数理解析研究所講究録
第 602 巻 1986 年 244-246

QUASI-ANOSOV DIFFEOMORPHISMS AND PSEUDO-ORBIT TRACING PROPERTY

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ABSTRACT

In this note we announce the result that every quasi-Anosov diffeomorphism with pseudo-orbit tracing property must be an Anosov diffeomorphism.

Let M be a compact boundaryless C^∞-manifold, and let $\mathrm{Diff}(M)$ be the space of C^1-diffeomorphisms of M endowed with the C^1-topology. An Axiom A diffeomorphism is said to satisfy the <u>strong transversality condition</u> if for every $x \in M$, $T_xM = T_xW^s(x) + T_xW^u(x)$. For an Axiom A diffeomorphism, the strong transversality is a sufficient condition to be structurally stable (i. e. there is a neighbourhood $\mathcal{U} \subset \mathrm{Diff}(M)$ of f such that for every $g \in \mathcal{U}$, there is a homeomorphism h on M satisfying $f \circ h = h \circ g$). We say that $f \in \mathrm{Diff}(M)$ is <u>topologically stable</u> if for every $\varepsilon > 0$, there is a neighbourhood $\mathcal{U}_\varepsilon$ of f in the set of homeomorphisms of M with the C^0-topology such that for every $g \in \mathcal{U}_\varepsilon$, there is a continuous surjection h on M satisfying $f \circ h = h \circ g$ and $d(h(x), x) < \varepsilon$ for $x \in M$ (here d denotes a metric compatible with the topology of M).

Let $g : X \to X$ be a homeomorphism of a compact metric space (X, d). A sequence of points $\{x_i\}_{i=a}^{b}$ $(-\infty \leq a < b \leq \infty)$ in X is called δ-pseudo-orbit of g if $d(g(x_i), x_{i+1}) < \delta$ for $a \leq i \leq b-1$. A sequence $\{x_i\}$ is called to be ε-traced by $x \in X$ if $d(g^i(x), x_i) < \varepsilon$ holds for $a \leq i \leq b$. We say that g has pseudo-orbit tracing property (abbrev. POTP) if for every $\varepsilon > 0$ there is $\delta > 0$ such that every δ-pseudo-orbit of g can be ε-traced by some point in X. We say that g is expansive if there exists $c > 0$ such that $d(g^n(x), g^n(y)) \leq c$ for every $n \in \mathbb{Z}$ implies $x = y$. Such a number c is called an expansive constant for g. For the materials of topological dynamics on compact manifolds, see Morimoto [4].

It is well known that every homeomorphism on M with expansivity and POTP is topologically stable, and that every topologically stable homeomorphism on M of dimension ≥ 2 has POTP (see [4]). Every Axiom A diffeomorphism f satisfying the strong transversality condition is topologically stable (thus every Anosov diffeomorphism is topologically stable) and so f has POTP.

We say that $f \in \mathrm{Diff}(M)$ is quasi-Anosov if for every $0 \neq v \in TM$, the set $\{\|(Tf)^n(v)\| : n \in \mathbb{Z}\}$ is unbounded. A quasi-Anosov diffeomorphism is equivalent to an Axiom A diffeomorphism satisfying $T_xW^s(x) \cap T_xW^u(x) = \{0_x\}$ for every $x \in M$ ([3]). Obviously every Anosov diffeomorphism is quasi-Anosov and its converse is true if $\dim M = 2$ ([3]). But it is known ([1]) that the converse is not true on a 3-dimensional manifold. Mañé proved the following

THEOREM ([3]). For $f \in \mathrm{Diff}(M)$ the following conditions are mutually equivalent;

(i) f is Anosov,

(ii) f is quasi-Anosov and satisfies the strong transversality condition,

(iii) f is quasi-Anosov and structurally stable.

The aim of this note is to announce the following theorem related to the above results.

THEOREM. Every quasi-Anosov diffeomorphism with POTP must be an Anosov diffeomorphism.

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