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Second order complete class in consistent estimators

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$x_1, x_2, \cdots, x_n$ は確率密度関数 $f(x,\theta)$ $(\theta \in \Theta \subset R^p)$ をもつ確率変数とする。Θ は開集合とし，$f(x,\theta)$ は次の条件を満たすものとする.

Condition 1. $f(x,\theta)$ は x, θ に関して可測とする.

Condition 2. $\{x \mid f(x,\theta) > 0\}$ は $\theta \in \Theta$ と独立である.

Condition 3. 各 x に対して，$f(x,\theta)$ は θ に関して3次まで偏微分可能で，それらの微係数は θ に関して連続である.

Condition 4. すべての $\theta \in \Theta$ に対して，

$$E_\theta(|\log f(x,\theta)|) < \infty$$

かつ $t\left(\frac{\partial}{\partial\theta_1}\log f(x,\theta), \frac{\partial}{\partial\theta_2}\log f(x,\theta), \cdots, \frac{\partial}{\partial\theta_p}\log f(x,\theta)\right)$ の共分散行列 $I_{1,1}(\theta)$ は正値行列である.

Condition 5. 各 $\theta_0 \in \Theta$ に対して，完備な近傍 $\Theta_0 \ni \theta_0$

および関数 $G(x), H(x)$ が存在して, $\theta \in \Theta_0$ に対して一様に

$$\left| \frac{\partial^i}{\partial \theta_\alpha^i} \log f(x, \theta) \right| \leqq G(x), \quad i=1,2,3, \quad \alpha=1,2,\cdots,p$$

$$\left| \frac{\partial^4}{\partial \theta_\alpha^4} \log f(x, \theta) \right| \leqq H(x)$$

を満たし, かつ

$$\sup_{\theta \in \Theta_0} E_\theta (G^3(X)) < \infty, \quad \sup_{\theta \in \Theta_0} E_\theta (H(x)) < \infty$$

Condition 6. 各 $\theta_0 \in \Theta$ に対して, 完備な近傍 $\Theta_0 \ni \theta_0$ が存在して, $n^{-\frac{1}{2}} I_{1,1}^{-\frac{1}{2}}(\theta) L_1(\theta)$, $n^{-\frac{1}{2}} C_{2,2}^{-\frac{1}{2}}(\theta) (L_2(\theta) - n e_{2,1}(\theta))$ は, $\theta \in \Theta_0$ に対して一様に, $o(n^{-\frac{1}{2}})$ まで Edgeworth－展開可能である. ただし $l(\theta) = \prod_{i=1}^n f(x, \theta)$, $L_i(\theta) = {}^t\left(\frac{\partial^i}{\partial \theta_1^i} \log l(\theta), \frac{\partial^i}{\partial \theta_2^i} \log l(\theta), \cdots, \frac{\partial^i}{\partial \theta_p^i} \log l(\theta) \right)$, $n e_{2,1}(\theta) = E_\theta (L_2(\theta))$,

$$n C_{2,2}(\theta) = E_\theta \left[(L_2(\theta) - n e_{2,1}(\theta))^t (L_2(\theta) - n e_{2,1}(\theta)) \right]$$

Condition 7. 最尤推定量 $\hat{\theta}_n$ は θ に関して局所一様に $o(n^{-\frac{1}{2}})$ まで Edgeworth－展開可能である.

ここで取扱う一致推定量 θ_n は任意の $b \in R^p$ に対して $P(\sqrt{n}\, {}^t b (\theta_n - \theta) \leqq 0)$ が θ に関して局所一様に $o(n^{-\frac{1}{2}})$ まで展開可能であるものとする. またそのような推

定量の全体を$\mathcal{F}$, $\mathcal{F}$の中で$o(n^{-\frac{1}{2}})$までEdgeworth-展開可能なものの全体を$\mathcal{F}_E$, また特定な$g(\theta, \frac{1}{\sqrt{n}}; b)$に対して$P(\sqrt{n}\,{}^t b(\theta_n-\theta)\leqq 0) = g(\theta, \frac{1}{\sqrt{n}}; b)$を満たす$\theta_n \in \mathcal{F}$の全体を$C(g(\theta, \frac{1}{\sqrt{n}}; b))$で表わす。また任意の$\theta_n \in \mathcal{F}$に対して,

$$E_\theta(\sqrt{n}(\theta_n-\theta)) = a(\theta) = a_1(\theta) + \frac{1}{\sqrt{n}} a_2(\theta) + o(n^{-\frac{1}{2}})$$

$$\begin{aligned} &E_\theta\left(n\left(\theta_n-\theta-\frac{1}{\sqrt{n}}a(\theta)\right){}^t\left(\theta_n-\theta-\frac{1}{\sqrt{n}}a(\theta)\right)\right) \\ &\qquad = C(\theta) = C_1(\theta) + \frac{1}{\sqrt{n}} C_2(\theta) + o(n^{-\frac{1}{2}}) \end{aligned}$$

なる記号を用いる。以上の条件の下で次の結果を得る.

定理1. $\theta_n \in C(g(\theta, \frac{1}{\sqrt{n}}; b))$に対して,

$$\begin{aligned} &P(-t_1 \leqq \sqrt{n}\,{}^t b(\theta_n-\theta) \leqq t_2) \\ &\quad \leqq F(t_2; \theta, \tfrac{1}{\sqrt{n}}; b; g(\theta, \tfrac{1}{\sqrt{n}}; b)) - F(-t_1; \theta, \tfrac{1}{\sqrt{n}}; b; g(\theta, \tfrac{1}{\sqrt{n}}; b)) \\ &\qquad + o(n^{-\frac{1}{2}}) \end{aligned}$$

$$(\forall t_1, t_2 \geqq 0, \ \forall b \in R^p)$$

ただし

$$\begin{aligned} &F(t; \theta, \tfrac{1}{\sqrt{n}}; b; g(\theta, \tfrac{1}{\sqrt{n}}; b)) = \Phi(u(g_{00}(\theta, b)) + t I_{1,1}^{\frac{1}{2}}(\theta, \xi)) \\ &+ \frac{1}{\sqrt{n}}\phi(u(g_{00}(\theta, b)) + t I_{1,1}^{\frac{1}{2}}(\theta, \xi))\Big[\{(g_{10}(\theta, b) + t g_{01}(\theta, \xi)) \\ &\quad / \phi(u(g_{00}(\theta, b)))\} + t\{(J_{1,1,1}(\theta, \xi) u(g_{00}(\theta, b)))/ \\ &\quad 6 I_{1,1}(\theta, \xi)\} + t^2\{(3 J_{2,1}(\theta, \xi) + 2 J_{1,1,1}(\theta, \xi))/ \\ &\quad 6 I_{1,1}(\theta, \xi)\}\Big] \end{aligned}$$

$$I_{\alpha,\beta}(\theta)=E_\theta\left(\frac{\partial}{\partial\theta_\alpha}\log f(x,\theta)\cdot\frac{\partial}{\partial\theta_\beta}\log f(x,\theta)\right),\quad I_{1,1}(\theta)=[I_{\alpha,\beta}(\theta)]$$

$$J_{\alpha\beta,\gamma}(\theta)=E_\theta\left(\frac{\partial^2}{\partial\theta_\beta\partial\theta_\alpha}\log f(x,\theta)\cdot\frac{\partial}{\partial\theta_\gamma}\log f(x,\theta)\right)$$

$$J_{\alpha,\beta,\gamma}(\theta)=E_\theta\left(\frac{\partial}{\partial\theta_\alpha}\log f(x,\theta)\cdot\frac{\partial}{\partial\theta_\beta}\log f(x,\theta)\cdot\frac{\partial}{\partial\theta_\gamma}\log f(x,\theta)\right)$$

$$I_{1,1}(\theta,\xi)=\sum_{\alpha,\beta=1}^{p}I_{\alpha,\beta}(\theta)\xi_\alpha\xi_\beta={}^t\xi I_{1,1}(\theta)\xi$$

$$J_{2,1}(\theta,\xi)=\sum_{\alpha,\beta,\gamma=1}^{p}J_{\alpha\beta,\gamma}(\theta)\xi_\alpha\xi_\beta\xi_\gamma$$

$$J_{1,1,1}(\theta,\xi)=\sum_{\alpha,\beta,\gamma=1}^{p}J_{\alpha,\beta,\gamma}(\theta)\xi_\alpha\xi_\beta\xi_\gamma$$

$$g\left(\theta,\frac{1}{\sqrt{n}};b\right)=g_{00}(\theta,b)+\frac{1}{\sqrt{n}}g_{10}(\theta,b)+o(n^{-\frac{1}{2}})$$

$$g_{01}(\theta;b,\xi)=\sum_{\alpha=1}^{p}\frac{\partial}{\partial\theta_\alpha}g_{00}(\theta,b)\xi_\alpha$$

$$u(g_{00}(\theta,b))=\Phi^{-1}(g_{00}(\theta,b)),\quad {}^tb\xi=1$$

$\theta_n\in\mathcal{F}_E$ の cumulants が次式で与えられるとしよう。

$$K_1(\sqrt{n}\theta_{n\alpha})=a_{1\alpha}(\theta)+\frac{1}{\sqrt{n}}a_{2\alpha}(\theta)+o(n^{-\frac{1}{2}})$$

$$K_2(\sqrt{n}\theta_{n\alpha},\sqrt{n}\theta_{n\beta})=C_{1\alpha\beta}(\theta)+\frac{1}{\sqrt{n}}C_{2\alpha\beta}(\theta)+o(n^{-\frac{1}{2}})$$

$$K_3(\sqrt{n}\theta_{n\alpha},\sqrt{n}\theta_{n\beta},\sqrt{n}\theta_{n\gamma})=\frac{1}{\sqrt{n}}d_{2\alpha\beta\gamma}(\theta)+o(n^{-\frac{1}{2}})$$

$$K_i(\sqrt{n}\theta_{n\alpha_1},\sqrt{n}\theta_{n\alpha_2},\cdots,\sqrt{n}\theta_{n\alpha_i})=o(n^{-\frac{1}{2}})\quad(i\geqq 4)$$

ただし $\theta_n={}^t(\theta_{n1},\theta_{n2},\cdots,\theta_{np})$.

そのとき

$$\bar{g}\left(\theta,\frac{1}{\sqrt{n}};b\right)=P(\sqrt{n}\,{}^tb(\theta_n-\theta)\leqq 0)$$

$$=\Phi\left(-{}^tb\,a_1(\theta)/(C_{1\alpha\beta}(\theta)b_\alpha b_\beta)^{\frac{1}{2}}\right)$$

$$-\frac{1}{\sqrt{n}}\phi\left(-{}^tb\,a_1(\theta)/(C_{1\alpha\beta}(\theta)b_\alpha b_\beta)^{\frac{1}{2}}\right)\Big[\{{}^tb\,a_2(\theta)/$$

$$(C_{1\alpha\beta}(\theta)b_\alpha b_\beta)^{\frac{1}{2}}\}-\{C_{2\alpha\beta}(\theta)b_\alpha b_\beta\cdot{}^tb\,a_1(\theta)/2(C_{1\alpha\beta}(\theta)b_\alpha b_\beta)^{\frac{3}{2}}\}$$

$$\left\{ d_{2\alpha\beta\gamma}(\theta)\, b_\alpha b_\beta b_\gamma \big/ 6\left(c_{1\alpha\beta}(\theta)\, b_\alpha b_\beta \right) \right\} \left\{ \left(\left({}^t b\, a_1(\theta)\right)^2 \big/ c_{1\alpha\beta}(\theta)\, b_\alpha b_\beta \right) - 1 \right\} + o(n^{-\frac{1}{2}})$$

従って　$b = \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, b'$　$(\|b'\| = 1)$ に対して,

$$F\left(t ; \theta, \tfrac{1}{\sqrt{n}} ; \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b', \mathrm{I}_{1,1}^{-\frac{1}{2}}(\theta) b' ; g\left(\theta, \tfrac{1}{\sqrt{n}} ; \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b'\right)\right)$$

$$= \Phi\left(-{}^t b' \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, a_1'(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b') + t\right)$$

$$- \frac{1}{\sqrt{n}} \phi\left(-{}^t b' \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, a_1'(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b') + t\right) \Big[{}^t b' \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, a_2'(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b')$$

$$- \frac{1}{2} {}^t b' \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, a_1'(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b') \times {}^t b' \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, C_2'(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b')$$

$$\times \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b' + \frac{1}{6} d'_{2\alpha\beta\gamma}(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b') (\mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b')_\alpha (\mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b')_\beta$$

$$\times (\mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b')_\gamma \left\{ \left({}^t b' \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, a_1'(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b')\right)^2 - 1 \right\}$$

$$+ t \Big\{ {}^t b' \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, A'(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b')\, \mathrm{I}_{1,1}^{-\frac{1}{2}}(\theta) b'$$

$$+ \frac{1}{3} {}^t b' \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta)\, a_1'(\theta, \mathrm{I}_{1,1}^{\frac{1}{2}}(\theta) b') \Big(3 J_{2,1}(\theta, \mathrm{I}_{1,1}^{-\frac{1}{2}}(\theta) b')$$

$$+ 2 J_{1,1,1}(\theta, \mathrm{I}_{1,1}^{-\frac{1}{2}}(\theta) b')\Big) \Big\}$$

$$+ t^2 \left\{ 3 J_{2,1}(\theta, \mathrm{I}_{1,1}^{-\frac{1}{2}}(\theta) b') + 2 J_{1,1,1}(\theta, \mathrm{I}_{1,1}^{-\frac{1}{2}}(\theta) b') \right\}$$

ただし

$$a_1'(\theta, b) = g^{\frac{1}{2}}(\theta, b)\, a_1(\theta), \quad a_2'(\theta, b) = g^{\frac{1}{2}}(\theta, b)\, a_2(\theta)$$

$$C_1'(\theta, b) = g(\theta, b)\, C_1(\theta), \quad C_2'(\theta, b) = g(\theta, b)\, C_2(\theta)$$

$$d'_{2\alpha\beta\gamma}(\theta, b) = g^{\frac{3}{2}}(\theta, b)\, d_{2\alpha\beta\gamma}(\theta)$$

$$A'(\theta, b) = {}^t\left(\frac{\partial}{\partial \theta_1} a_1'(\theta, b), \frac{\partial}{\partial \theta_2} a_1'(\theta, b), \ldots, \frac{\partial}{\partial \theta_p} a_1'(\theta, b) \right)$$

$$g(\theta, b) = {}^t b\, \mathrm{I}_{1,1}^{-1}(\theta)\, b \big/ {}^t b\, C_1(\theta)\, b.$$

さらに

$$C_2''(\theta, b) = C_2'(\theta, b) - 2 A'(\theta, b)\, \mathrm{I}_{1,1}^{-1}(\theta)$$

$$d_2''(\theta, b) = {}^t\!\left(d_{21}''(\theta, b),\ d_{22}''(\theta, b),\ \cdots,\ d_{2p}''(\theta, b)\right)$$

$$d_{2r}''(\theta, b) = \sum_{\alpha,\beta} d_{2\alpha\beta r}''(\theta, b)\, b_\alpha b_\beta \qquad , r = 1, 2, \cdots, p$$

$$\begin{aligned} d_{2\alpha\beta r}''(\theta, b) = {} & d_{2\alpha\beta r}'(\theta, b) + \left(J_{\gamma_1\gamma_2,\gamma_3}(\theta) + J_{\gamma_1\gamma_3,\gamma_2}(\theta) \right. \\ & \left. + J_{\gamma_2\gamma_3,\gamma_1}(\theta) + 2 J_{\gamma_1,\gamma_2,\gamma_3}(\theta) \right) I^{-1}_{\gamma_1,\alpha}(\theta) \\ & \times I^{-1}_{\gamma_2,\beta}(\theta)\, I^{-1}_{\gamma_3,r}(\theta) \end{aligned}$$

と置き,

$$\begin{aligned} \theta_n' = {} & \hat\theta_n + \frac{1}{\sqrt{n}} a_1'(\hat\theta_n, I_{1,1}^{\frac{1}{2}}(\hat\theta_n) b') + \frac{1}{n}\Big[a_2'(\hat\theta_n, I_{1,1}^{\frac{1}{2}}(\hat\theta_n) b') - \bar a_2(\hat\theta_n) \\ & - \frac{1}{2} a_1'(\hat\theta_n, I_{1,1}^{\frac{1}{2}}(\hat\theta_n) b') \cdot {}^t b' I_{1,1}^{\frac{1}{2}}(\theta)\, C_2''(\theta, I_{1,1}^{\frac{1}{2}}(\hat\theta_n) b') I_{1,1}^{\frac{1}{2}}(\hat\theta_n) b' \\ & + \frac{1}{6} d_2''(\hat\theta_n, I_{1,1}^{\frac{1}{2}}(\hat\theta_n) b') \left\{ \left({}^t b' I_{1,1}^{\frac{1}{2}}(\hat\theta_n) a_1'(\hat\theta_n, I_{1,1}^{\frac{1}{2}}(\hat\theta_n) b')\right)^2 - 1 \right\}\Big] \end{aligned}$$

と置くならば,

$$\begin{aligned} & F\left(t\,;\, \theta, \frac{1}{\sqrt{n}}\,;\, I_{1,1}^{\frac{1}{2}}(\theta) b',\ I_{1,1}^{-\frac{1}{2}}(\theta) b'\,;\, g\left(\theta, \frac{1}{\sqrt{n}}\,;\, I_{1,1}^{\frac{1}{2}}(\theta) b'\right)\right) \\ & \quad = P\left(\sqrt{n}\, {}^t b' I_{1,1}^{\frac{1}{2}}(\theta)(\theta_n' - \theta) \leqq t\right) + o(n^{-\frac{1}{2}}) \qquad ({}^\forall t \in R) \end{aligned}$$

任意の $b \in R^p$ に対しては,

$$\begin{aligned} \theta_n' = {} & \hat\theta_n + \frac{1}{\sqrt{n}} a_1'(\hat\theta_n, ({}^t b I_{1,1}^{-1}(\hat\theta_n) b)^{-\frac{1}{2}} b) + \frac{1}{n}\Big[a_2'(\hat\theta_n, ({}^t b I_{1,1}^{-1}(\hat\theta_n) b)^{-\frac{1}{2}} b) \\ & - \bar a_2(\hat\theta_n) - \frac{1}{2} a_1'(\hat\theta_n, ({}^t b I_{1,1}^{-1}(\hat\theta_n) b)^{-\frac{1}{2}} b) \times ({}^t b I_{1,1}^{-1}(\theta) b)^{-1} \\ & \times {}^t b\, C_2''(\hat\theta_n, ({}^t b I_{1,1}^{-1}(\hat\theta_n) b)^{-\frac{1}{2}} b)\, b \\ & + \frac{1}{6} d_2''(\hat\theta_n, ({}^t b I_{1,1}^{-1}(\hat\theta_n) b)^{-\frac{1}{2}} b) \\ & \quad \times \left\{ ({}^t b I_{1,1}^{-1}(\hat\theta_n) b)^{-1} \left({}^t b\, a_1(\hat\theta_n, ({}^t b I_{1,1}^{-1}(\theta) b)^{-\frac{1}{2}} b)\right)^2 - 1 \right\} \end{aligned}$$

ただし $\bar a_2(\theta)$ は,

$$E_\theta\left(\sqrt{n}(\hat\theta_n - \theta)\right) = \frac{1}{\sqrt{n}} \bar a_2(\theta) + o(n^{-\frac{1}{2}})$$

を満たす.

$C_1(\theta) = I_{1\cdot1}^{-1}(\theta)$ ならば

$$a_1'(\theta, b) = a_1(\theta),\ a_2'(\theta, b) = a_2(\theta),\quad C_2'(\theta, b) = C_2(\theta)$$

$$d_2''(\theta, b) = 0,\quad A'(\theta, b) = A(\theta)$$

であり，$C_1(\theta) \neq I_{1\cdot1}^{-1}(\theta)$ ならば，少くとも1つは0でない t_1, t_2 に対して，$\theta_n'' = \hat{\theta}_n + \frac{1}{\sqrt{n}} a_1'(\theta, ({}^t b I_{1\cdot1}^{-1}(\theta) b)^{-\frac{1}{2}} b)$

と置くとき，

$$P(-t_1 \leqq \sqrt{n}\,{}^t b(\theta_n - \theta) \leqq t_2) < P(-t_1 \leqq \sqrt{n}\,{}^t b(\theta_n'' - \theta) \leqq t_2) + o(1)$$

が成り立つことに注意して次の結果が得られる。

定理2. 各 $\theta_n \in \mathfrak{F}_E$ に対して，$a_1(\theta, b)$, $a_2'(\theta)$ および半正値行列 $D(\theta)$ が存在して，$\theta_n' = \hat{\theta}_n + \frac{1}{\sqrt{n}} a_1(\hat{\theta}_n, b) + \frac{1}{n} a_2'(\hat{\theta}_n) - \frac{1}{n} ({}^t b I_{1\cdot1}^{-1}(\hat{\theta}_n) b)^{-1} ({}^t b D(\hat{\theta}_n) b) a_1(\theta)$ は θ に関して局所一様に，

$$P(-t_1 \leqq \sqrt{n}(\theta_n - \theta) \leqq t_2) \leqq P(-t_1 \leqq \sqrt{n}\,{}^t b(\theta_n' - \theta) \leqq t_2) + o(n^{-\frac{1}{2}})$$

$$({}^\forall b \in R^p,\ {}^\forall t_1, t_2 \geqq 0,\ \text{少くとも1つは正})$$

が成り立つ。

定理3. $\theta_n \in \mathfrak{F}_E$ に対して，$a(\theta) = o(1)$ が成り立つならば，$a_2'(\theta)$ が存在して，$\theta_n' = \hat{\theta}_n + \frac{1}{n} a_2'(\hat{\theta}_n)$ は，θ に関して局所一様に，

$$P(-t_1 \leqq \sqrt{n}\,{}^t b(\theta_n - \theta) \leqq t_2) \leqq P(-t_1 \leqq \sqrt{n}\,{}^t b(\theta_n' - \theta) \leqq t_2) + o(n^{-\frac{1}{2}})$$

$(\forall b \in R^p, \forall t_1, t_2 \geqq 0$, 少くとも1つは正$)$

が成り立つ.

定理4. $\theta_n \in \mathcal{F}_E$ に対して, $C(\theta) = I_{1,1}^{-1}(\theta) + \frac{1}{\sqrt{n}}\left(A(\theta) I_{1,1}^{-1}(\theta) + I_{1,1}^{-1}(\theta) A(\theta)\right) + o(n^{-\frac{1}{2}})$ が成り立つならば, $a_2'(\theta)$ が存在して, $\theta_n' = \hat{\theta}_n + \frac{1}{\sqrt{n}} a_1(\hat{\theta}_n) + \frac{1}{n} a_2'(\hat{\theta}_n)$ は, θ に関して局所一様に

$$P_\theta(\sqrt{n}(\theta_n - \theta) \in B) = P_\theta(\sqrt{n}(\theta_n' - \theta) \in B) + o(n^{-\frac{1}{2}})$$
$$(\forall B \in \mathcal{B}^1)$$

が成り立つ.