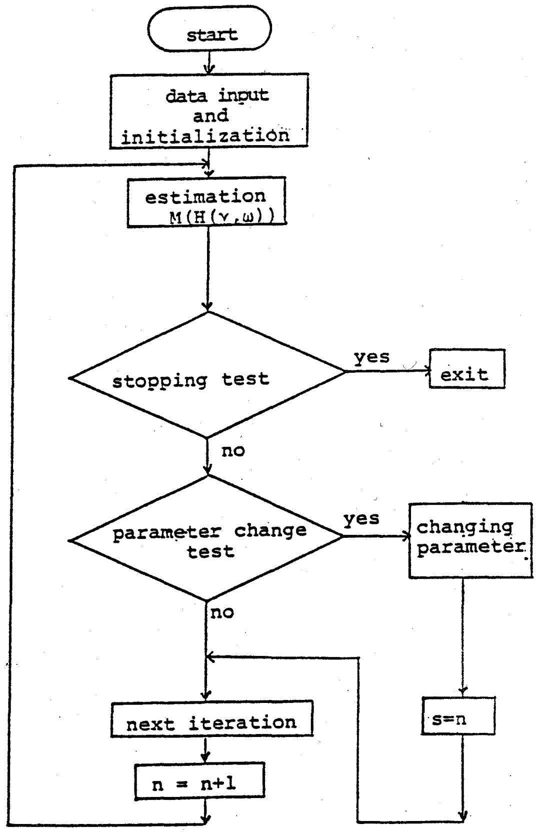

Figure 3. Flowchart of Adaptive SAOR-CG algorithm.