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Differentials of Prym maps and counterexamples of the infinitesimal Torelli theorem

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In this note, we shall announce new examples for which the infinitesimal period map (for the definition, see §3) is not injective. These are certain smooth (n+1)-folds X for $n \geq 1$ which have the structures of the fibre spaces $\tau: X \longrightarrow C$ over hyperelliptic curves of genus $g \geq 1$. In case $n = 1$, these examples give surfaces of general type or elliptic surfaces for which the infinitesimal Torelli theorem does not hold even if they have sufficiently high geometric genera. We also explain the relation between these example and the (co-)differential of the Prym maps for hyperelliptic curve.

All the varieties in this note will be defined over the field of complex numbers $\mathbb{C}$ and by a hyperelliptic curve of genus $g \geq 1$, we mean a smooth complete curve which has a linear system g^1_2.

Details will be published elsewhere.

1. Prym maps and its (co-)differentials.

In this section, we shall review some known facts on Prym varieties due to Mumford [M] and some results on (co-)differential of Prym maps due to Beauville [B].

Let

(1.1) $$\pi:\tilde{C} \longrightarrow C$$

be a double covering where $\tilde{C}$ and C are nonsingular complete curves and π is a morphism of degree 2. To each such double covering, we associate the data $(C,\delta,\mathcal{L})$ consisting of

(1.2):

(i) C ; *a nonsingular complete curve of genus* g

(ii) δ ; *an effective divisor of* **degree 2m** *of* C *without multiple components, the ramification divisor of* π,

(iii) $\mathcal{L}$; *an invertible sheaf on* C *such that* $\mathcal{L}^2 \simeq \mathcal{O}(\delta)$ *and*

(1.3) $$\pi_*(\mathcal{O}_{\tilde{C}}) = \mathcal{O}_C \oplus \mathcal{L}^{-1}.$$

(We say that the data $(C,\delta,\mathcal{L})$ in (1.2) is of *type* (g,m).) Conversely, if the data $(C,\delta,\mathcal{L})$ is given as in (1.2), we can construct the double covering $\pi:\tilde{C} \longrightarrow C$ branched at the divisor δ which satisfies (1.3). Since genus of C = g and degree of δ = 2m, by Hurwitz's formula, the genus $\tilde{g}$ of $\tilde{C}$ is equal to 2g+m-1.

Let $J = \mathrm{Pic}^0(C)$ and $\tilde{J} = \mathrm{Pic}^0(\tilde{C})$ be the Jacobians of the curves C and $\tilde{C}$ in (1.1) and let Θ and $\tilde{\Theta}$ be the theta divisors of J and $\tilde{J}$ which give canonical principal polarizations:

(1.4) $$\lambda_{\Theta} : J \longrightarrow \hat{J} = \mathrm{Pic}^0(J),$$

(1.5) $$\lambda_{\tilde{\Theta}} : \tilde{J} \longrightarrow \hat{\tilde{J}} = \mathrm{Pic}^0(\tilde{J}).$$

(Here for any divisor D on an abelian variety A, we define $\lambda_D : A \longrightarrow$

$\hat{A} = \mathrm{Pic}^{0}(A)$ by $\lambda_D(x) =$ [divisor class of $T_x(D) - D$].)
The double covering (1.1) induces two homomorphisms of Jacobians

$$\varphi = \pi^{*}: J \longrightarrow \tilde{J} \quad \text{and} \quad Nm = \pi_{*}: \tilde{J} \longrightarrow J.$$

and it is easy to see that $Nm \circ \varphi = 2_J$ and $\varphi \circ Nm = 1_{\tilde{J}} + \iota^{*}$ where $\iota: \tilde{C} \longrightarrow \tilde{C}$ is the involution corresponding to the double covering (1.1). The Prym variety of the double covering (1.1) is the odd part of $\tilde{J}$ defined by

(1.6) $$P = (\ker Nm)^{0} = \ker(1_{\tilde{J}} + \iota^{*})^{0} = \mathrm{Im}\,(1_{\tilde{J}} - \iota^{*}).$$

Since $\dim.J = g$ and $\dim.\tilde{J} = 2g + m - 1$, we have $\dim.P = g + m - 1$. If $\sigma = (\varphi, 1): J \times P \longrightarrow \tilde{J}$ is the canonical isogeny, the polarization $\rho: P \longrightarrow \hat{P}$ defined by the condition $\sigma^{*}(\lambda_{\tilde{\theta}}) = 2\lambda_{\theta} \times \rho$. In [M], Mumford proved that if m = 0 (unramified case) or m = 1 there exists a theta divisor Ξ which gives a principal polarization $\lambda_{\Xi}: P \xrightarrow{\sim} \hat{P}$ and such that $\rho = 2\lambda_{\Xi}$. Thus if m = 0 or 1, the Prym variety (P, Ξ) is a pricipally polarized Abelian variety.

Let R_g^m denote the set of isomorphism class of the double coverings $\pi: \tilde{C} \longrightarrow C$ which correspond to some data $(C, \delta, \mathcal{L})$ of type (g,m) in (1.2), A_{g+m-1} the set of isomorphism class of principally polarized Abelian varieties of dim. g+m-1. For m = 0 or 1, we can define the natural map

(1.7) $$\mathbf{Prym}: R_g^m \longrightarrow A_{g+m-1}$$

by $\mathbf{Prym}(\pi: \tilde{C} \longrightarrow C) = (P, \Xi)$, which we call the Prym map. (We will assume that m = 0 or 1 for a moment.) It is well-known that both R_g^m and A_{g+m-1} have natural structures of algebraic varieties and **Prym** is the morphism of algebraic varieties. But since both $\mathcal{R}_g^m$ and $\mathcal{A}_{g+m-1}$ have singularities, when we calculate the differencial of the Prym map, it is more convenient to consider the following moduli functors:

$$R_g^m : S \longrightarrow \left\{\begin{array}{l}\text{isomorphism class of smooth curve } \mathcal{C} \text{ and } \tilde{\mathcal{C}} \text{ over } S \\ \text{and } S\text{-morphism } \Phi:\tilde{\mathcal{C}} \longrightarrow \mathcal{C} \text{ of degree 2 such that} \\ \text{for each } s \in S \text{ the fibre } \Phi_s:\tilde{\mathcal{C}}_s \longrightarrow \mathcal{C}_s \text{ is of type} \\ (g,m) \text{ in } (1.2)\end{array}\right\}$$

and

$$A_{g+m-1} : S \longrightarrow \left\{\begin{array}{l}\text{isomorphism class of principally polarized Abelian} \\ \text{variety of rel.dim.= } g+m-1 \text{ over } S\end{array}\right\}.$$

Then we have a natural morphism of functors

(1.8) $$\mathrm{Pr}: R_g^m \longrightarrow A_{g+m-1} .$$

For any functor F, we denote by $T_{F,p}$ (resp. $T^*_{F,p}$) the tangent space (resp. cotangent space) at $p \in F(\mathbb{C})$ (cf. 7.2,[B]). Then as in 7.3 in [B] at we can show that for a point $(C,\delta,\mathcal{L}) \in R_g^m(\mathbb{C}) = \mathcal{R}_g^m$ and $(P,\Xi) = \mathbf{Prym}\,(C,\delta,\mathcal{L}) \in A_{g+m-1}(\mathbb{C}) = \mathcal{A}_{g+m-1}$ there exist isomorphisms

(1.9) $$T_{R_g^m,(C,\delta,\mathcal{L})} \simeq H^1(C,\Theta_C \otimes \mathcal{O}(-\delta))$$

(1.10) $$T_{A_{g+m-1},(P,\Xi)} \simeq S^2(H^1(C,\mathcal{L}^{-1})) \text{ (symmetric tensor).}$$

Moreover, by Serre duality, we have

(1.11) $$T^*_{R_g^m,(C,\delta,\mathcal{L})} \simeq H^0(C,(\omega_C)^2 \otimes \mathcal{O}(\delta))$$

(1.12) $T^{*}_{A_{g+m-1}, (P,\Xi)} \simeq S^2(H^0(C, \omega_C \otimes \mathscr{L}))$ (symmetric tensor).

(Here we denoted by θ_C and ω_C the tangent sheaf and the dualizing sheaf respectively.) The following proposition was proved by Beauville (cf.Proposition 7.5 in [B]) for the case m = 0 (unramified case) and can be proved for the case m = 1 in the same way.

Proposition 1.1

For m = 0,1 the codifferential of the Prym map Pr: $R^m_g \longrightarrow A_{g+m-1}$ at $(C,\delta,\mathscr{L}) = (\pi:\tilde{C} \longrightarrow C) \in R^m_g(\mathbb{C})$ can be identified with the natural cup product map

(1.13) $\varphi_{C,\pi} : S^2(H^0(C, \omega_C \otimes \mathscr{L})) \longrightarrow H^0(C, (\omega_C)^2 \otimes \mathcal{O}(\delta))$

induced by the isomorphism $(\omega_C \otimes \mathscr{L})^2 \simeq \omega_C{}^2 \otimes \mathcal{O}(\delta)$.

Remark 1.2. Considering the moduli space of Abelian varieties with a polarization (not always principal), we can define the Prym map for $m \geq 2$ and get the same result as in Proposition 1.1.

2. Infinitesimal Torelli theorem for Prym maps

We say that *the infinitesimal Torelli theorem for the Prym map* holds for $\pi:\tilde{C} \longrightarrow C = (C,\delta,\mathscr{L}) \in R^m_g(\mathbb{C}) = \mathcal{R}^m_g$ if *the differential of the map* Pr *in* (1.8) *is injective at* the point. By Proposition 1.1, the infinitesimal Torelli for the Prym map holds for $(C,\delta,\mathscr{L}) \in R^m_g(\mathbb{C})$

if and only if the cup product $\varphi_{C,\pi}$ in (1.13) is surjective. In case m = 0, Beauville proved the following theorem (cf. Proposition 7.8 in [B]).

Theorem 2.1.

If $g \geq 7$, there exists a member $(\pi:\tilde{C} \longrightarrow C)=(C,\delta,\mathcal{L})$ in $R_g^0(\mathbb{C})$ such that $\varphi_{C,\pi}$ in (1.13) is surjective. In particular, the infinitesimal Torelli holds for this member and hence for general members in $R_g^0(\mathbb{C})$ for $g \geq 7$.

In case m = 1, we can prove the following theorem.

Theorem 2.2.

If $g \geq 8$, there exists a member $(\pi:\tilde{C} \longrightarrow C) = (C,\delta,\mathcal{L})$ in $R_g^1(\mathbb{C})$ such that $\varphi_{C,\pi}$ in (1.13) is surjective. In particular, the infinitesimal Torelli holds for this member and hence for general members in $R_g^1(\mathbb{C})$ for $g \geq 8$.

Remark 2.3. Our proof of Theorem 2.2 is based on the proof of Beauville in [B] but we need a little change to complete the proof.

Next let us consider the data $(C,\delta,\mathcal{L}) \in R_g^1$ where C is a hyperelliptic curve of genus g with the morphism $\tau:C \longrightarrow \mathbb{P}^1$ and $\delta = P + Q \in \mathrm{Div}(C)$ and $\mathcal{L} \in \mathrm{Pic}(C)$ such that $\mathcal{L}^2 \simeq \mathcal{O}(\delta)$. The following theorem gives counterexamples of infinitesimal Torelli theorem of Prym map and we shall use this theorem to construct counterexamples of infinitesimal Torelli theorem for higher dimensional varieties.

Theorem 2.4.

Let $\tau: C \longrightarrow \mathbb{P}^1$ be a hyperelliptic curve of genus g and let $\pi: \tilde{C} \longrightarrow C$ be the double covering constructed from the data $(C, \delta, \mathcal{L})$ p R_g^1 as above. Then we have

$$\begin{array}{c}\text{codim.of Im. } \varphi_{C,\pi} \text{ in} \\ H^0(C, \omega_C{}^2 \otimes \mathcal{O}(\delta)) \\ \text{(See (1.13))}\end{array} = \begin{cases} g & \text{if } \dim H^0(\mathcal{L}) = 1 \\ 2 & \text{if } \dim H^0(\mathcal{L}) = 0 \end{cases}$$

Remark 2.5. Let C be a hyperelliptic curve of genus g and consider the unramified double covering $\pi: \tilde{C} \longrightarrow C$ constructed from a data $(C, \delta, \mathcal{L})$ where $\delta = \phi$ and $\mathcal{L}^2 \simeq \mathcal{O}_C$. In this case, we can also prove that the map $\varphi_{C,\pi}$ in (1.13) is *never surjective*.

Remark 2.6. We should mention that even if infinitesimal Torelli theorem for usual period maps does not hold for hyperelliptic curves local Torelli thoerem holds for them, that is, the period map $p: M_g \longrightarrow A_g$ is immersion at the hyperelliptic locus. But in this case, the Prym map **Prym** = $Pr(\mathbb{C})$: $R_g^1 \longrightarrow A_g$ really has a positive dimensional fibre through the point $(C, \delta, \mathcal{L}) \in R_g^1$ in theorem 2.4. The reason why **Prym** has positive dimensional fibres in this case was studied by Dalaljan in [D] in view of the towers of curves.

3. Certain fiber spaces over curves and counterexamples of the infinitesiaml Torelli theorem.

Let X be a smooth projective variety of dimension $n + 1$. The cup product map $H^1(X,\Theta_X)\otimes H^0(X,K_X) \longrightarrow H^1(X,\Omega_X^n)$ induces the infinitesimal period map

$$(3.1) \qquad p_*: H^1(X,\Theta_X) \longrightarrow \mathrm{Hom}\,(H^0(X,K_X),\ H^1(X,\Omega_X^n)).$$

It is wellknown that the map p_* is the edge part of the differential of period map (with respect to the middle cohomology of X) and by Serre duality the dual of (3.1) is given by the cup product

$$(3.2) \qquad H^0(X,K_X)\otimes H^n(X,\Omega_X^1) \longrightarrow H^0(X,\Omega_X^1 \otimes K_X) .$$

* As we mention in introduction, we construct the examples for which the map (3.1) are not injective, or equivalently the map (3.2) are not surjective. First we shall give some technical lemmas.

Let $\tau:X \longrightarrow C$ be a proper surjective morphism from a smooth (n+1)-fold X to a smooth complete curve C. Let S denote the set of points of C such that π: $X - \tau^{-1}(S) \longrightarrow C-S$ is smooth. Set $D = \tau^*(S)$ (pull-back of the divisor S) and let $D = \Sigma\, m_i\, D_i$ be the irreducible decomposition of D. Moreover we assume that the reduced structure $D_{red} = \Sigma\, D_i$ is a normal crossing divisor. The following lemma is easy to prove (see for example [S]).

<u>**Lemma 3.1.**</u> Under the same notation as above, set $\overline{D} = \Sigma\,(m_i-1)D_i$.

Then we have an exact sequence

$$(3.3) \qquad 0 \longrightarrow \tau^*(\Omega_C^1)\otimes \mathcal{O}(\bar{D}) \longrightarrow \Omega_X^1 \longrightarrow \mathcal{F} \longrightarrow 0.$$

Here $\mathcal{F}$ is the sheaf of X satisfying $\mathcal{F}_{|X-\tau^{-1}(S)} \simeq \Omega^1_{X/C|X-\tau^{-1}(S)}$.

Next we have the following lemma.

<u>**Lemma 3.2.**</u> Under the same notation as above, let us assume moreover that:

(i)the fibre space $\tau:X \longrightarrow C$ is a locally trivial fibration,

(ii) the sheaf $R^n\tau_*O(\bar{D})$ (resp. $R^n\tau_*\omega_{X/C}(\bar{D})$) is torsion free.

Then we have a natural injection

$$(3.4) \qquad 0 \longrightarrow \Omega_C^1 \otimes R^n\tau_*O(\bar{D}) \longrightarrow R^n\tau_*\Omega_X^1$$

$$(3.5) \quad (\text{resp. } 0 \longrightarrow (\Omega_C^1) \otimes R^n\tau_*\omega_{X/C}(\bar{D}) \longrightarrow R^n\tau_*(\Omega_X^1 \otimes \omega_{X/C})).$$

Proof. Tensoring $\omega_{X/C}$ to the exact sequence (3.3) and taking the direct images of it, we get the long exact sequence

$$R^{n-1}\tau_*\Omega_X^1 \otimes \omega_{X/C} \longrightarrow R^{n-1}\tau_*\mathcal{F}\otimes \omega_{X/C} \xrightarrow{t_{X/C}} \Omega_C^1 \otimes R^n\tau_*\omega_{X/C}(\bar{D}) \to R^n\tau_*\Omega_X^1 \otimes \omega_{X/C}$$

On $C' = C-S$, by relative duality theorem, this sequence is dual to

$$\tau_*\theta_X \longrightarrow \theta_C \xrightarrow{t^*_{X/C}} R^1\tau_*\theta_{X/C} \longrightarrow R^1\tau_*\theta_X$$

(Note that on $X - \tau^{-1}(S)$, $\mathcal{F} \simeq \Omega^1_{X/C}$ and $\omega_{X/C}(\bar{D}) \simeq \omega_{X/C}$.) But the map $t^*_{X/C}$ is nothing but a Kodaira-Spencer map and therefore zero map by assumption (i). Hence its dual $t_{X/C|C-S}$ is also zero map, so the support of the image of $t_{X/C}$ in $\Omega^1_C \otimes R^n \tau_* \omega_{X/C}$ is contained in S. Then by assumption of the torsion freeness (ii), the image of $t_{X/C}$ is zero and this completes the proof of the second assertion. A proof of the first assertion is similar and left for the reader.

Now let us construct a smooth (n+1)-fold X which have a fibration $\tau : X \longrightarrow C$ over a curve. For nonnegative integers g and m, take a data $(C, \delta, \mathcal{L}) \in R^m_g$ as in (1.2) and construct a corresponding double covering $\pi : \tilde{C} \longrightarrow C$ and put $\delta = \sum_{i=1}^{2m} p_i$. We take a smooth double cover $f : Z \longrightarrow \mathbb{P}^n$ whose branch locus is a smooth hypersurface of degree 2d and assume that $\ell = p_g(Z) = \dim. H^0(Z, K_Z)$ is positive. (Note that this is equiavalent to the condition that $d \geq n+1$.) There exist the involutions $\iota_1 : Z \longrightarrow Z$ and $\iota_2 : \tilde{C} \longrightarrow \tilde{C}$ coresponding to these double coverings and hence $Z/(\iota_1) \simeq \mathbb{P}^n$ and $\tilde{C}/(\iota_2) \simeq C$. Now if we set $\bar{X} = (Z \times \tilde{C})/(\iota_1 \times \iota_2)$ and X = the blowing up of $\bar{X}$ along the image of $\Sigma\ D \times p_i$, we get a smooth (n+1)-folds X and a natural fibration

$$\tau : X \longrightarrow C. \tag{3.6}$$

Then we have the following theorem.

<u>**Theorem 3.3.**</u> Under the same notaion as above, if the cup product map $\varphi_{C,\pi}$ in (1.13) is not surjective, the cup product map (3.2) is not surjective and hence the infinitesimal period map in (3.1) is not injective.

Proof. Considering the Leray spectral sequence of the morphism $\tau: X \longrightarrow C$, we get the following exact sequences:

(3.7)

$$0 \longrightarrow H^1(C, R^{n-1}\tau_*\Omega_X^1) \longrightarrow H^n(X,\Omega_X^1) \longrightarrow H^0(C, R^n\tau_*\Omega_X^1) \longrightarrow 0$$

(3.8)

$$0 \longrightarrow H^1(C, R^{n-1}\tau_*\Omega_X^1 \otimes K_X) \longrightarrow H^n(X,\Omega_X^1 \otimes K_X) \longrightarrow H^0(C, R_*^n\tau\ \Omega_X^1 \otimes K_X) \longrightarrow$$

and an isomorphism

(3.9) $$H^0(X,K_X) \simeq H^0(C,\tau_*K_X)$$

The natural map $R^n\tau_*\Omega_X^1 \otimes \tau_*K_X \longrightarrow R^n\tau_*(\Omega_X^1 \otimes K_X)$ induces the map

(3.10) $$H^0(C, R^n\tau_*\Omega_X^1) \otimes H^0(C,\tau_*K_X) \longrightarrow H^0(C, R_*^n\tau\ \Omega_X^1 \otimes K_X).$$

Moereover by an easy argument, we can show the following isomorphisms;

(3.11) $$\tau_*\omega_{X/C} \simeq \mathcal{L}^{\oplus \ell},$$

(3.12) $$R^n\tau_*\mathcal{O}(\bar{D}) \simeq \mathcal{L}, \quad R^n\tau_*\omega_{X/C}(\bar{D}) \simeq \mathcal{L}^{\otimes 2} \simeq \mathcal{O}(\delta).$$

By definition, $\tau: X \longrightarrow C$ is a local trivial fibration. Thus together with (3.12), by Lemma 3.3, we have the following commutative diagram:

(3.13)

$$\begin{array}{ccc} & 0 & & 0 \\ & \downarrow & & \downarrow \\ H^0(\Omega_C^1 \times \tau_*\omega_{X/C}) \otimes H^0(\Omega_C^1 \times R^n\tau_*O(\bar{D})) & \xrightarrow{\rho_1} & H^0((\Omega_C^1)^2 \otimes R^n\tau_*\omega_{X/C}(\bar{D})) \\ \downarrow \lambda_1 & & \downarrow \lambda_2 \\ H^0(\tau_*K_X) \otimes H^0(R^n\tau_*(\Omega_X^1)) & \xrightarrow{\rho_2} & H^0(R^n\tau_*(\Omega_X^1 \otimes K_X)) \end{array}$$

By exact sequences (3.9) and (3.10), if the cup product ρ_2 in (3.12) is not surjective, the map (3.2) is not surective. In (3.13), since λ_1, λ_2 is injective, if ρ_1 is not surjective, ρ_2 is not surjective. But by (3.11) and (3.12), the map ρ_2 is nothing but the cup product

$$H^0(\Omega_C^1 \otimes \mathcal{L})^{\oplus \ell} \otimes H^0(\Omega_C^1 \otimes \mathcal{L}) \longrightarrow H^0((\Omega_C^1)^2 \otimes \mathcal{L}^2).$$

But the last is nothing but ℓ-times repetition of $\varphi_{C,\pi}$ in (1.13) and this completes the proof.

Corollary 3.4. Let C be a hyperelliptic curve of genus g and consider a data $(C,\delta,\mathcal{L})$ in R_g^m for m = 0 or 1. For all double cover $f:Z \longrightarrow \mathbb{P}^n$, let $\tau:X \longrightarrow C$ be the corresponding fibre space as in (3.6). Then the infinitesimal period map in (3.1) is not injective for X.

Remark 3.5. The infinitesimal period map (3.1) is a part of the differential of period map. So the failure of the injectivity of (3.1) does not imply the failure of the injectivity of period map if $n \geq 2$. In case $n = 1$, the map in (3.1) coincides with the whole of differential, so the examples in corollary 3.4 give counterexample of the infinitesimal Torelli theorem for surfaces of general type and elliptic surfaces.

Example 3.6. We conclude this note by giving two examples.

(i) Let $f:Z \longrightarrow \mathbb{P}^2$ be a K3 surface of degree 2 and C a hyperelliptic curve of genus 2. We take δ as the empty set. Then the constructed X has $p_g(X) = 1$ and the fibration $\tau:X \longrightarrow C$ is smooth.

(ii) Let $f:E \longrightarrow \mathbb{P}^1$ be an elliptic curve and C another elliptic curve and take δ as distinct two points $P + R \in C$. Then the corresponding $\tau:X \longrightarrow C$ is an elliptic surface with $p_g(X) = q(X) = 1$. In [S], the author found that the period map has one dimensional fibre for this surface. Moreover this surface appears as a main component of a degeneration of Kunev surface (a surface of general type with $p_g = c_1^2 = 1$ and special automorphisms). Such a phenomenan was recently studied by Usui in [U]. By using the correspondence of curves, we can show that there exists another elliptic curve C' such that the Hodge structure of X above is isomorphic to that of the Kummer surface Km(E×C') of E×C'.

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