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Large-time behavior of solutions for the equations of a viscous gas

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1. Introduction

We consider one-dimensional flow of a compressible fluid. In the Lagrange mass coordinate (t,x), the motion of the fluid is described by the following equations.

$$
(1.1)\qquad \begin{aligned} & v_t - u_x = 0\ , \qquad u_t + p_x = (\mu u_x/v)_x\ , \\ & (e + u^2/2)_t + (pu)_x = (\kappa\theta_x/v + \mu u u_x/v)_x\ . \end{aligned}
$$

Here $v > 0$ is the specific volume, u the velocity, $\theta > 0$ the absolute temperature, e the internal energy, μ the coefficient of viscosity and κ the coefficient of heat-conductivity. Let us denote the entropy by s. It is known that among five thermodynamic variables v, θ, p, e and s, only two of them are independent. In fact they may all be considered as smooth functions of (v,θ), (v,s), (v,e) or (p,s). We write $p = p(v,\theta) = \hat{p}(v,s)$ and $e = e(v,\theta)$ and assume that

$$
(1.2)_1 \qquad \partial p(v,\theta)/\partial v < 0\ , \quad \partial p(v,\theta)/\partial\theta > 0\ , \quad \partial e(v,\theta)/\partial\theta > 0\ ,
$$

$$
(1.2)_2 \qquad \partial^2\hat{p}(v,s)/\partial v^2 > 0\ .
$$

Notice that these conditions are satisfied for the case of an ideal polytropic gas:

$$(1.3) \qquad p = R\theta/v = \hat{R}v^{-\gamma}e^{(\gamma-1)s/R}, \qquad e = R\theta/(\gamma-1) + \text{constant},$$

where $R > 0$ is the gas constant, $\gamma > 1$ is the adiabatic exponent and $\hat{R}$ is a positive constant. We also assume that μ and κ are smooth functions of two independent thermodynamic variables and satisfy one of the following two conditions.

$$(1.4)_1 \qquad \mu > 0, \quad \kappa > 0 \quad \text{(viscous heat-conductive fluid)},$$

$$(1.4)_2 \qquad \mu \equiv 0, \quad \kappa > 0 \quad \text{(inviscid heat-conductive fluid)}.$$

We shall study the large-time behavior of solutions to the initial value problem for (1.1). Our main result is as follows: If the initial data are close to a given constant state, then a unique smooth solution of (1.1) exists for all time $t \geq 0$ and approaches the superposition of the nonlinear and linear diffusion waves constructed in terms of the self-similar solutions of the Burgers equation and the linear heat equation as $t \to \infty$.

We remark that the same asymptotic result has been obtained in [5] for a wide class of systems including (1.1).

Notations

We introduce several function spaces. Let $p \in [1,\infty]$, $\beta \in \mathbb{R}$ and $s \geq 0$. L^p denotes the usual Lebesgue space on $\mathbb{R}$, with the norm $|\cdot|_p$. L^p_β denotes the space of functions $f = f(x)$ such that $(1+|x|)^\beta f \in L^p$, with the norm $|\cdot|_{p,\beta}$. H^s denotes the space of functions $f = f(x)$ such that $\partial_x^\ell f \in L^2$ for $0 \leq \ell \leq s$, with the norm $\|\cdot\|_s$. Note that $H^0 = L^2$

and $\|\cdot\|_0 = |\cdot|_2$. $C^0([0,\infty); H^s)$ is the space of continuos functions on $[0,\infty)$ with values in H^s.

2. Preliminaries

We first choose v and θ as independent thermodynamic variables and write $p = p(v,\theta)$, $e = e(v,\theta)$ and $s = s(v,\theta)$. The thermodynamic law $de = \theta ds - pdv$ gives

$$(2.1) \qquad e_v = -(p - \theta p_\theta), \quad s_v = p_\theta, \quad s_\theta = e_\theta/\theta,$$

where we used abbreviations such as $e_v = \partial e(v,\theta)/\partial v$. When v and s are regarded as independent variables, we write $\theta = \hat{\theta}(v,s)$, $p = \hat{p}(v,s)$ and $e = \hat{e}(v,s)$. Using (2.1), we obtain

$$(2.2) \qquad \begin{aligned} &\hat{\theta}_v = -\theta p_\theta/e_\theta, \quad &&\hat{p}_v = p_v - \theta p_\theta^2/e_\theta, \quad &&\hat{e}_v = -p, \\ &\hat{\theta}_s = \theta/e_\theta, &&\hat{p}_s = \theta p_\theta/e_\theta, &&\hat{e}_s = \theta, \end{aligned}$$

where $\hat{\theta}_v = \partial\hat{\theta}(v,s)/\partial v$, etc. In particular, we have $\hat{p}_v < 0$ by $(1.2)_1$. Similarly, choosing v and e as independent variables and writing $\theta = \tilde{\theta}(v,e)$, $p = \tilde{p}(v,e)$ and $s = \tilde{s}(v,e)$, we obtain

$$(2.3) \qquad \begin{aligned} &\tilde{\theta}_v = (p - \theta p_\theta)/e_\theta, \quad &&\tilde{p}_v = (p_v - \theta p_\theta^2/e_\theta) + pp_\theta/e_\theta, \quad &&\tilde{s}_v = p/\theta, \\ &\tilde{\theta}_e = 1/e_\theta, &&\tilde{p}_e = p_\theta/e_\theta, &&\tilde{s}_e = 1/\theta, \end{aligned}$$

where $\tilde{\theta}_v = \partial\tilde{\theta}(v,e)/\partial v$, etc. In particular, we have $\tilde{p}_v - p\tilde{p}_e = \hat{p}_v$.

3. Vector form of the system

Put $E = e + u^2/2$. Then (1.1) is regarded as a system for (v,u,E)

and is rewritten in the vector form

$$(3.1)\qquad w_t + f(w)_x = (G(w)w_x)_x\ ,$$

where $w = (v,u,E)^T$, $f(w) = (-u,p,pu)^T$, and $G(w)$ is the matrix given by (3.3) below. We denote by $A(w)$ the Jacobian of $f(w)$ with respect to w. Then (3.1) is equivalent to

$$(3.1)'\qquad w_t + A(w)w_x = (G(w)w_x)_x\ .$$

$A(w)$ and $G(w)$ are given explicitly as follows.

$$(3.2)\qquad A(w) = \begin{pmatrix} 0 & -1 & 0 \\ \tilde{p}_v & -u\tilde{p}_e & \tilde{p}_e \\ u\tilde{p}_v & p - u^2\tilde{p}_e & u\tilde{p}_e \end{pmatrix},$$

$$(3.3)\qquad G(w) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mu/v & 0 \\ \kappa\tilde{\theta}_v/v & \mu u/v - \kappa u\tilde{\theta}_e/v & \kappa\tilde{\theta}_e/v \end{pmatrix}.$$

By straightfoward calculations, using (2.2) and (2.3), we know that the eigenvalues of $A(w)$ are given by

$$(3.4)\qquad \lambda_1(w) = -(-\hat{p}_v)^{1/2}\ ,\qquad \lambda_2(w) = 0\ ,\qquad \lambda_3(w) = (-\hat{p}_v)^{1/2}\ .$$

These are all real and distinct since $\hat{p}_v < 0$ by $(1.2)_1$. This means that the inviscid system $w_t + f(w)_x = 0$ is strictly hyperbolic. The corresponding right and left eigenvectors, $r_j(w)$ and $\ell_j(w)$, are

$$(3.5)\qquad \begin{aligned} r_j(w) &= a_j(1,\ -\lambda_j,\ -u\lambda_j - p)^T\ ,\quad j=1,3, \\ r_2(w) &= a_2(\tilde{p}_e,\ 0,\ -\tilde{p}_v)^T\ , \end{aligned}$$

$$(3.6)\qquad \begin{aligned} \ell_j(w) &= b_j(-\tilde{p}_v,\ -\lambda_j + u\tilde{p}_e,\ -\tilde{p}_e)\ , \qquad j=1,3, \\ \ell_2(w) &= b_2(p,\ -u,\ 1)\ , \end{aligned}$$

where $a_j b_j \neq 0$, $j=1,2,3$. We choose a_j and b_j such that $2a_j b_j = 1/(-\hat{p}_v)$, $j=1,3$, and $a_2 b_2 = 1/(-\hat{p}_v)$. In this case we have

$$(3.7)\qquad < \ell_j(w), r_k(w) > = \delta_{jk}\ , \qquad j,k=1,2,3,$$

where $<\ ,\ >$ denotes the standard inner product of $\mathbb{R}^3$. When $(1.2)_2$ is assumed, we determine a_j such that $a_j = -2\lambda_j/\hat{p}_{vv}$, $j=1,3$, and $a_2 = \theta/(-\hat{p}_v)$, where $\hat{p}_{vv} = \partial^2\hat{p}(v,s)/\partial v^2$. Then we have

$$(3.8)\qquad < \nabla\lambda_j(w), r_j(w) > = 1\ , \qquad j=1,3,$$

$$(3.9)\qquad < \nabla s(w), r_2(w) > = 1\ .$$

Here the gradient ∇ is with respect to w, and $s = s(w)$ is the entropy. Since $< \nabla\lambda_j(w), r_j(w) > \neq 0$, $j=1,3$, the first and the third characteristic fields are genuinely nonlinear in the sense of Lax [7]. While the second field is linearly degenerate ([7]) because we have $< \nabla\lambda_2(w), r_2(w) > = 0$ by $\lambda_2(w) = 0$.

4. Global existence and decay of solution

We consider (3.1) with the initial condition

$$(4.1)\qquad w(0,x) = w_0(x)\ ,$$

where $w_0 = (v_0, u_0, E_0)^T$ with $E_0 = e_0 + u_0^2/2$. We seek a solution of (3.1), (4.1) in a neighborhood of a constant state $\bar{w} = (\bar{v}, \bar{u}, \bar{E})^T$, where $\bar{v} > 0$, $\bar{u} \in \mathbb{R}$, and $\bar{E} = \bar{e} + \bar{u}^2/2$ with $\bar{\theta} = \tilde{\theta}(\bar{v}, \bar{e}) > 0$. We have the following global existence result.

Theorem 4.1. ([6], see also [4]) *Assume* $(1.2)_1$, *and* $(1.4)_1$ *or* $(1.4)_2$. *If* $w_0(x) - \bar{w}$ *is small in* H^s, $s \geq 2$, *then the initial value problem* (3.1), (4.1) *has a unique global solution* $w(t,x)$ *in an appropriate function space. In particular, we have* $w - \bar{w} \in C^0([0,\infty); H^s)$ *and* $\|w(t) - \bar{w}\|_s \leq C\|w_0 - \bar{w}\|_s$ *for* $t \in [0,\infty)$, *where* C *is a constant. Moreover, the solution* $w(t,x)$ *converges to the constant state* $\bar{w}$ *uniformly in* $x \in \mathbb{R}$ *as* $t \to \infty$.

This result is proved by an energy method which makes use of the following properties: The system (3.1) has an entropy function and is transformed into a symmetric system of hyperbolic-parabolic type which satisfies the stability condition. We refer the reader to [4],[5] for the details. See also [1],[2].

Next we study a decay rate of the difference $w(t,x) - \bar{w}$ for $t \to \infty$. The linearized system of (3.1) around the constant state $\bar{w}$ is

$$(4.2) \qquad w'_t + A(\bar{w})w'_x = G(\bar{w})w'_{xx} .$$

Denote by e^{tR} the semigroup of (4.2). We have

$$(4.3) \qquad |\partial_x^\ell(e^{tR}f)|_2 \leq Ce^{-ct}|\partial_x^\ell f|_2 + C(1+t)^{-(1/2+\ell-k)/2}|\partial_x^k f|_1 ,$$

where $0 \leq k \leq \ell$, C and c are positive constants, and $f = f(x)$ is a function such that the norms on the right hand side of (4.3) are finite (see [9]). Making use of (4.3), we obtain the following

Theorem 4.2. ([4]) *Assume* $(1.2)_1$, *and* $(1.4)_1$ *or* $(1.4)_2$. *If* $w_0(x) - \bar{w}$ *is small in* $H^s \cap L^1$, $s \geq 3$, *then the solution* $w(t,x)$ *of* (3.1) *constructed in Theorem* 4.1 *satisfies*

$$(4.4) \qquad |\partial_x^{\ell}(w(t) - \bar{w})|_2 \le CN_s(1+t)^{-(1/2+\ell)/2}, \qquad t \in [0,\infty),$$

where $\ell \ge 0$, $3\ell \le s-2$, C *is a constant and* $N_s = \|w_0 - \bar{w}\|_s + |w_0 - \bar{w}|_1$.

5. Approximation by uniformly parabolic system

We first note that the matrix $A(w)$ has the spectral resolution $A(w) = \sum \lambda_j(w)P_j(w)$, where $P_j(w) = r_j(w)\ell_j(w)$ and the summation is taken over all $j = 1,2,3$. We then define the matrix $D(w)$ by

$$(5.1) \qquad D(w) = \sum_{j=1}^{3} \kappa_j(w)P_j(w),$$

where $\kappa_j(w) = \langle \ell_j(w), G(w)r_j(w) \rangle$ with $G(w)$ given by (3.3). By straightforward calculations we have

$$(5.2) \qquad \begin{aligned} \kappa_j(w) &= (-\mu\hat{p}_v + \kappa\hat{p}_s^2/\theta)/(-2v\hat{p}_v), \qquad j = 1,3, \\ \kappa_2(w) &= (-\kappa p_v)/(-ve_\theta\hat{p}_v). \end{aligned}$$

Note that these coefficients are all positive by $(1.2)_1$, and $(1.4)_1$ or $(1.4)_2$.

Now we consider the system

$$(5.3) \qquad z_t + f(z)_x = D(\bar{w})z_{xx},$$

with the initial condition $z(0,x) = w_0(x)$. The system (5.3) is semilinear and uniformly parabolic, and hence has a unique global solution $z(t,x)$, provided that $w_0(x) - \bar{w}$ is small in H^s, $s \ge 1$. The linearized system of (5.3) around the constant state $\bar{w}$ is

$$(5.4) \qquad z'_t + A(\bar{w})z'_x = D(\bar{w})z'_{xx}.$$

Denote by e^{tS} the semigroup of (5.4). We easily obtain the estimate

(5.5) $$|\partial_x^{\ell}(e^{tS}f)|_2 \le Ce^{-ct}|\partial_x^{\ell}f|_2 + C(1+t)^{-(1/2+\ell-k)/2}|\partial_x^k f|_1 ,$$

where $0 \le k \le \ell$, C and c are positive constants. Making use of (5.5), we know that if $w_0(x) - \bar{w}$ is small in $H^s \cap L^1$, $s \ge 1$, then the solution $z(t,x)$ of (5.3) satisfies

(5.6) $$|\partial_x^{\ell}(z(t) - \bar{w})|_2 \le CN_s(1+t)^{-(1/2+\ell)/2} , \qquad t \in [0,\infty),$$

where $0 \le \ell \le s$ and C is a constant.

Furthermore, we can show that for $t \to \infty$, the solution $w(t,x)$ of (3.1) is well approximated by the solution $z(t,x)$ of (5.3). More precisely, we have the following

<u>Theorem 5.1</u>. ([5]) *Assume* $(1.2)_1$, *and* $(1.4)_1$ *or* $(1.4)_2$. *If* $w_0(x) - \bar{w}$ *is small in* $H^s \cap L^1$, $s \ge 5$, *then we have*

(5.7) $$|\partial_x^{\ell}(w(t) - z(t))|_2 \le CN_s(1+t)^{-(3/2+\ell)/2+\alpha} , \qquad t \in [0,\infty),$$

where $\ell \ge 0$, $3\ell \le s-5$, C *is a constant, and* $\alpha > 0$ *is a small fixed constant.*

This approximation result is based on the following better decay estimate for the difference between the semigroups e^{tR} and e^{tS}.

(5.8) $$|\partial_x^{\ell}(e^{tR} - e^{tS})f|_2 \le Ce^{-ct}|\partial_x^{\ell}f|_2 + C(1+t)^{-(3/2+\ell-k)/2}|\partial_x^k f|_1 ,$$

where $0 \le k \le \ell$, C and c are positive constants.

6. Diffusion waves

Following Liu [8], we shall construct the diffusion waves. First we determine the coefficients $\delta_j(\bar{w})$, $j = 1,2,3$, by

$$(6.1)\qquad \int_{-\infty}^{\infty}(w_0(x)-\overline{w})dx = \sum_{j=1}^{3}\delta_j(\overline{w})r_j(\overline{w}) .$$

Put $\delta(\overline{w}) = (\delta_1(\overline{w}),\delta_2(\overline{w}),\delta_3(\overline{w}))$ and assume that $\delta(\overline{w}) \neq 0$. Next we introduce the Riemann invariant. A function of w which are constant in the direction of $r_j(w)$ is called j-Riemann invariant. For each j, we have two independent j-Riemann invariants given below.

$$(6.2)\qquad \begin{aligned} &s \ \text{ and } \ u + \int^{v}\lambda_j(v,s)dv \qquad \text{for } j=1,3, \\ &p \ \text{ and } \ u \qquad \text{for } j=2. \end{aligned}$$

Here the eigenvalue $\lambda_j(w)$ is regarded as a function of v and s.

Now, for the genuinely nonlinear field $\lambda_j(w)$, $j=1$ or $j=3$, we define j-diffusion wave $W_j(t,x)$, $W_j = (v_j,u_j,E_j)^T$ with $E_j = e_j + u_j^2/2$, by

$$(6.3)_1\qquad \begin{aligned} &s_j(t,x) = \overline{s}\ , \quad u_j(t,x) = \overline{u} - \int_{\overline{v}}^{v_j(t,x)}\lambda_j(v,\overline{s})dv\ , \\ &\lambda_j(v_j(t,x),\overline{s}) - \lambda_j(\overline{v},\overline{s}) = Y(t+1,x-\lambda_j(\overline{v},\overline{s})(t+1)\,;\kappa_j(\overline{w}),\delta_j(\overline{w}))\ . \end{aligned}$$

Here $\overline{s} = \tilde{s}(\overline{v},\overline{e})$, $\kappa_j(\overline{w})$ and $\delta_j(\overline{w})$ are given by (5.2) and (6.1), respectively, and

$$(6.4)_1\qquad Y(t,x\,;\kappa,\delta) = \sqrt{\kappa}\,t^{-1/2}\frac{(e^{\delta/2\kappa}-1)e^{-\xi^2}}{\sqrt{\pi}+(e^{\delta/2\kappa}-1)\int_{\xi}^{\infty}e^{-\eta^2}d\eta}\ , \qquad \xi = x/\sqrt{4\kappa t}\ .$$

The function Y in $(6.4)_1$ is the self-similar solution of the Burgers equation $y_t + yy_x = \kappa y_{xx}$ and satisfies

$$(6.5)\qquad \int_{-\infty}^{\infty}Y(t,x\,;\kappa,\delta)dx = \delta\ , \qquad t \in (0,\infty).$$

See [3],[8]. Note that $W_j(t,x)$ lies on the curve $R_j(\overline{w})$ defined by $dw/d\tau = r_j(w)$ and $w=\overline{w}$ at $\tau = 0$. Since $\lambda_j(w)$ is monotone along $R_j(\overline{w})$ by (3.8), the relations in $(6.3)_1$ uniquely determine $v_j(t,x)$ and

$s_j(t,x)$ and therefore all other thermodynamic variables.

For the linearly degenerate field $\lambda_2(w) = 0$, we define 2-diffusion wave $W_2(t,x)$, $W_2 = (v_2,u_2,E_2)^T$ with $E_2 = e_2 + u_2^2/2$, by

$$(6.3)_2 \qquad \begin{aligned} &p_2(t,x) = \bar{p}\,, \qquad u_2(t,x) = \bar{u}\,, \\ &s_2(t,x) - \bar{s} = Y(t+1,x\,;\,\kappa_2(\bar{w}),\delta_2(\bar{w}))\,, \end{aligned}$$

where $\bar{p} = \tilde{p}(\bar{v},\bar{e})$, etc., and

$$(6.4)_2 \qquad Y(t,x\,;\,\kappa,\delta) = \delta(4\pi\kappa t)^{-1/2}e^{-\xi^2}\,, \qquad \xi = x/\sqrt{4\kappa t}\,.$$

This Y is the self-similar solution of the linear heat equation $y_t = \kappa y_{xx}$ and satisfies (6.5). Notice that $W_2(t,x)$ lies on the curve $R_2(\bar{w})$. The relations in $(6.3)_2$ define $p_2(t,x)$ and $s_2(t,x)$ and therefore all other thermodynamic variables.

Finally, we define $W(t,x)$, the superposition of the diffusion waves, by

$$(6.6) \qquad W(t,x) - \bar{w} = \sum_{j=1}^{3}(W_j(t,x) - \bar{w})\,.$$

By straightforward calculations, using $(6.3)_{1,2}$ and $(6.4)_{1,2}$, we have

$$(6.7) \qquad W_t + f(W)_x = D(\bar{w})W_{xx} + r_x(t,x) - q(t,x)\,,$$

where $r(t,x)$ and $q(t,x)$ are known functions such that

$$(6.8) \qquad \begin{aligned} &|\partial_x^\ell r(t,x)| \le C|\delta(\bar{w})|e^{-c(t+|x|)}\,, \\ &|\partial_x^\ell q(t,x)| \le C|\delta(\bar{w})|^2(1+t)^{-(4+\ell)/2}\sum_{j=1}^{3} e^{-c\xi_j^2}\,, \end{aligned}$$

where $\ell \ge 0$, $\xi_j = (x - \lambda_j(\bar{w})(t+1))/\sqrt{t+1}$, and C and c are positive constants. For the details, see [5],[8].

7. Large-time behavior

We shall show that $W(t,x)$ defined by (6.6) is an asymptotic solution for $t \to \infty$ of the uniformly parabolic system (5.3). To this end we construct the linear hyperbolic wave $\zeta(t,x)$ as the solution of

$$(7.1) \qquad \zeta_t + A(\bar{w})\zeta_x = q(t,x) \ ,$$

with the following condition: $\zeta(t,x) \to 0$ uniformly in $x \in \mathbb{R}$ as $t \to \infty$. Here $q(t,x)$ is the function in (6.7). By the characteristic method, we have a unique smooth solution $\zeta(t,x)$ satisfying

$$(7.2) \qquad |\partial_x^\ell \zeta(t,x)| \le C|\delta(\bar{w})|^2 \sum_{j=1}^{3} \{(t+1+|x-\lambda_j(\bar{w})(t+1)|^2)^{-(2+\ell)/2} + (t+1+|x-\lambda_j(\bar{w})(t+1)|)^{-(3+\ell)/2}\} \ ,$$

where $\ell \ge 0$ and C is a constant. Also, it is shown that for $t \in [0,\infty)$,

$$(7.3) \qquad \int_{-\infty}^{\infty} (W(t,x) - \bar{w} + \zeta(t,x))dx = \sum_{j=1}^{3} \delta_j(\bar{w}) r_j(\bar{w}) \ .$$

From (6.1) and (7.3) we know that $w_0(x) - W(t,x) - \zeta(t,x)$ has zero integral for each $t \in [0,\infty)$. By virtue of this property, we have a desired conclusion.

<u>Theorem 7.1</u>. ([5]) *Assume* $(1.2)_{1,2}$, *and* $(1.4)_1$ *or* $(1.4)_2$. *Suppose that* $w_0(x) - \bar{w}$ *is small in* $H^s \cap L^1_\beta$, $s \ge 1$ *and* $\beta \ge 1/2$. *Let* $z(t,x)$ *be the solution of* (5.3) *and let* $W(t,x)$ *be the superposition of the diffusion waves defined by* (6.6). *Then we have*

$$(7.4) \qquad |\partial_x^\ell (z(t) - W(t))|_2 \le CM_s(1+t)^{-(1+\ell)/2+\alpha} , \qquad t \in [0,\infty),$$

where $0 \le \ell \le s$, C *is a constant,* $M_s = \|w_0 - \bar{w}\|_s + |w_0 - \bar{w}|_{1,1/2}$, *and* $\alpha > 0$ *is a small fixed constant.*

In the proof of this theorem, the following estimate for the semigroup e^{tS} plays an essential role: If $f \in L^1_\beta$, $\beta \in [0,1]$, and $f(x)$ has zero integral, then we have

$$(7.5) \qquad |\partial_x^\ell(e^{tS}f)|_p \le Ct^{-(1-1/p+\beta+\ell)/2}|f|_{1,\beta}, \qquad t \in (0,\infty),$$

where $\ell \ge 0$, $p \in [1,\infty]$ and C is a constant.

We remark that (7.4) is a meaningful asymptotic relation for $t \to \infty$, because for large t, the L^2-norm of $\partial_x^\ell(W(t,x) - \overline{w})$ is bounded from below by $c|\delta(\overline{w})|t^{-(1/2+\ell)/2}$ with a positive constant c.

A combination of Theorems 5.1 and 7.1 gives the main result of this paper.

Theorem 7.2. ([5]) *We assume the conditions of Theorem* 7.1 *with* $s \ge 1$ *replaced by* $s \ge 5$. *Then the solution* $w(t,x)$ *of* (3.1) *satisfies*

$$(7.6) \qquad |\partial_x^\ell(w(t) - W(t))|_2 \le CM_s(1+t)^{-(1+\ell)/2+\alpha}, \qquad t \in [0,\infty),$$

where $\ell \ge 0$, $3\ell \le s-5$ *and* C *is a constant;* M_s *and* α *are the same as those in Theorem* 7.1.

This theorem means that the superposition of the diffusion waves defined by (6.6) is also an asymptotic solution for $t \to \infty$ of the system (3.1).

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