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DIFFERENTIABLE SINGULAR COHOMOLOGY RELATED TO FOLIATION

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Introduction

Let $(M,\mathcal{F})$ be a C^∞-foliation of codimension q on a paracompact Hausdorff manifold of dimension n. T(M) be the tangent bundle of M and $F = T(\mathcal{F})$ the subbundle of T(M) consisting of tangent vectors of leaves of $\mathcal{F}$. Let V denote the normal bundle of $\mathcal{F}$ with respect to a Riemannian metric on M. Then we have the splitting of T(M) into a Whitney sum:

$$T(M) = F \oplus V$$

and the dual splitting of cotangent bundle,

$$T^*(M) = V^* \oplus F^*.$$

On a local foliation chart, $(x,u)\colon U \to R^p \times R^q$ one chooses differential forms

$$\theta_j = dx_j + \sum_{\alpha=1}^{q} a_{j\alpha} du_\alpha \qquad 1 \leq j \leq p,$$

$$\nu_\alpha = \partial/\partial u_\alpha - \sum_{j=1}^{p} a_{j\alpha} \partial/\partial x_j \qquad 1 \leq i \leq q$$

such that $\{\theta_1,\cdots,\theta_p,du_1,\cdots,du_q\}$, $\{\partial/\partial x_1,\cdots,\partial/\partial x_p,v_1,\cdots,v_q\}$ are dual bases of $T_m^*(M)$ and $T_m(M)$, $m \in U$.

Let $A^{r,s}$ be the vector space of differential forms which are locally

$$\sum a_{i_1\cdots i_r j_1\cdots j_s} du_{i_1}\cdots du_{i_r}\theta_{j_1}\cdots\theta_{j_s}.$$

Let A(M) denote the vector space of C^∞-differential forms on M. Then we have

$$A(M) = \oplus A^{r,s}.$$

The exterior derivative of an $\omega \in A^{r,s}$ is of the form

$$d\omega = d_1\omega + d_2\omega + d_{\mathcal{F}}\omega$$

where $d_1\omega \in A^{r+2,s-1}$, $d_2\omega = A^{r+1,s}$ and $d_{\mathcal{F}}\omega \in A^{r,s+1}$ are uniquely determined.

From the relation $d^2 = 0$, it follows that

$$d_1^2 = 0,\ d_{\mathcal{F}}^2 = 0,\cdots\cdots.$$

In particular $d_{\mathcal{F}}: A^{r,s} \to A^{r,s+1}$ defines a cohomology vector space $H^{r,s}_{\mathcal{F}DR}(M)$ with transverse degree r and leaf degree s.

Let $C^{\infty}_{\mathcal{F}}$ denote the sheaf of germs of real valued C^{∞}-functions on M constant along leaves of $\mathcal{F}$ and $\check{H}^{s}(M;C^{\infty}_{\mathcal{F}})$, the Čech cohomology vector space of M with the coefficient sheaf $C^{\infty}_{\mathcal{F}}$. We have already a de Rham type isomorphism

$$H^{0,s}_{\mathcal{F}DR}(M) \overset{\cong}{\to} H^{s}(M;C^{\infty}_{\mathcal{F}}),$$

as a part of the Dolbealt isomorphism for foliation. (Cf., e.g., [V, Théorèm 3,2], [S, Theorem 4.2].)

In the present paper, we establish a singular cohomology version of this isomorphism. Let $C^{\mathcal{F}}_{*}(M;R)$ be the chain complex with the coefficient group R generated by differentiable singular simplexes in leaves of $\mathcal{F}$. Then we introduce a <u>differentiable</u> singular cochain complex $C^{*}_{\mathcal{F}}(M,R)$ for $C^{\mathcal{F}}_{*}(M;R)$. (See Section 2 below.)

Let $H^{s}_{\mathcal{F}D}(M;R)$ denote the cohomology vector space of $C^{*}_{\mathcal{F}D}(M;R)$ and $\Lambda: A^{0,s} \to C^{s}_{\mathcal{F}D}(M;R)$, the homomorphism defined by the integration

$$\int_{\sigma_s^{\mathcal{F}}} \omega$$

of $\omega \in A^{0,s}$ on a simplex $\sigma_s^{\mathcal{F}} \in C_s^{\mathcal{F}}(M;R)$.

MAIN THEOREM. Λ <u>induces an isomorphism</u>

$$\Lambda^*: H^{0,s}_{\mathcal{F}DR}(M) \xrightarrow{\cong} H^s_{\mathcal{F}D}(M;R).$$

(See Section 3.)

This relation can be used to give Weil operators [HH, DEFINITION 1.6] a meaning as cohomology classes (cf. [S. Theorem 5.4]).

All manifolds, maps and foliations are assumed to be of class C^∞ and manifolds are without boundaries.

1. Differentiable singular chains in leaves

Let $(M,\mathcal{F})$ be the C^∞-foliation of codimension q on the paracompact Hausdorff manifold of dimension n. Let $\sigma_s^{\mathcal{F}}$ be a C^∞-singular s-simplex such that the image of $\sigma_s^{\mathcal{F}}$ is contained in a leaf of $\mathcal{F}$.

Let $C_s^{\mathcal{F}}(M;R)$ denote the vector space over R with the basis $\{\sigma_s^{\mathcal{F}}\}$. Then we have obviously

$$\partial C_s^{\mathcal{F}}(M;R) \subset C_{s-1}^{\mathcal{F}}(M;R)$$

for the boundary operator ∂, and obtain a chain complex

$$C_*^{\mathcal{F}}(M;R): \cdots \xrightarrow{\partial} C_s^{\mathcal{F}}(M;R) \xrightarrow{\partial} C_{s-1}^{\mathcal{F}}(M;R) \xrightarrow{\partial} \cdots \xrightarrow{\partial} C_0^{\mathcal{F}}(M;R) \to 0,$$

$$C_0^{\mathcal{F}}(M;R) = \sum_{m\in M} R_m \qquad (R_m \cong R).$$

Let $(M',\mathcal{F}')$ be a codimension q foliation on a manifold M' of dimension n'. Let $f: M \to M'$ be a C^∞-map which is transverse to $\mathcal{F}'$ and preserves leaves, i.e., for each leaf L of $\mathcal{F}$, there is a leaf L' of $\mathcal{F}'$ such that $f(L) \subset L'$. One can see that $\mathcal{F}$ coincides with the pullback $f^*\mathcal{F}'$ of $\mathcal{F}'$. f induces a chain map

$$f_\#: C_*^{\mathcal{F}}(M;R) \to C_*^{\mathcal{F}'}(M';R).$$

Let f_0, f_1: $M \to M'$ be C^∞-maps transverse to $\mathcal{F}'$ so that $f_0^*\mathcal{F}' = f_1^*\mathcal{F}' = \mathcal{F}$. If there is a C^∞-map $H: M\times R \to M'$ transverse to $\mathcal{F}'$ such that

$$f_i(m) = H(m,i) \qquad i = 0, 1,$$

$$H^*\mathcal{F}' = \pi^*\mathcal{F}$$

where π: $M \times R \to M$ is the first factor projection, then f_0, f_1 are called C^∞-homotopic by leaf preserving map and denoted by $f_0 \; {}_{\mathcal{F}'} \simeq_{\mathcal{F}} \; f_1$. H is called leaf preserving C^∞-homotopy of f_0 and f_1.

Let $H_s^{\mathcal{F}}(M;R)$ denote the s-dimensional homology vector space of $C_*^{\mathcal{F}}(M;R)$. The chain map $f_\#$: $C_*(M;R) \to C_*^{\mathcal{F}'}(M';R)$ induces the homomorphism of homology vector spaces, f_*: $H_*^{\mathcal{F}}(M;R) \to H_*^{\mathcal{F}'}(M';R)$.

Since affine simplexes are differentiable ones and the homotopy H: $M \times R \to M'$ preserves leaves, prism operators are well defined in $C_*^{\mathcal{F}}(M;R)$ and one obtains,

PROPOSITION 1.1. *If f_0, f_1: $M \to M'$ are homotopic by leaf preserving map, then $f_{0\#}$ and $f_{1\#}$ are chain homotopic and hence we have*

$$f_{0\#} = f_{1\#}: H_s^{\mathcal{F}}(M;R) \to H_s^{\mathcal{F}'}(M';R).$$

Again since affine simplexes are differentiable ones, as well as prism operator, one can construct subdivision operators

$Sd_s^{\mathcal{F}}$: $(C_s^{\mathcal{F}}(M;R) \to C_s^{\mathcal{F}}(M;R)$ which are chain maps and one obtains the chain homotopies $\Phi_s^{\mathcal{F},Sd}$: $C_s^{\mathcal{F}}(M;R) \to C_{s+1}^{\mathcal{F}}(M;R)$ between the identity operator and $Sd_s^{\mathcal{F}}$ by the usual manner (see, e.g., [G, p.65]).

An open set $X \subset M$ has the restricted foliation of $\mathcal{F}$ denoted by the same symbol. $C_s^{\mathcal{F}}(X;\mathcal{F})$ has an excision property as follows:

PROPOSITION 1.2. *If X_1 and X_2 are open sets of M, then the natural inclusion map* ι: $C_*^{\mathcal{F}}(X_1;R)+C_*^{\mathcal{F}}(X_2;R) \to C_*^{\mathcal{F}}(X_1 \cup X_2;R)$ *gives a chain homotopy equivalence*.

Let λ_k: $X_1 \cap X_2 \to X_k$ and μ_k: $X_k \to X_1 \cup X_2$ be the natural inclusion maps for $k = 0, 1$, which induce chain maps $\lambda_{k\#}$: $C_*^{\mathcal{F}}(X_1 \cap X_2;R) \to C_*^{\mathcal{F}}(X_k;R)$ and $\mu_{k\#}$: $C_*^{\mathcal{F}}(X_k;R) \to C_*^{\mathcal{F}}(X_1 \cup X_2;R)$.

We define chain maps

$$\lambda: C_*^{\mathcal{F}}(X_1 \cap X_2;R) \to C_*^{\mathcal{F}}(X_1;R) \oplus C_*^{\mathcal{F}}(X_2;R),$$

$$\mu: C_*^{\mathcal{F}}(X_1;R) \oplus C_*^{\mathcal{F}}(X_2;R) \to C_*^{\mathcal{F}}(X_1;R) + C_*^{\mathcal{F}}(X_2;R)$$

by $\lambda(c) = (\lambda_{1\#}c, -\lambda_{2\#}c)$ and $\mu(c_1, c_2) = \mu_{1\#}c_1+\mu_{2\#}c_2$. One obtains a short exact sequence of chain complexes,

$$0 \to C_*^{\mathcal{F}}(X_1 \cap X_2;R) \xrightarrow{\lambda} C_*^{\mathcal{F}}(X_1;R) \oplus C_*^{\mathcal{F}}(X_2;R) \xrightarrow{\mu} C_*^{\mathcal{F}}(X_1;R)+C_*^{\mathcal{F}}(X_2;R) \to 0.$$

Let λ_* and μ_* be homology homomorphisms induced by λ and μ respectively.

COROLLARY 1.3. *If X_1 and X_2 are open sets of M, then we have the Mayer-Vietoris exact sequence of* $H_*^{\mathcal{F}}$:

$$\cdots \to H_i^{\mathcal{F}}(X_1 \cap X_2;R) \xrightarrow{\lambda_*} H_i^{\mathcal{F}}(X_1;R) \oplus H_i^{\mathcal{F}}(X_2;R)$$
$$\xrightarrow{\mu_*} H_i^{\mathcal{F}}(X_1 \cup X_2;R) \xrightarrow{\partial_*} H_{i-1}^{\mathcal{F}}(X_1 \cap X_2;R) \to \cdots,$$

where ∂_* is the connecting homomorphism.

2. Differentiable singular cochains restricted leaves

Let Δ^s be the standard s-simplex and $D^q(\varepsilon) \subset R^q$ be an open ε-ball around the origin for sufficiently small number $\varepsilon > 0$ and $\hat{\sigma}_s^{\mathcal{F}}: D^q(\varepsilon)\times\Delta^s \to M$ any differentiable map such that

$$\hat{\sigma}_s^{\mathcal{F}}\{x\} = \hat{\sigma}_s^{\mathcal{F}} \,|_{\{x\}\times\Delta^s} \in C_s^{\mathcal{F}}(M;R),$$

for each $x \in D^q(\varepsilon)$. An s-cochain ξ for $C_s^{\mathcal{F}}(M;R)$ is called <u>differentiable</u> with $\mathcal{F}$ if the value $\xi(\hat{\sigma}_s^{\mathcal{F}}\{x\})$ is differentiable with respect to x and $\hat{\sigma}_s^{\mathcal{F}}$ is called an ε-<u>thickening</u> of $\sigma_s^{\mathcal{F}}$ if $\sigma_s^{\mathcal{F}} = \hat{\sigma}_s^{\mathcal{F}}(0)$.

Let δ denote the usual coboundary operator for cochains of $C_s^{\mathcal{F}}(M;R)$. We denote the set of differentiable cochains of $C_s^{\mathcal{F}}(M;R)$ by $C_{\mathcal{F}D}^s(M;R)$. This is a vector subspace of the s-cochain vector space $C_{\mathcal{F}}^s(M;R)$.

Let $e_i: \Delta^{s-1} \to \Delta^s$, $i = 0,\cdots,s$ be the standard face map and $\hat{\sigma}_s^{\mathcal{F}}$, a differentiable ε-thickening of $\sigma_s^{\mathcal{F}}$. Then the map $\widehat{\partial_i\sigma}_s^{\mathcal{F}}: D^q(\varepsilon)\times\Delta^{q-1} \to M$ defined by

$$\widehat{\partial_i\sigma}_s^{\mathcal{F}} = \hat{\sigma}_s^{\mathcal{F}}\cdot(id_{D^q(\varepsilon)}\times e_i)$$

is obviously a C^∞ ε-thickening of $\partial_i\sigma_s^{\mathcal{F}}$. So, if $\xi \in C_{\mathcal{F}D}^{s-1}(M;R)$ and $\varepsilon > 0$ is sufficiently small, then $\xi(\widehat{\partial_i\sigma}_s^{\mathcal{F}}\{x\})$ is differentiable with respect to $x \in D^q(\varepsilon)$ for $i = 0,\cdots,s$.

Therefore we have

$$\delta\xi(\hat{\sigma}_s\{x\}) = \xi(\partial\hat{\sigma}_s^{\mathcal{F}}\{x\})$$

$$= \sum_{i=0}^{s} (-1)^i \xi(\partial_i(\hat{\sigma}_s^{\mathcal{F}}\{x\}))$$

$$= \sum_{i=0}^{s} (-1)^i \xi(\widehat{\partial_i \sigma}_s^{\mathcal{F}}\{x\}).$$

The last formula shows $\delta\xi(\hat{\sigma}_s^{\mathcal{F}}\{x\})$ is differentiable with respect to $x \in D^q(\varepsilon)$, and so we have $\delta\xi \in C^s_{\mathcal{F}D}(M;R)$. Thus one obtains,

LEMMA 2.1. $C^*_{\mathcal{F}D}(M;R) = \{C^s_{\mathcal{F}D}(M;R),\delta\}$ _is a cochain complex_.

$C^*_{\mathcal{F}D}(M;R)$ is called the _differentiable singular cochain complex_ for the foliation $(M,\mathcal{F})$. In the rest of this section we introduce cochain maps induced by transverse C^∞-maps for foliations, cochain homotopies induced by leaf preserving C^∞-homotopies between transverse C^∞-maps and cochain homotopies between cochain subdivision operators. These are obtained by checking that the images satisfy differentiability condition for ε-thickenings of differentiable singular simplexes contained in leaves.

Let $(M,\mathcal{F})$ and $(M',\mathcal{F}')$ be C^∞-foliations of the same codimension q and $f: M \to M'$ a C^∞-map transverse to $\mathcal{F}'$ such that $f^*\mathcal{F}' = \mathcal{F}$.

LEMMA 2.2. f <u>induces a cochain map</u> $f^\#: C^*_{\mathcal{F}'D}(M';R) \to C^*_{\mathcal{F}D}(M;R)$.

Let f_0, f_1: $M \to M'$ be C^∞-maps transverse to $\mathcal{F}'$ so that $f_0^*\mathcal{F}' = f_1^*\mathcal{F}' = \mathcal{F}$. Assume that $f_0 \;{}_{\mathcal{F}'}\!\simeq_{\mathcal{F}}\; f_1$ by a leaf preserving C^∞-homotopy H: $M\times R \to M'$.

LEMMA 2.3. <u>If we have</u> $f_0 \;{}_{\mathcal{F}'}\!\simeq_{\mathcal{F}}\; f_1$, <u>then</u> $f_0^\#$, $f_1^\#$: $G^*_{\mathcal{F}'D}(M';R) \to C^*_{\mathcal{F}D}(M;R)$ <u>are cochain homotopic</u>.

Let $Sd^{\mathcal{F}} = \{Sd_s^{\mathcal{F}}\}$: $C_*^{\mathcal{F}}(M;R) \to C_*^{\mathcal{F}}(M;R)$ be the subdivision operator and $\Phi^{\mathcal{F},Sd} = \{\Phi_s^{\mathcal{F},Sd}\}$: $C_*^{\mathcal{F}}(M;R) \to C_{*+1}^{\mathcal{F}}(M;R)$ the chain homotopy operator between the identity operator and $Sd^{\mathcal{F}}$. We define cochain map $Sd_{\mathcal{F}} = \{Sd^s_{\mathcal{F}}\}$: $C^*_{\mathcal{F}}(M;R) \to C^*_{\mathcal{F}}(M;R)$ by the formula,

$$(Sd^s_{\mathcal{F}}\xi)(\sigma^s_{\mathcal{F}}) = \xi(Sd_s^{\mathcal{F}}\sigma_s^{\mathcal{F}}),\quad \xi \in C^s_{\mathcal{F}}(M;R),\ \sigma_s^{\mathcal{F}} \in C_s^{\mathcal{F}}(M;R)$$

and also define a homomorphism

$\Phi_{\mathcal{F},Sd} = \{\Phi^s_{\mathcal{F},Sd}\}: C^*_{\mathcal{F}}(M;R) \to C^{*-1}_{\mathcal{F}}(M;R)$ by the formula,

$$(\Phi^s_{\mathcal{F},Sd}\xi)(\sigma^{\mathcal{F}}_{s-1}) = \xi(\Phi^{\mathcal{F},Sd}_{s-1}\sigma^{\mathcal{F}}_{s-1}),\ \xi \in C^s_{\mathcal{F}}(M;R),$$

$$\sigma^{\mathcal{F}}_{s-1} \in C_{s-1}(M;R).$$

LEMMA 2.4. $Sd_{\mathcal{F}}$ _is a cochain map_ $C^*_{\mathcal{F}D}(M;R) \to C^*_{\mathcal{F}D}(M;R)$ _and_ $\Phi_{\mathcal{F},Sd}$ _is a cochain homotopy_ $C^*_{\mathcal{F}D}(M;R) \to C^{*-1}_{\mathcal{F}D}(M;R)$ _between the identity map and_ $Sd_{\mathcal{F}}$.

3. Differentiable singular cohomology for foliation

The cohomology vector space $H^*_{\mathcal{F}D}(M;R) = \bigoplus_{s\geq 0} H^s_{\mathcal{F}D}(M;R)$ of the differentiable singular cochain complex $C^*_{\mathcal{F}D}(M;R)$ in LEMMA 2.1 is called the _differentiable singular cohomology for the foliation_ $(M,\mathcal{F})$.

Let $(M',\mathcal{F}')$ be another foliation and $f: M \to M'$, a C^∞-map transverse to $\mathcal{F}'$ such that $f^*\mathcal{F}' = \mathcal{F}$. By LEMMA 2.2, f induces a homomorphism of differentiable singular cohomology vector spaces:

$$f^*: H^*_{\mathcal{F}'D}(M';R) \to H^*_{\mathcal{F}D}(M;R).$$

Let $f_0, f_1: M' \to M$ be C^∞-maps transverse to $\mathcal{F}'$ so that $f_0^*\mathcal{F}' = f_1^*\mathcal{F}' = \mathcal{F}$. Assume that $f_0 \simeq_{\mathcal{F}',\mathcal{F}} f_1$. Then by LEMMA 2.3, we have

$$f_0^* = f_1^*: H^*_{\mathcal{F}'D}(M';R) \to H^*_{\mathcal{F}D}(M;R).$$

If X_1 and X_2 are open sets of M, then the natural inclusion map $\iota: C^{\mathcal{F}}_*(X_1;R)+C^{\mathcal{F}}_*(X_2;R) \to C^{\mathcal{F}}_*(X_1 \cup X_2;R)$ induces a cochain map

$$\iota^\#: C^*_{\mathcal{F}D}(X_1 \cup X_2;R) \to C^*_{\mathcal{F}D}(X_1;R) + C^*_{\mathcal{F}D}(X_2;R).$$

By making use of a chain homotopy equivalence of PROPOSITION 1.2 and LEMMA 2.4, one obtains,

LEMMA 3.1. $\iota^\#$ _is a cochain homotopy equivalence_.

Let $X_k \subset M$ $k = 1, 2$ be open sets and let $\lambda_k: X_1 \cap X_2 \to X_k$ and $\mu_k: X_k \to X_1 \cup X_2$ be the natural inclusion maps. They induce cochain maps $\lambda_k^\#: C^*_{\mathcal{F}D}(X_k;R) \to C^*_{\mathcal{F}D}(X_1 \cap X_2;R)$ and $\mu_k^\#: C^*_{\mathcal{F}D}(X_1 \cup X_2;R) \to C^*_{\mathcal{F}D}(X_k;R)$.

We define cochain maps

$$\lambda^\#: C^*_{\mathcal{F}D}(X_1;R) \oplus C^*_{\mathcal{F}D}(X_2;R) \to C^*_{\mathcal{F}D}(X_1 \cap X_2;R),$$

$$\mu^{\#}: C^{*}_{\mathcal{F}D}(X_1;R)+C^{*}_{\mathcal{F}D}(X_2;R) \to C^{*}_{\mathcal{F}D}(X_1;R)\oplus C^{*}_{\mathcal{F}D}(X;R)$$

by $\lambda^{\#}(\xi_1, \xi_2) = \lambda^{\#}_1(\xi_1)-\lambda^{\#}_2(\xi_2)$ and $\mu^{\#}(\xi) = (\mu^{\#}_1(\xi), \mu^{\#}_2(\xi))$.

One obtains a short exact sequence of cochain complexes

$$0 \to C^{*}_{\mathcal{F}D}(X_1;R)+C^{*}_{\mathcal{F}D}(X_2;R) \xrightarrow{\mu^{\#}} C^{*}_{\mathcal{F}D}(X_1;R)\oplus C^{*}_{\mathcal{F}D}(X_2;R)$$

$$\xrightarrow{\lambda^{\#}} C^{*}_{\mathcal{F}D}(X_1\cap X_2;R) \to 0.$$

Let λ^{*} and μ^{*} be cohomology homomorphism induced by $\lambda^{\#}$ and $\mu^{\#}$ respectively. By LEMMA 3.1, we obtain,

THEOREM 3.2. _If_ X_1 _and_ X_2 _are open sets of_ M, _then we have the Mayer-Vietoris exact sequence of_ $H^{*}_{\mathcal{F}D}$:

$$\cdots\to H^{i}_{\mathcal{F}D}(X_1\cup X_2;R) \xrightarrow{\mu_*} H^{i}_{\mathcal{F}D}(X_1;R)\oplus H^{i}_{\mathcal{F}D}(X_2;R)$$

$$\xrightarrow{\lambda_*} H^{i}_{\mathcal{F}D}(X_1\cap X_2;R) \xrightarrow{\delta_*} H^{i+1}_{\mathcal{F}D}(X_1\cup X_2;R) \to\cdots$$

where δ^{*} _is the connecting homomorphism_.

We call codimension q foliation $(M,\mathcal{F})$ $\mathcal{F}$-_contractible_ if there exists a q-dimensional submanifold N transverse to $\mathcal{F}$ and a map f: $M \to N \subset M$ transverse to $\mathcal{F}$ which is C^{∞}-homotopic to the identity map id_M by leaf preserving map, i.e., $f \;{}_{\mathcal{F}'}\!\simeq_{\mathcal{F}}\; id_M$.

Let $\mathcal{U} = \{U_\alpha\}$ be an open cover of M. If an intersection of finite open sets of $\mathcal{U}$ is $\mathcal{F}$-contractible, then we call $\mathcal{U}$ an $\mathcal{F}$-simple cover of $(M, \mathcal{F})$. If $(M, \mathcal{F})$ is a foliation on a paracompact Hausdorff manifold, then by [S, Lemma 4.1], every open cover $\mathcal{U}$ of M admits a refinement $\mathcal{U}' = \{U_i'\}$ which is $\mathcal{F}$-simple.

Moreover, by taking sufficiently small neighborhood of foliation chart as U_i', one can assume $\bar{U}_i'$ is compact. One constructs, by induction, an increasing sequence $\{V_j\}$ of open sets in M such that $\bar{V}_j$ is compact, $\bar{V}_j \subset V_{j+1}$ and $\bigcup_j V_j = M$.

The integral operator Λ: $A^{0,s} \to C^s_{\mathcal{F}D}(M;R)$ defined by

$$\Lambda(\omega)(\sigma_s^{\mathcal{F}}) = \int_{\sigma_s^{\mathcal{F}}} \omega, \quad \omega \in A^{0,s}, \ \sigma_s^{\mathcal{F}} \in C_s^{\mathcal{F}}(M;R)$$

is a cochain map [S, Lemma 5.2] and defines a natural homomorphism,

$$\Lambda^*: H^{0,s}_{\mathcal{F}DR}(M) \to H^s_{\mathcal{F}D}(M;R).$$

If $U_i' \subset M$ is on $\mathcal{F}$-contractible open set, there exists a q-dimensional submanifold $N \subset U_i'$ transverse to $\mathcal{F}$ and by LEMMA 2.3,

$$H^s_{\mathcal{F}D}(U_i';R) = \begin{cases} C^\infty(N) & s = 0 \\ 0 & s > 0 \end{cases}$$

where $C^\infty(N)$ is the vector space of C^∞-function on N. By [S, Corollary 3.2], Λ^* gives the isomorphism $H^s_{\mathcal{F}DR}(U_i') \cong H^s_{\mathcal{F}D}(U_i';R)$.

By making use of the Mayer-Vietoris sequence of $H^*_{\mathcal{F}D}$ obtained in THEOREM 3.2, that of $H^*_{\mathcal{F}DR}$ and the five lemma, we have,

LEMMA 3.3. <u>For each</u> j, Λ^* <u>is an isomorphism of vector spaces</u>: $H^{0,s}_{\mathcal{F}DR}(V_j) \xrightarrow{\cong} H^s_{\mathcal{F}D}(V_j;R)$.

PROOF OF THE MAIN THEOREM. Both sequences $\{H^{0,s}_{\mathcal{F}DR}(V_j)\}$ and $\{H^s_{\mathcal{F}D}(V_j;R)\}$ satisfy the Mittag-Leffler condition since $\bar{V}_j \subset V_{j+1}$ is compact. Therefore, the isomorphism Λ^* of LEMMA 3.3 gives, by the arguments [M1, §§A.3-A.4] and [M2, Appendix §3], the isomorphism

$$H^{0,s}_{\mathcal{F}DR}(M) \xrightarrow{\cong} H^s_{\mathcal{F}D}(M;R).$$

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