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A Note on Three-Way Two-Dimensional Alternating Turing Machines

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ABSTRACT In this paper, we investigate the properties of three-way two-dimensional alternating Turing machines (TR2-ATMs) which are restricted versions of ordinary four-way two-dimensional Turing machines (2-ATMs). We first show that 2-ATMs are more powerful than TR2-ATMs with spaces below log m. Then, we investigate the closure properties of TR2-ATMs on general rectangular input tapes. Finally, we show that o(log m) space-bounded TR2-ATMs cannot accept the set of all square connected pictures.

1. Introduction

Recently [1], two-dimensional alternating Turing machines (2-ATMs) were introduced as a generalization of two-dimensional nondeterministic Turing machines (2-TMs) and as a mechanism to model parallel computation. In the subsequent papers [2,3], several investigations of this automata have been continued.

In this paper, we investigate a three-way two-dimensional alternating Turing machines (TR2-ATMs) which is a restricted version of the 2-ATM. The input head of the TR2-ATM can only move left, right, or down.

It is known [1] that for spaces greater than or equal to log m, TR2-ATMs are equivalent to 2-ATMs on square input tapes. Section 3 shows for space smaller than log m, TR2-ATMs are weaker than 2-ATMs.

Section 4 investigates the closure properties under such as row catenation or row closure of the class of sets of general rectangular input tapes accepted by TR2-ATMs.

It is also known [1,4] that a four-way two-dimensional alternating finite automaton (i.e., 2-ATM with zero space) can accept the set of all squre connected pictures (T_c), and m space is necessary and sufficient for three-way two-dimensional deterministic or nondeterministic Turing machines to accept T_c. Section 5 show that log m space is necessary and sufficient for TR2-ATMs to accept T_c.

2. Preliminaries

[Definition 2.1] Let Σ be a finite set of symbols. A two-dimensional rectangular array of elements of Σ.

The set of all two-dimensional tapes over Σ is denoted $\Sigma^{(2)}$. Given a tape $x \in \Sigma^{(2)}$, we let $\ell_1(x)$ be the number of rows of x and $\ell_2(x)$ be the number of columns of x. The set of all $x \in \Sigma^{(2)}$ with $\ell_1(x)=m$ and $\ell_2(x)=n$ ($\ell_1(x)=m$, $\ell_2(x)=n$) is denoted by $\Sigma^{(m,n)}$ ($\Sigma^{(m,*)}$, $\Sigma^{(*,n)}$, respectively). If $1 \leq i \leq \ell_1(x)$ and $1 \leq j \leq \ell_2(x)$, we let x(i,j) denote the symbol in x with coordinates (i,j). Furthermore, we define

$$x[i:i',j:j'],$$

when $1 \leq i \leq i' \leq \ell_1(x)$ and $1 \leq j \leq j' \leq \ell_2(x)$, as the two-dimensional tape z satisfying the following:

(i) $\ell_1(z)=i'-i+1$ and $\ell_2(z)=j'-j+1$.

(ii) for each $k,r[1 \leq k \leq \ell_1(z), 1 \leq r \leq \ell_2(z)]$, $z(k,r)=x(k+i-1,r+j-1)$.

When i=i' (j=j') above, x[i:i',j:j'] is abbreviated by x[i,j:j'] (x[i:i',j]). For $x \in \Sigma^{(m,n)}$, the ith row x[i,1:n] and jth column x[1:m,j] are simply denoted by x[i,*] and x[*,j], respectively.

Two-dimensional alternating Turing machines were introduced in [1]. We recall the definition.

[Definition 2.2] A <u>two-dimensional alternating Turing machine</u> (2-ATM) is a seven-tuple

$$M=(Q,q_0,U,F,\Sigma,\Gamma,\delta),$$

where

(1) Q is a finite set of <u>states</u>,

(2) $q_0 \in Q$ is the <u>initial state</u>,

(3) $U \subseteq Q$ is the set of <u>universal states</u>,

(4) $F \subseteq Q$ is the set of <u>accepting states</u>,

(5) Σ is a finite <u>input alphabet</u> ($\# \notin \Sigma$ is the <u>boundary symbol</u>),

(6) Γ is a finite <u>storage tape alphabet</u> ($B \in \Gamma$ is the <u>blank symbol</u>), and

(7) $\delta \subseteq (Q\times(\Sigma\cup\{\#\})\times\Gamma)\times(Q\times(\Gamma-\{B\})\times\{$left,right,up,down,no move$\}\times\{$left,right,no move$\})$ is the <u>next move relation</u>.

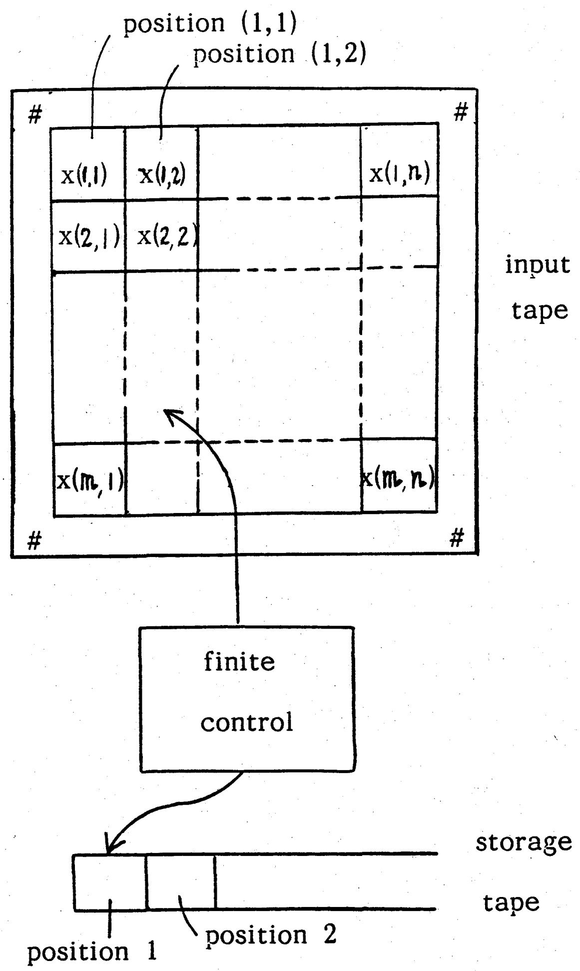

Fig.1 Two-dimensional alternating Turing machine.

A state q in Q-U is said to be existential. As shown in Fig. 1, the machine M has read-only rectangular input tape with boundary symbol "#" and one semi-infinite storage tape, initially blank. Of course, M has a finite control, an input head, and a storage tape head. A position is assigned to each cell of the storage tape, as shown in Fig.1. A step of M consists of reading one symbol from each tape, writing a symbol on the storage tape, moving the input and storage heads in specified directions, and entering a new state, in accordance with the next move relation δ. Note that the machine cannot write the blank symbol. If the input head falls off the input tape, or if the storage head falls off the storage tape (by moving left), then the machine M can make no further move.

[Definition 2.3] A <u>configuration</u> of a 2-ATM $M=(Q,q_0,U,F,\Sigma,\Gamma,\delta)$ is an element of

$$\Sigma^{(2)}\times(N\cup\{0\})^2\times S_M,$$

where $S_M=Q\times(\Gamma-\{B\})^*\times N$, and N denotes the set of all positive integers. The first component of a configuration $c=(x,(i,j),(q,\alpha,k))$[↓] represents the input to M. The second component (i,j) of c represents the input head position. The third component (q,α,k) of c represents the state of the finite control, nonblank contents of the storage tape, and the storage-head position. If q is the state associated with configuration c, then c is said to be universal (existential, accepting) state. The <u>initial configuration</u> of M on input x is

$$I_M(x)=(x,(1,1),(q_0,\lambda,1)).$$

[Definition 2.4] Given $M=(Q,q_0,U,F,\Sigma,\Gamma,\delta)$, we write

$$c \vdash_M c'$$

and say c' is a successor of c if configuration c' follows from configuration c in one step of M, according to the transition rules δ. The relation $\vdash_M$ is not necessarily single valued, since δ is not.

A <u>computation tree</u> of M is a finite, nonempty labeled tree with the properties,

(1) each node π of the tree is labeled with a configuration

↓ We note that $0\leq i\leq \ell_1(x)+1$, $0\leq j\leq \ell_2(x)+1$, and $1\leq k\leq|\alpha|+1$, where for any string w, $|w|$ denotes the length of w (with $|\lambda|=0$, where λ is the null string).

$\ell(\pi)$,

(2) if π is an internal node (a nonleaf) of the tree, $\ell(\pi)$ is universal and

$$\{c \mid \ell(\pi) \vdash_{M} c\} = \{c_1, \cdots, c_k\},$$

has exactly k children $\rho_1, \cdots, \rho_k$ such that $\ell(\rho_i) = c_i$,

(3) if π is an internal node of the tree and $\ell(\pi)$ is existential, then π has exactly one child such that

$$\ell(\pi) \vdash_{M} \ell(\rho).$$

An <u>accepting computatiion tree</u> of M on x is a computation tree whose root is labeled with $I_M(x)$ and whose leaves are all labeled with accepting configurations. We say that M <u>accepts</u> x if there is an accepting computation tree of M on x. Define

$$T(M) = \{x \in \Sigma^{(2)} \mid M \text{ accepts } x\}.$$

We next recall the definition of three-way 2-ATM.

[Definition 2.5] A <u>three-way two-dimensional alternating Turing machine</u> (TR2-ATM) is a 2-ATM $M=(Q,q_0,U,F,\Sigma,\Gamma,\delta)$ such that

$$\delta \subseteq (Q \times (\Sigma \cup \{\#\}) \times \Gamma) \times (Q \times (\Gamma - \{B\}) \times \{\text{left,right,down,no move}\} \times \{\text{left,right,no move}\}).$$

That is, a TR2-ATM is a 2-ATM whose input head can move left, right, or down, but not up.

In this paper, we shall investigate the properties of TR2-ATMs whose storage tapes are bounded (in length) to use.

Let $L: N \times N \rightarrow R$ be a function with two variables m and n, where R denotes all non-negative real numbers. With each 2-ATM (TR2-ATM) M we associate a space complexity function SPACE which takes configuration $c=(x,(i,j),(q,\alpha,k))$ to natural numbers. Let SPACE(c) $=|\alpha|$. We say that M is "L(m,n) space-bounded" if for all m,n and for all x with $\ell_1(x)=m$ and $\ell_2(x)=n$, if x is accepted by M then there is an accepting computation tree of M on input x such that for each node π of the tree $SPACE(\ell(\pi)) \leq \lceil L(m,n) \rceil$[↓]. By "2-ATM(L(m, n))" ("TR2-ATM(L(m,n))") we denote an L(m,n) space bounded 2-ATM (L(m,n) space bounded TR2-ATM). Define

$\mathcal{L}$[2-ATM(L(m,n))]={T|T=T(M) for some 2-ATM(L(m,n)) M}, and

$\mathcal{L}$[TR2-ATM(L(m,n))]={T|T=T(M) for some TR2-ATM(L(m,n)) M}.

In section 3 and 5 of this paper, we concentrate on the properties of TR2-ATMs whose input tapes are restricted to square ones. Let $L:N \to R$ be a function with one variable m. We say that M is "L(m) space-bounded" if for all m and for all x with $\ell_1(x)=\ell_2(x)=m$, if x is accepted by M, then there is an accepting computation tree of M on x such that, for each node π of the tree, $SPACE(\ell(\pi)) \leq L(m)$. By "$2\text{-}ATM^s(L(m))$" ("$TR2\text{-}ATM^s(L(m))$") we denote an L(m) space-bounded 2-ATM (TR2-ATM) whose input tapes are restricted to square ones. Define

$\mathcal{L}[2\text{-}ATM^s(L(m))]$={T|T=T(M) for some $2\text{-}ATM^s(L(m))$ M}, and

$\mathcal{L}[TR2\text{-}ATM^s(L(m))]$={T|T=T(M) for some $TR2\text{-}ATM^s(L(m))$ M}.

By using well-known techniques, it is easily proved that for any constant $k \geq 0$, $\mathcal{L}[2\text{-}ATM^s(k)]=\mathcal{L}[2\text{-}ATM^s(0)]$. We especially denote a $2\text{-}ATM^s(0)$ by '$2\text{-}AFA^s$'. A two-dimensional deterministic finite automaton ($2\text{-}DA^s$) [1,5] is a special case of $2\text{-}AFA^s$ whose configurations each have at most one successor.

3. Three-way vs. Four-way

[↓] $\lceil r \rceil$ means the smallest integer greater than or equal to r. From now on, we omitt $\lceil\ \rceil$, if no confusion occurs.

In this section, we investigate the relationship between TR2-ATMs and 2-ATMs or other four-way automata on square input tapes. Following is the key lemma.

[Lemma 3.1] Let $T=\{x\in\{0,1\}^{(2m,2m)} \mid m\geq 1$ & $\exists w\in\{0,1\}^{+}(x[1,*]=ww^{R})$ & (the other part of x consists of 0's)$\}$[↓]. Then

1) $T\in\mathcal{L}[\text{2-DA}^{S}]$, and

2) $T\notin\mathcal{L}[\text{TR2-ATM}^{S}(L(m))]$ for any function $L:N\to R$ such that $\lim_{m\to\infty}[L(m)/\log m]=0$.

[Note 3.1] It is easily shown that the row reflection of T, that is, the set $\{x\in\{0,1\}^{(2m,2m)} \mid m\geq 1$ & $\exists w\in\{0,1\}^{+}(x[2m,*]=ww^{R})$ & (the other part of x consists of 0's)$\}$ can be accepted by a $\text{TR2-ATM}^{S}(0)$.

It was unknown in [1] that $\text{TR2-ATM}^{S}(L(m))$'s are less powerful than $\text{2-ATM}^{S}(L(m))$'s for $L:N\to R$ such that $\lim_{m\to\infty}[L(m)/\log m]=0$. From Lemma 3.1 and additional arguments, we can solve the problem as follows.

[Theorem 3.1] For any function $L:N\to R$ such that $\lim_{m\to\infty}[L(m)/\log m]=0$,

$$\mathcal{L}[\text{TR2-ATM}^{S}(L(m))]\subsetneq\mathcal{L}[\text{2-ATM}^{S}(L(m))].$$

From the fact that $\mathcal{L}[\text{TR2-ATM}^{S}(\log m)]=\mathcal{L}[\text{2-ATM}^{S}(\log m)]$ [1] and $\mathcal{L}[\text{2-DA}^{S}]\subseteq\mathcal{L}[\text{2-AFA}^{S}]$, and from Lemma 3.1, we get the following.

[Theorem 3.2] log m space is necessary and sufficient for TR2-ATM^{S} to simulate 2-DA^{S} and 2-AFA^{S}.

4. <u>Closure Properties of $\mathcal{L}$[TR2-ATM(L(m,n))]</u>.

In this section, we investigate the closure properties of [TR2-ATM(L(m,n))] under several operations.

[↓]For any string w, w^{R} denotes the reversal of w.

We assume that the reader is familiar with definitions of the operations "⊖"(row catenation), "(subscript +)"(row closure), "RC"(row cyclic closure), and "projection", which are defined over two-dimensional tapes or over sets of two-dimensional tapes. (If necessary, see reference [5].)

[Definition 4.2] Let $g:N \to R$ be a fully space constractible function[↓]. Let x be in $\Sigma^{(*,n)}$ ($n \geq 1$). When $\ell_1(x)$ is divided by $2^{\lceil g(n) \rceil}$, we call

$$x[(j-1)2^{\lceil g(n) \rceil}+1:j2^{\lceil g(n) \rceil},*]$$

the <u>jth g(n)-block of x</u>, for each $j(1 \leq j \leq \ell_1(x)/2^{\lceil g(n) \rceil})$.

We now give several lemmas which will be used below.

[Lemma 4.1] Let $g:N \to R$ be a fully space constructible function, and let

$T(g)=\{x \in \{0,1\}^{(k2^{g(n)},n)} | n \geq 1, k \geq 1$ (i.e., x has exactly k g(n)-blocks)$\}$,

$T'(g)=\{x \in \{0,1\}^{(k2^{g(n)},n)} | (n \geq 1, k \geq 2)$ & the first and kth g(n)-blocks of x are identical$\}$, and

$T''(g)=T(g) \ominus \{2\}^{(1,*)} \ominus T'(g)$.

Then, $T(g)$, $T'(g)$, and $T''(g)$ are all in $\mathcal{L}$[TR2-ATM(g(n))].

[Lemma 4.2] Let $g:N \to R$ be a monotonic nondecreasing function which is fully space constructible, and $f:N \to R$ be a function such that $\lim_{m \to \infty}[f(m)/\log m]=0$. Furthermore, let $T(g)$, $T'(g)$, and $T''(g)$ be sets described in Lemma 4.1. Then,

$T'(g) \ominus T(g)=\{x \in \{0,1\}^{(k2^{g(n)},n)} | (n \geq 1, k \geq 3)$ & $\exists j(2 \leq j \leq k-1)$(the first and jth g(n)-block of x are identical)$\}$

[↓] See reference [1] or [6] for the definition of fully space constructible function.

and T''(g)^{RC} are not in $\mathfrak{L}$[TR2-ATM(L(m,n))], where L(m,n)=f(m) •g(n).

Lemma 4.1, 4.2, and some additional arguments lead us to the following theorem.

[Theorem 4.1] Let f(m) and g(n) be the function described in Lemma 4.3. Then $\mathfrak{L}$[TR2-ATM(f(m)•g(n))] are not closed under row catenation, row closure, row cyclic closure, or projection.

[Note 4.1] It is not so difficult to show that for any function L:N×N→R such that L(m,n)≧log m, $\mathfrak{L}$[TR2-ATM(L(m,n))] (=$\mathfrak{L}$[2-ATM(L(m, n))]) is closed under row catenation and row closure.

5. Recognizability of Connected Pictures

It is well-known [1] that a 2-AFA^s can accept the set of all the square connected pictures. (We denote it by "T_c".) It is, however, an open problem whether or not a three-way 2-AFA^s can accept T_c. We solve the problem as follows.

[Theorem 5.1] log m space is necessary and sufficient for TR2-ATM^s to accept the set of all the square connected pictures.

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