

On the Uniformization of Analytic Sets  
with countable sections

(Abstract)

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In (Ya 1) we announced the following results:

- (I) Every analytic set in the plane with countable sections can be uniformized by the difference of two analytic sets.
- (II) There is an analytic set in the plane with countable sections which cannot be uniformized by an analytic set.

Clearly we have

- (III) There is an analytic set in the plane with countable sections which cannot be uniformized by a co-analytic set.

But we cannot decide the following proposition:

- (\*) There is an analytic set in the plane with countable sections which cannot be uniformized by the sum of an analytic set and a co-analytic set.

In my visiting Berkeley at the March of 1984, Professor Addison pointed out me the following more weak proposition:

- (\*\*) There is an analytic set in the plane with countable sections which cannot be uniformized by either an analytic set or a co-analytic set.

For (\*\*) we can show

THEOREM 1. There is a  $\sum_1^1$  set  $A$  in the plane with the following properties:

- (i) For each real  $\alpha$   $A^{<\alpha>} = \{\beta : A(\alpha, \beta)\}$  is nonempty and at most two elements.
- (ii)  $A$  cannot be uniformized by either an analytic set or a co-analytic set.

Assuming projective determinacy, we can extend the theorem to

THEOREM 2. There is a  $\sum_{2n+1}^1$  set  $A$  in the plane with the following properties:

- (i) For each real  $\alpha$   $A^{<\alpha>}$  is nonempty and has at most two elements.
- (ii)  $A$  cannot be uniformized by either a  $\sum_{2n+1}^1$  set or a  $\prod_{2n+1}^1$  set.

For the proofs of these, see (Ya 2). Also full uniformization problem of analytic sets the reader can be found in (Mo) and (Ya 3).

#### References

- (Mo) Y.N. Moschovakis, Descriptive Set Theory, North-Holland, 1980.
- (Ya 1) Y. Yasuda, On the uniformization of analytic sets with countable sections and related results, RIMS Kokuroku, 480(1983), 204-208.

(Ya 2) Y.Yasuda, An answer of a question of Addison concerning the uniformization of analytic sets, to appear in Comment. Math. St. Paul. in 1986.

(Ya 3) Y.Yasuda, Some properties of thin  $\prod_{2n+1}^1$  sets, to appear in Z. Math. Logik Grundlag. Math. in 1987.