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SPATIO-TEMPORAL PATTERNS OF A PLASTIC NETWORK SYSTEM

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ABSTRACT

Spatio-temporal patterns of two-dimensional plastic networks have been studied numerically. Each constituent element evolves following a threshold dynamics and the interactions between elements are temporally changed according to the current states of elements. Such a network system is highly adaptive and thus it is expected to be useful for a model to describe central nervous systems. The evolution of the system from disordered states yields cyclic motions spatially separated. These phenomena may give an example of temporal order.

1. INTRODUCTION

Non-equiliblium statistical physics has made a great progress and fields covered by it have been spreading very rapidly up to living systems.

Neural networks are quite interesting systems to be studied from a viewpoint of non-equilibrium statistical physics. In the traditional physics, interactions are assumed to be time-independent. In order to study cooperative phenomena of living systems, it is essential to introduce " time-dependent " interactions among elements in living systems, as was emphasized by one of the present authors in a specific model.(1) Such time-dependent interactions may be called to be "plastic".

In the present paper, we introduce a typical " plastic cellular automata(CA) " as a model of neural systems.*) Usually, in modeling central nervous systems (CNS), information put into the relevant system is symbolized using spatio-temporal patterns of neural firing. That is, spatial locations of neurons in active states and temporal behaviors of neurons (firing rate of neurons) would play a role of carriers of information in the system. However, there have been reported very few studies concerning its firing spatial patterns and most of previous works have been performed on its statistical properties (such as the number of firing neurons).3),4)

We study here especially on its dynamical patterns. A real CNS is characterized by its plastic structure5) constantly tuned by external and internal signals. The purpose of the present paper is to study such a network consisting of modifiable wires(synapses) and elements(neurons). Such a plastic structure is a necessary factor for the system to possess an adaptability to its surroundings.6)

To clarify and embody the word "plastic", some basic terms are introduced in Section 2. In Section 3, simulations of a plastic network are presented. In Section 4 and Section 5, we analyze the mechanism which causes a cyclic motion. We report here several typical spatial patterns which give a long cyclic mode mostly independent of network parameters.

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2. BASIC MODELING OF NEURAL NETWORK

States of Neural Network A central nervous system (CNS) is a network of interconnected neurons. Connections between neurons are called "synapses", and neurons are interacting with each other by sending signals in electrical and chemical forms through these synapses.

Several assumptions have been proposed for constructing models for CNS.5),6),7) "All or none principle" is the first assumption. The principle is as follows: at a given time t, each neuron is either in a firing state or a non-firing state and no intermediate state exists. A "firing state" is an active state in which a neuron can transmit signals to coupled neurons. Analogously to the Ising spin system, we assign to each neuron (at time t) following two-valued state-variables

$$S_x(t) = \begin{cases} 1 & \text{(firing state)} \\ -1 & \text{(non-firing state)} \end{cases} \qquad (2.1)$$

A signal is transmitted from "j-th neuron" to "i-th neuron" in forms of electrical pulses and chemical substances through synaptic connections which are expressed as

$$\sum_j J_{i\leftarrow j}(S_j+1)/2 \qquad (2.2)$$

in our models where $J_{i\leftarrow j}$ represents the strength of coupling. $J_{i\leftarrow j}$ can be either positive or negative corresponding to excitatory or inhibitory signal, respectively. An excitatory signal is positively integrated and thus works as a factor to fire the neuron which receives it, whereas an inhibitory signal has an opposite role.

State of a network at time t is described by two qualitatively different types of variables as below:

$$\Phi(t) = (S_1(t), S_2(t), \cdots, S_N(t)) \quad (2.3)$$

$$\Psi(t) = (J_{11}(t), J_{12}(t), \cdots, J_{NN}(t)) \quad (2.4)$$

The first one is binary sequences of neural states and the second one is the set of strength of synapses.

In dealing with CNS as a deterministic dynamical system of a discrete time, a relevant mapping from Φ(t) and Ψ(t)intoΦ(t+1) and Ψ(t+1), respectively, is introduced. This is one of the finest form to describe an evolutional way of the network. Hereafter we restrict ourselves to a discrete time ($t \rightarrow n$) and discrete space dynamics.

Here we introduce the following two further assumptions peculiar to our model. We neglect here which signal is positive or negative and only treat its integrated value which determines the state of relevant neuron at the succeeding time- step. If the integrated value exceeds the threshold, a neuron which receives signal fires (transmit the signal to coupled neurons). Otherwise, it is eliminated.

For the simplicity, the threshold value H_x is taken to be proportional to its synaptic strength.

i) $J_{x \leftarrow y} = J_x$; $J_{x \leftarrow y}$ only depends on site x.

ii) $H_x = V \cdot J_x$; the threshold value is proportional to J_x.

By these assumptions, synapses become site-random fields and (2.4) is reduced to (2.5) as the next page:

$$\Psi(n) = (J_1(n), J_2(n), \cdots, J_N(n)) \qquad (2.5)$$

It should be noted that the asymmetrical condition ($J_{x\leftarrow y} \neq J_{y\leftarrow x}$) is still kept in (2.5).

Dynamics of States Each firing decision of a neuron is determined by a threshold mechanism. These neurons are able to change their states synchronously, spontaneously, or in other ways. We express these "scanning" methods by introducing the following "scanning operator" $T_i (i=1,2,..,n)$ where T_i operates on ith site:

$$S_x(n+1) = T_x S_x(n) \qquad (2.6)$$

Generally, T_x's are non-commuting operators($T_1T_2\Phi \neq T_2T_1\Phi$). More explicitly, T_x in (2.6) is defined as follows:

$$\text{if } \sum_y S_y(m) > V;\ S_x(n+1) = \mathrm{sgn}(J_x(n)) \qquad (2.7)$$

$$\text{otherwise } ;\ S_x(n+1) = -\mathrm{sgn}(J_x(n))$$

with

$$\mathrm{sgn}(x) = \begin{cases} 1 & (x > 0) \\ -1 & (x \leq 0) \end{cases} \qquad (2.8)$$

Where m=n+1 if T_y operates before T_x and m=n otherwise. The symbol $\sum_y$ denotes the summation over all neurons coupled to x and V is a threshold value at the site x.

By such a mean-field-assumption, a neuron who receives signals from coupled neurons will follow the induced mean-field $\{\mathrm{sgn}(J_x(n))\}$, if the number of coupled neurons in an active state exceeds the threshold value V.

Typical three scanning methods are listed below:

TYPE 1 (Synchronized way); T_y operate simultaneously on all the sites.

TYPE 2 (Spontaneous way); $\prod_j T_j = T_N T_2 T_5 T_7 T_1 T_{23} \cdots\cdot\cdot$ etc

Operating orders are chosen randomly.

TYPE 3 (Regular way); $\prod_j T_j = T_N T_{N-1} T_{N-2} \cdots\cdot T_2 T_1$

T_y operate in order from 1st site to Nth site.

A mapping of network state $\Psi(n)$, namely an evolutional way of distribution form of synapses, should be carefully determined. One of the most fundamental forms comes from Hebb's hypothesis.$^{10)}$ His original idea of the hypothesis is that the coupling coefficient $J_{x \leftarrow y}(n+1)$ between two neurons (x and y) is increased when $S_x(n+1)=1$ and $S_y(n)=1$.

Here the Hebb hypothesis is modified and is extended to a more symmetrical form. That is, if $S_x(n)\sum_y S^{(m)}_y=1$ then $J_x(n)$ is increased and, it is otherwise decreased. A momentary pre-postsynaptic correlation $(S_x(n)\sum_y S_y(n))$ determines the tuning mechanism of the strength of synapses in a synchronous scanning type. By using the scanning method defined previously, $J_x(n)$ is mapped into $J_x(n+1)$ as follows:

$$J_x(n+1) = T'_x J_x(n) \tag{2.9}$$

For the numerical calculation, the following functional form is used in the present paper. This is a natural extension of so-called Cooper-construction:$^{11)}$

$$T'_x J_x(n) = F(J_x(n)) + G(\{S_\cdot\}) \qquad (2.10)$$

$$F(J) = (1-\gamma)J$$

$$G(\{S\}) = \alpha_1 S_x(n) \sum_y S_y(m) - \alpha_2 \langle S_p S_q \rangle^3 \qquad (2.11)$$

with some constants γ , α_1 and α_2,

where $\langle S_p S_q \rangle = \frac{1}{N} \sum_x S_x(n) \sum_y S_y(n)$

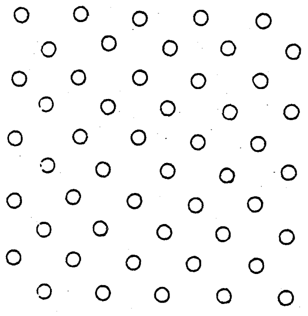
(Average value of a correlation at time n)

3. SIMULATIONS

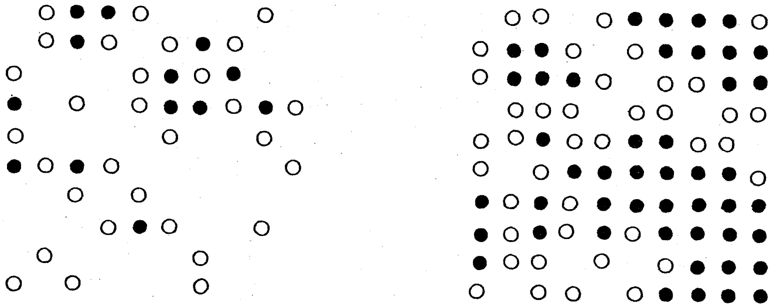
Attractors A neural network has its neurons in a rigid spatial site. Indeed, in a cerebral cortex of a mammalian, neurons are not randomly distributed but they organize a regular network structure.5),7) We use here a square lattice network of 100 sites with a periodic boundary condition in which each neuron is assigned in a two-dimensional plane and couples to its four nearest neighbors. In this square lattice network, a threshold value V=3 would be a value favorable to causing a cyclic motion as is shown in the next subsection.

The asymptotic behavior of this network can be classified roughly into three categories from its spatio-temporal patterns, essentially in the same way as Wolfram classified in cellular automata:2),12),13)(See Fig.1)

1) the same firing pattern is repeated for all time steps,
2) finite numbers(≥ 2) of $\Phi(n)$ and $\Psi(n)$ are repeated,
    i) Spatially localized periodic structure corresponding

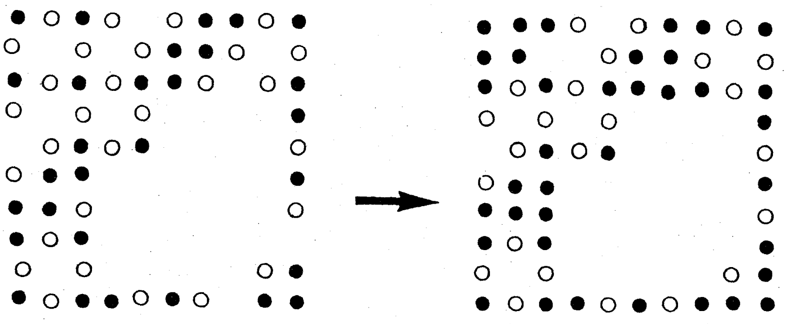


1) Fixed point like attractor; No eligible neurons exist.



2) i)Spatially localized oscillating areas coexist (K=84). (Right figure)

ii)Catastrophic structure; Amost all the oscillating areas are connected(K=2). (Left figure)



3) Neighboring epileptic or dead neurons of eligible one (left figure) become eligible after 1000 time steps (right figure).

Fig.1 Phenomenological classification of network behaviors in an attractor region, except for 3) which is assumed to be in a long transient region. Each site with eligible, epileptic and dead neurons is represented by black point, white point and blanks, respectively. The temporal evolution of these networks are defined by Eqs.(2.6),(2.9) and (2.10). A simulation is performed on a square lattice of 100 sites. The initial strength of synapse at each site is taken to be random, namely, takes an independent value from$[-1,1]$ with equal probability. The initial state of a neuron at each site is taken as $S_x(0)=-\mathrm{sgn}(J_x(0))$ (e.g. values 1 or -1 independently).

to fixed points,

ii) Catastrophic structure... periodic but not spatially locarized,

3) A localized aperiodic structure survives, or such an aperiodic region intrudes into other neighboring regions.

It should be noted that the recurrence of the same firing patterns does not necessarily mean no more that the system enters into cyclic modes for a plastic network.

We identify the occurrence of a cyclic mode K when the next conditions are satisfied:

$$S_x(n+K) - S_x(n) = 0$$

and

$$|J_x(n+K) - J_x(n)| < \delta \; (\sim 0.001) \qquad (3.1)$$

for all sites x

Here the step number K is a minimal length of the recurrence of the same state $\Phi(n)$ and $\Psi(n)$. We identify K with a cyclic mode of attractor and will be called either a reducible or irreducible cycle. A reducible cycle is defined as a superposition of subcycles ($K_1, K_2, K_3, \ldots$). In such a case, K=L.C.M. of (K_1, K_2, K_3). On the other hand, an irreducible cycle can not be decomposed into subcycles.(See the next section.)

Each neuron is also categorized into the following three types according to its behaviors as Clark et al. 5)discussed in their paper:

1) dead neuron ....neuron which never fires,

2) epileptic neuron ....neuron which always fires, (shows cyclic behavior, but its cyclic mode is the same as in the unit

of discrete time steps)

3) eligible neuron ....neuron which is not locked into either extreme behavior.

Firing Loops The purpose of this simulation is to find the linked stable firing loop of eligible neurons for varying the relevant parameters and initial conditions. A definition of loop is that each eligible neuron couples to at least one eligible neuron in its nearest neighbors.

It is widely believed that there are two different kinds of mechanism that correspond to a short term and a long term memory.6)-9) The first idea is to relate those loops to memory carriers. It is a natural and fascinating idea to imagine some relations between "associative memories" or "thought" and "loop" or "loop-loop coupling".

A computer simulation has shown the existence of firing loops but with a fairy different mechanism in comparison with a static network system (a system of fixed values of synapses). In most cases, the attained attractors shows a cyclic motion (namely class 2 behavior by the previous definition). This turns that a cyclic mode k of the network is L.C.M. of each mode of these localized periodic structures.

The total cyclic mode K will be decomposed into

1) $\otimes$ (simple subcycles)

and

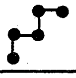
2) (simple subcycles) $\otimes$ (rich subcycles) (See Table.I).

where the symbol $\otimes$ means a direct product.

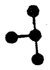
A simple subcycle is always a simple irreducible one which only one eligible neuron participates in the cyclic mode.(This is not a loop by definition). A spatially localized pattern of a simple subcycle is an eligible neuron surrounded by three epileptic neurons and one dead one. In such a case, a possible cyclic mode of simple

Table.I Examples of cyclic modes. Each cycle is decomposed into simple and rich subcycles. Upper suffix denotes the number of the same pattern and mode. Whereas lower one denotes the number of eligible neuron constructing the cyclic motion. Spatial patterns of rich subcycles are also depicted.

Scanning type 3 (Sequential)

Period	Decomposition into subcycles	Number of eligible neurons
12	$6_1^6 \times 4_1^1 \times 1^{39}$	7
18	9_4^1() $\times\ 9_2^1$() $\times\ 6_1^9 \times 1^{33}$	15
66	66_5^1() $\times\ 6_1^4 \times 1^{30}$	9
84	12_3^1() $\times\ 7_3^1$() $\times\ 6_1^7 \times 1^{29}$	13
288	32_3^1() $\times\ 9_2^3$() $\times\ 6_1^6 \times 1^{29}$	15

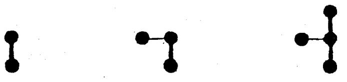
Scanning type 1 (Parallel)

12	12_3^1() $\times\ 6_1^6 \times 1^{29}$	9
66	66_4^1() $\times\ 6_1^7 \times 1^{29}$	11
462	77_2^1() $\times\ 7_3^2$() $\times\ 6_1^5 \times 1^{31}$	13

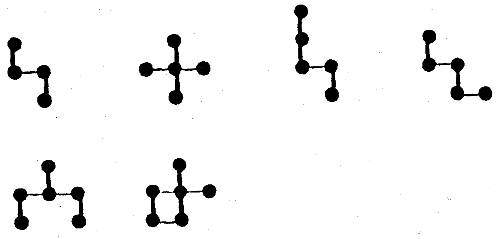
subcycle is 2, 4 and 6.

On the other hand, the mode of K which is larger than 12 (=L.C.M. of (6,4,2)) should contain rich subcycles. Rich subcycles which contain more than two eligible neurons are composed of minimum firing loops by definition.

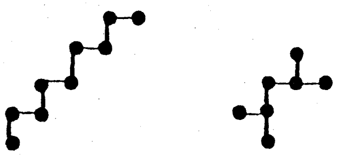
For random initial conditions, well observed spatially localized patterns are listed up in Fig 2. Although the cyclic mode they carry depends on the network parameters (γ , α_1,α_2), and also on the



1) Geometrical patterns of most probable rich sub-cycles.



2) Rare but observed patterns.



3) Patterns generated from a special initial state.

Fig.2 The total cyclic mode K will be decomposed into products of subcycles. A rich subcycle is defined as a subcycle containing more than two eligible neurons. Examples of spatial patterns of these rich subcycles are listed as in 1) $\sim$ 3).

location of epileptic and dead neurons surrounding the eligible neurons, these geometrical patterns are scanning-type independent.

As it is depicted in figures 2 and 3, these rich subcycles are "twisted strings" which consist of nearly one-dimensional coupled eligible neurons. In the case of one-dimensionally connected case (e.g. each neuron in rich subcycles has exactly two coupled eligible neurons except the edging neuron of a string), we can assign the numbers from one end of a string to the other.

Such rich subcycles are able to be compared with one-dimensional CA by writing a spatio-temporal pattern. Although some patterns are similar to that of one-dimensional cellular automata (1DCA)12),13),

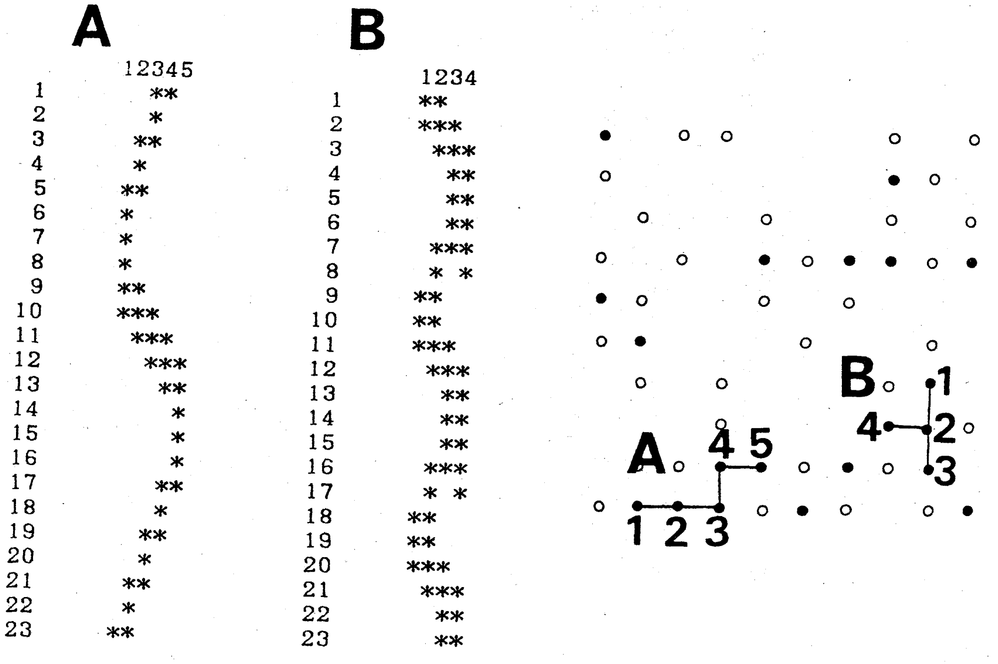
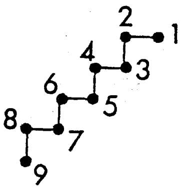


Fig.3 Space-time section of rich sub-cycles. After a network is detected to a cyclic motion, oscillating areas are picked out and the constituent neurons are numbered and expanded horizontally. The configurations of successive steps are shown on successive horizontal lines. Neurons on firing states are represented by stars, and neurons on a non-firing state are by blanks.

the mechanisms that cause those patterns are different. In 1DCA, particular rules create propagating patterns, on the other hand, spatial inhomogeneity of synaptic distribution generates such patterns in our model.

Another important aspect of rich subcycles is that there exist particular eligible neurons which simultaneously cooperate in more



```
    123456789
 1    *****
 2   **   **
 3  **  *  **
 4  **  *  **
 5  ** *** **
 6     * *
 7    ** **
 8    ** **
 9   **   **
10  *********
11  * ** ** *
12  *       *
13  *   *   *
14  *  ***  *
15  * ** ** *
16  *  * *  *
17  ** * * **
18    ** **
19    *   *
20   *******
21  ** * * **
22  * ** ** *
23  ***   ***
24  **  *  **
25   *  *  *
26     ***
27    ** **
28   *** ***
29   *     *
30  **  *  **
31  ** *** **
32  ** * * **
33     * *
34    ** **
35    ** **
36   **   **
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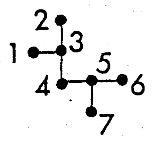

```
    1234567
 1   ** **
 2    ***
 3
 4  * * * *
 5    ***
 6   ** **
 7   *****
 8   ** **
 9    ***
10
11  * * * *
12    ***
13   ** **
14   *****
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18  * * * *
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32  * * * *
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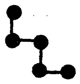
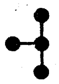
35   *****
36   ** **
```

Fig.4 Space-time sections of obtained large local cyclic motion are depicted. The configurations of succesive steps are shown on succesive horizontal lines. Neurons on a firing state are represented by stars, and neurons on non-firing states are by blanks. It shows that processing signals (firing state) propagate symmetrically relative to the central neuron.

than two subcycles. Some of obtained large subcycles can be taken as linking of smaller size of rich subcycles.

Examples of such large subcycles containing 9 and 7 eligible neurons are depicted in Fig 4. It has a symmetrical form and the processing signals(sites of neurons in a firing state) propagate symmetrically to both the ends of a string. Namely, a central neuron in these subcycles links two rich subcycles()and(), respectively.

Unfortunately, these large subcycles are rather rare for systems starting from a random initial state and can be obtained under setting special initial conditions.

4. MECHANISM OF SUBCYCLES

Mechanism which produces subcycles are owing to the dynamical behavior of J_n. Basically, the mechanism which causes a cyclic motion (for a threshold value V =3) is that if one non-active neuron(S_x) is surrounded by three epileptic and one dead neurons, then J_x should gradually change its value towards firing neuron(S_x). Starting with a randomly distributed initial state, the average value of correlation $\langle S_pS_q\rangle^3$ (the third term in J_x)is negligible in the attractor region and is approximated as a small constant value (ε).

A maximum period of a simple subcycle is K=6 (Namely, S=1 for 3 steps, S=-1 for the next three steps, it explains period K=6).(See the proof in the full paper.) On the contrary, the rich subcycles are brought about, when the surrounding neurons change their states. In such a case, the situation is not simple enough to analyze its mechanism.

The following five different mappings are possible :

$$J_x(n+1)=(1-\gamma)J_x(n)+\alpha_1 C-\varepsilon \qquad (4.1)$$

with C = -4,-2,0,2,4. These correspond to I_1, I_2, I_3, I_4, I_5. They differ in the second term which is the summation of states of coupled neurons.) Temporal evolution of J equals to temporal ordering of those mappings. (For example, $I_2 \rightarrow I_2 \rightarrow I_5 \rightarrow I_3 \rightarrow I_1 \ldots$).

These orderings are determined by the states of the surrounding neurons.(e.g. the second term C) Inversely, the phase difference of a neighboring J determines the temporal evolution of neurons.

There seems two types of long cyclic behaviors in a rich subcycle. One is caused essentially by smallness of ε and the other is independent of ε . For the former type, even two eligible neurons can achieve a long periodic behavior by using two mappings in (4.1). Mapping I_3, namely C=0 is dominant, which causes long iteration steps to change the sign of $J_x(n)$. As ε decreases, a longer period should be attained. The latter type of motion is essentially due to the multiple spatial connection among neurons. Mappings with $C \neq 0$ are dominant, and consequently $J_x(n)$ changes its sign easily . The propagation of patterns is observed in this case.(See Fig.3-4)

Rich subcycles are unstable against the noise (anti-deterministic flip of a single neural site). On the other hand, simple subcycles are not so. This is because a rich subcyclic motion comes from a subtle phase difference of J s between coupled neurons, whereas simple one can oscillate by itself.

5. DISCUSSIONS

In the present paper, we have investigated the spatio-temporal behavior of plastic network systems in two dimensions from a phenomenological point of view. Our first aim was to find a long and stable loop of eligible neurons using plasticity. Cyclic behaviors obtained are decomposed into rich and simple subcyclic motions and the former subcycle is indeed a string of eligible neurons.(Which is called a "loop" here.)

The attained loops are grouped into typical localized geometrical patterns. Each loop has its own cyclic mode according to the spatial location of surrounding epileptic and eligible neurons. Geometrical patterns of these loops are twisted strings and some of them are one-dimensionally connected. A space-time section of loop shows a behavior qualitatively similar to the section of one-dimensional CA12). Especially, breathing patterns are worthnoting to study. The length of one pitch of breathing motion is determined from the phase difference of synapses participating in the motion.

In recent years, lots of works have been published on problems of memory storaging from the view point of spin glass models (SG).3),11),14),15) Attractors of SG are fixed points and cyclic behaviors are infrequently observed, however, the advantage of SG is that spatial pattern of attractor can be expressed in terms of the distribution of synapses. The problem is how to controle the basin structure of these attractors. (e.g. Most of the initial states go to "wanted attractors" and the rest of initial states go to "unwanted attractors".)

In our model, one firing pattern does not only correspond to a unique spatial distribution of synapses but also correspond to the to the temporal evolution of strength of synapses. To this point, the capacity of memory storaging should be much larger than that of SG. However, an algorithm how to construct the initial distribution to get the "wanted cyclic attractor" is still missing.

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