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THE PARAMETER SPACE OF A PARALLEL BLOWER SYSTEM

Hideaki Okazaki*, Hideo Nakano** and Takehiko Kawase*
* Dept. of Mech. Eng., Waseda University
3-4-1 Okubo, Shinjuku, 160 Japan
** Dept. of Mech. Eng., Sagami Institute of Technology
1-1-25 Nishikaigan, Tsujido, Kanagawa, 251 Japan

ABSTRACT

Global system stability and instability conditions in terms of the parameters of a parallel blower system are discussed and fundamental aspects of manifold nonlinear phenomena observed in the system are studied through computer simulation.

1. INTRODUCTION

There sometimes occur large oscillations of the mass flow in the flow passage of a certain type of turbomachine while one operates the machine in the flow region less than the designed flow point giving the maximal efficiency of the machine. This is often called "surge", which is a global system instability meaning a self-excited oscillation caused by the positive slope of the turbomachine under some system parameters such as fluid inertances (i.e. duct lengths) and capacitances (i.e. plenum volume). The above was first found by Stodola[1)], Fujii[2)], and Emmons et al.[3)] investigated the surge from dynamical viewpoint. Since then many turbomachine models, which play the crucial role in the system behavior, have been proposed among which the parallel compressor model and the multi-stage compressor model are notable.

The global system stability and instability of turbomachine systems having only one machine (surveyed by Greitzer[4)]), have thoroughly been studied especially by Greitzer but those of the systems having series or parallel machines are seldom studied. Emmons[5] investigated the static instability of a parallel compressor system and Fujii[2] studied the properties of the equilibrium points for a parallel pumping system. However the global system stability and instability conditions (in particular dynamic) of the parallel turbomachine systems are not found.

The positive slope of the turbomachine essentially comes from the flow separation in the blade surface usually called stall and hence the unsteady internal flow along the blade passages should be more thoroughly investigated for the machine design purposes. However, it is also very much important to clarify the global system behavior for the system design view point. Here we focus our attention on the global system stability and instability of a parallel blower system and investigate the parameter space of the above system.

The present paper consists of four sections. In section 2 we consider a very simple parallel blower system but should notice that the system is encountered in many process applications such as gas handling systems in the chemical industries or exhaust gas systems for power plants etc. We derive the dynamic equations of the present system and confirm that the dynamics is described by Brayton-Moser equation under some coordinate transformation.In section 3 complete stability and LaSalle's theorem (which is useful for proving the above stability) are introduced. Next we have three theorems which are needed for providing the complete stability conditions in terms of the parameters of the parallel blower system and have completely stable region on the system parameters. In section 4 conditions on the existence of attractors, which are unstable conditions, are introduced.These conditions are given using the stable manifold theorem and boundedness of the system. Next we have the unstable region on the system parameters. In section 5 fundamental aspects of manifold nonlinear phenomena observed in the system are studied through computer simulation. The dynamical behavior is classified into the static equilibrium states the periodic attractor, the saddle type closed orbit and the non-periodic attractor.

2. PARALLEL BLOWER MODEL

We consider the very simple parallel blower system illustrated in Fig.1. As the ratio of the wave length of the oscillation to the representative dimension or duct length is much greater than unity. The present system is modelled by a lumped parameter model shown in Fig.2. In the above model we assume the following conditions.

(1) The thermodynamic process of the fluid in the plenum is adiabatic.

(2) The ratio of the fluid inertia of the throttle duct to that of each blower duct is much smaller than unity and the inertia of the fluid in the throttle duct is also neglected.

(3) While surge occurs in the system, the blower characteristics does not change severely and is practically similar to that of the static characteristics (without surge).

Under these assumptions each blower is modelled as an actuator disk which has the function denoting the blower characteristics and the system dynamics is described below.

The equation of the fluid motion in each blower duct:

$$\frac{dq_k}{dt} = B_k(f_k(q_k) - p) \qquad (k=1,2) \tag{1}$$

The equation of the continuity in the plenum:

$$\frac{dp}{dt} = F\left(\sum_{k=1}^{2} q_k - g(p)\right) \qquad F \equiv 1/\left(\sum_{k=1}^{2} B_k\right) \tag{2}$$

where

$$q_k \equiv m_k/\rho U\sqrt{A_1A_2}\ , \quad p \equiv \rho U^2/2\ , \quad t \equiv \omega\tau\ , \quad \tau\text{: time}$$

$$B_k \equiv \frac{U}{2\omega L_k}\sqrt{A_1A_2/A_k^2}\ , \quad k = 1,2 \qquad \omega \equiv c_0\sqrt{\left(\sum_{k=1}^{2} A_k / L_k\right) / V_P}$$

m_k:mass flow rate, p:plenum pressure, V_p:plenum volume,
L_k:blower duct length, A_k:cross sectional area, ρ:density,
c_0:sound velocity, U:peripheral velocity of the rotor,
f_k:blower characteristics, g:throttle characteristics.

The blower characteristics $f_k(k=1,2)$ and the throttle characteristics g have the properties described below.

$f_k : R \to R, C^1$ class, $(k=1,2)$

(i) There exists a unique $q_{0k}(>0)$ such that $f_k(q_{0k})=0$.

(ii) $\forall q_k(\neq q_{0k}), (q_k-q_{0k})f_k(q_k)<0$.

(iii) There exists $M_k(>0)$ such that $\forall q_k$, $|q_{0k}-q_k|>M_k$ implies $df_k/dq_k<0$

(iv) $q_k f_k(q_k) \to -\infty$ as $|q_k| \to \infty$.

$g: R \to R$, differentiable

$g(p)=(1/\sqrt{G})\,\mathrm{sgn}(v)\sqrt{|v|}$

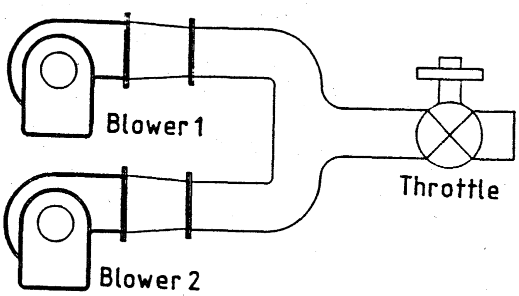


Fig.1 Parallel blower system

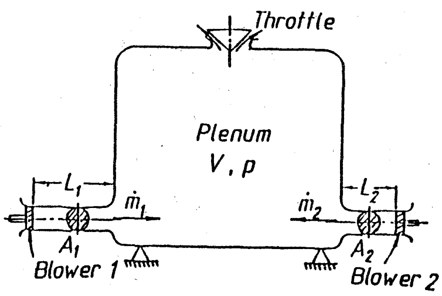


Fig.2 Parallel blower model

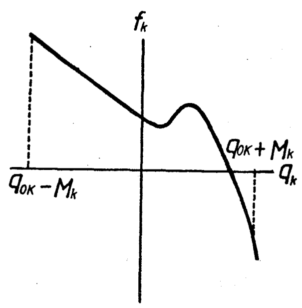


Fig.3 Blower characteristics

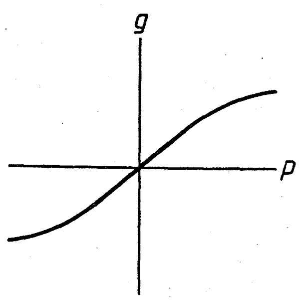


Fig.4 Throttle characteristics

Generally speaking, blower characteristics satisfies the properties (i),(ii),(iii) and (iv). We assume that f_k is C^1 class function in order to make the discussion easy. The throttle characteristics is described by a differentiable function and it is usually described by a square root function of the pressure difference across the throttle.

2.1 The equivalent bistable system

The active element of the parallel blower system of our concern is the blower f_k (k=1,2). We introduce the coordinate transformations so that the blower is equivalently represented as the parallel connection of an ideal flow source and a nonlinear resistor:

translation and reflexion

$$\begin{pmatrix} q_k \\ p \end{pmatrix} \longmapsto \begin{pmatrix} i_k \\ v \end{pmatrix} \quad : \begin{cases} i_k = -q_k + i_{0k} \\ v = p \end{cases} \quad (k=1,2) \qquad (3)$$

We write q_{0k} as i_{0k} in the new coordinate system.

Writing the dynamics(1),(2) in terms of eq.(3), we have the following bistable system:

$$L_k \frac{di_k}{dt} = v - h_k(i_k) \quad (k=1,2) \ , \qquad (4)$$

$$C \frac{dv}{dt} = I_0 - g(v) - \sum_{k=1}^{2} i_k \ , \qquad (5)$$

where $h_k(i_k) \equiv f_k(i_{0k} - i_k)$, $L_k \equiv 1/B_k$, $C \equiv \sum_{k=1}^{2} B_k$ and $I_0 = \sum_{k=1}^{2} i_{0k}$

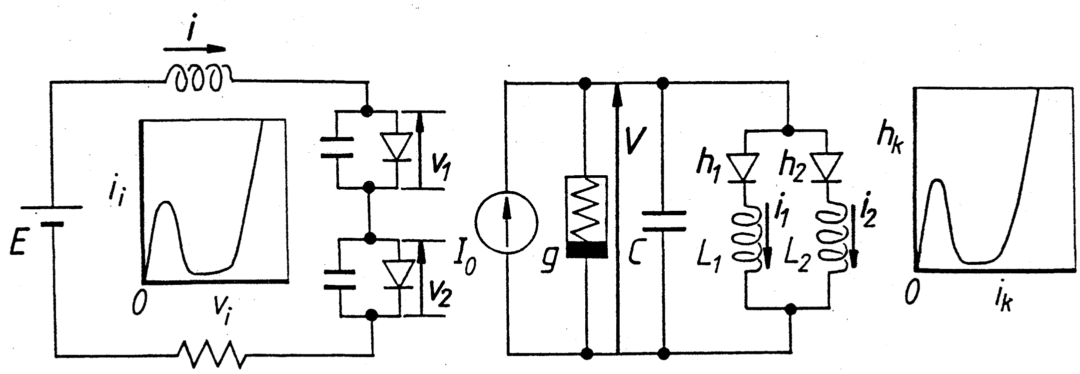


Fig.5 Circuit models

The functions h_k (k=1,2) and g have the properties described below.

(a) h_k is c^1 class . $\forall i_k$ ($\neq 0$) , $i_k h_k(i_k) > 0$ and $h_k(0)=0$ iff $i_k=0$.

(b) $\exists M_k > 0$ such that , $|i_k| \geqq M_k$ implies $dh_k/di_k > 0$.

(c) $i_k h_k(i_k) \to \infty$ as $|i_k| \to \infty$.

(d) $v(\neq 0)$, $vg(v) > 0$ and $g(0)=0$ holds iff $v=0$.

(e) $dg/dv > 0$.

(f) $vg(v) \to \infty$ as $v \to \infty$.

It is easily seen that the bistable system (4), (5) is dual under some correspondence to the representative flip-flop circuit studied by Moser [8] and its dynamics is hence analogous to that of the above circuit.(See Fig.5)

correspondence of variables(See Fig.5)

equivalent circuit of the parallel blower system		flip-flop circuit
voltage across the capacitor	⟷	current through the inductor
current through the nonlinear resister	⟷	voltage across the nonlinear register

The above correspondence results in the interchange of (1) the parallel connection with the series connection, (2) the capacitor with the inductor, and (3) the current source with the voltage source.

2.2 Brayton-Moser equation

The essential parts of the following discussion was given in ref. (8).So we follow the Moser's line. We first define the following coordinate transformation .

$$I_k(i,v) = (v - h_k(i_k))/L_k \ , \ V(i,v) = (I_0 - g(v) - \sum_{k=1}^{2} i_k)/C \qquad (6)$$

Then we consider the next differential form.

$$F = CV(i,v)dv - \sum_{k=1}^{2} I_k(i,v)di_k \qquad (7)$$

Since R^3 is connected and

$$\frac{\partial}{\partial i_k}(CV) = -\frac{\partial}{\partial v}(L_k I_k) = -1 \qquad (8)$$

,

F is the total differentiation of some function.Therefore if F is integrated along an arbitrary closed curve c,

$$\int_c F = 0$$

is found.The function $P:R^3 \to R$ defined as

$$P \equiv \int_{(0,0,0)}^{(i_1,i_2,v)} F \quad . \qquad (9)$$

P is called a scalar potential (mixed potential[10]).In fact P is given as

$$P = I_0 v - v\sum_{k=1}^{2} i_k + \sum_{k=1}^{2}\int_0^{i_k} h_k di_k - \int_0^v g dv \ . \qquad (10)$$

In terms of the mixed potential P,the bistable system(4),(5) is described by the following equations.

$$L_k \frac{di_k}{dt} = -\frac{\partial P}{\partial i_k} \quad (k=1,2) \ (4)', \quad C\frac{dv}{dt} = \frac{\partial P}{\partial v} \qquad (5)'$$

The set of the equations(4)'and(5)' are called the Brayton-Moser equation[11].

3. COMPLETE STABLE REGION ON THE PARAMETERS OF PARALLEL SYSTEM

3.1 Complete stability

It is never completely satisfactory (without finding the region size of asymptotic stability in the phase space) to know only that the system is asymptotically stable.In engineering field, particularly ,it is very much important to find conditions on the parameters of the system which guarantee that the equilibrium state is globally and asymptotically stable.Global and asymptotical stability means that no matter what initial conditions are given to the differential equation of the system, the resulting solution approaches one of the equilibrium states as t goes to infinitely. If the system is in this condition, the equilibrium states are called to have the complete stability. One of the most useful results for proving the complete stability of any system described by a differential equation is that by LaSalle[12),13)]. We consider a system of autonomous differential equations

$$\frac{dx}{dt} = f(x) \quad , x \in R^n \tag{11}$$

where the function f is C^1 class and satisfies $f(0)=0$. Let $V(x):R^3 \longrightarrow R$ be C^1 class and $E = [x \mid dV/dt = 0]$ and let M be the largest invariant set contained in E.

Theorem (LaSalle)

1) $^\forall x(\neq 0)$, $V(x) > 0$

2) $^\forall x$, $dV/dt \leqq 0$

If the scalar function V(x) satisfies 1) and 2) described above, then every solution of (11) bounded for $t \geqq 0$ approaches M as $t \rightarrow \infty$.

The problem of establishing complete stability can be broken up into two parts: The first part is to construct a Liapunov function V(x) satisfying 1) and 2) such that the only solution remaining in E for all t is the trivial one (x=0). The second is to show that all the solutions are bounded for $t \geqq 0$. It is sometimes possible to do both parts by constructing a single Liapunov function. However it is often more convenient to discuss the two problems separately and the second part is of much importance.

3.2 Global dynamical behavior

Now we first define the energy function to discuss the boundedness of the solution.

Let $W: R^3 \longrightarrow R$ be the energy function defined below.

$$W \equiv \frac{1}{2}\sum_{k=1}^{2} L_k i_k{}^2 + \frac{1}{2} Cv^2 \tag{12}$$

By differentiating W along the solution of the bistable system(4),(5),one finds

$$-\frac{dW}{dt} = -\sum_{k=1}^{2} i_k L_k \frac{di_k}{dt} - vC\frac{dv}{dt} = -\left(\sum_{k=1}^{2} i_k L_k I_k + vCV\right) \qquad (13)$$

Thus the time derivative of W vanishes at the origin of the new coordinate, that is , at the equilibrium points of the system.

From eq.(13) we have

$$-dW/dt = -\left(\sum_{k=1}^{2} i_k L_k I_k + vCV\right) = -\sum_{k=1}^{2} i_k(v-h_k) + v\left(I_0-g(v)-\sum_{k=1}^{2} i_k\right)$$
$$= \sum_{k=1}^{2} h_k i_k + v(g(v)-I_0) \qquad (14)$$

Now we assume , besides the properties of h_k stated previously, that for $|i_k| > M_k$, $h_k i_k > GI_0^3$.

We restrict our discussion to sufficiently large value of W :

$$W > W_0 = Cv_0^2 + \sum_{k=1}^{2} L_k M_k^2/2 = C(GI_0^2)^2/2 + \sum_{k=1}^{2} L_k M_k^2/2$$

where $v_0 = g^{-1}(I_0)$.

From the expression of W , we easily see that the above implies that at least one of the three coordinates v and i_k (k=1,2) is very large , and so we can assume

$$v > v_0 = GI_0^2 \text{ or } |i_k| > M_k \ (k=1,2) .$$

If the first holds,then,$-dW/dt > 0$ is obvious . If the first is violated,$|i_k| > M_k$ holds for at least one k. Then $i_k h_k > GI_0^3$. This results in

$$-dW/dt > GI_0^3 + v(g(v) - I_0) .$$

Now if $v < 0$, $-dW/dt > vg(v) \geqq 0$, and if $v > 0$,

$$GI_0^3 - vI_0 + vg(v) \geqq GI_0^3 - v_0 I_0 + vg(v) = vg(v) \geqq 0.$$

The above proves that

$$-dW/dt > 0 \text{ for } W > W_0. \qquad (15)$$

Thus from the condition (15), the solutions of the system not only remain bounded but also penetrate into the region D defined below.

$$D = \left\{ (i,v)\in R^3 \ \middle|\ 0 \leqq |v| < GI_0^2,\ |i_k| \leqq M_k \ , \ k=1,2 \right\}$$

Now we get theorem 1.

Theorem 1

If $|i_k| > M_k$, $i_k h_k(i_k) > GI_0^3$ then all the solutions of the system(4),(5) are bounded.

Now we can restrict our discussion to the dynamics within the region D. We can easily show that the equilibrium points of the system(4),(5) are contained in the above domain D. As stated below, the function f_k (k=1,2) and g(p) **satisfy** the properties described previously, so all the equilibrium points must lie in the first quadrant in the (q,p) space.

Let (q^*,p^*) be the equilibrium points. Then we have

$$q_k^* < q_{0k}, \ (k=1,2)$$
$$p^* < G(\sum_{k=1}^{2} q_k^*)^2 < G(\sum_{k=1}^{2} q_{0k})^2 = GI_0^2 \qquad (16)$$

since the equilibrium states must also satisfy the throttle characteristics. By using the above relations in terms of (i,v) coordinates, we have

$$i^*_k < i_{0k}, \ (k=1,2)$$
$$v^* < G(i_{0k})^2 = GI_0^2 \qquad (17)$$

where (i^*,v^*) is the (i,v) expression of (q^*,p^*). Thus we easily see that the equilibrium points are contained in D by taking M_k such that $M_k > i_{0k}$.

Now we define another scalar function S as

$$S = \sum_{k=1}^{2} L_k I_k^2/2 + CV^2/2 + \lambda P \qquad (18)$$

where P is the mixed potential and λ a positive constant .

By differentiating S with respect to v and i_k, we easily have

$$\frac{\partial S}{\partial v} = - C\left(\frac{1}{C}g' - \lambda\right)\frac{dv}{dt} + \sum_{k=1}^{2}\frac{di_k}{dt} ,$$

$$\frac{\partial S}{\partial i_k} = - \frac{dv}{dt} - \sum_{k=1}^{2} L_k\left(\frac{1}{L_k}h_k' + \lambda\right)\frac{di_k}{dt} .$$

So we have

Theorem 2.

The extreme points of S coincide with the equilibrium points, which are included in the domain D, of the system(4),(5).

Using the expression of eq.(18) and noting $g(v) = (1/\sqrt{G})\,\mathrm{sgn}(v)\sqrt{|v|}$, we have the another expression for P as

$$P = -(3G/4)\,\mathrm{sgn}(v)g(v)(CV)^2 + K(i,v) \qquad (19)$$

where $K(i,v) = (3G/4)\,\mathrm{sgn}(v)g(v)(CV + (2/3)g(v))^2 + \sum_{k=1}^{2}\int_0^{i_k} h_k\,di_k$

Putting above relation of S to eq.(18) and noting the functional relation described below

$$g(v)g'(v) = \mathrm{sgn}(v)/2G,$$

we finally have

$$S = \frac{1}{2Cg'(v)}\left(g'(v) - \frac{3}{4}\lambda C \right)(CV)^2 + \frac{1}{2}\sum_{k=1}^{2} L_k I_k^{\,2} + \lambda K(i,v), \qquad (20)$$

On the other hand, easy calculation gives the expression of the time derivative of S,

$$-\frac{dS}{dt} = -\left(\sum_{k=1}^{2} \frac{\partial S}{\partial i_k}\frac{di_k}{dt} + \frac{\partial S}{\partial v}\frac{dv}{dt} \right) = \sum_{k=1}^{2} (h_k' + \lambda L_k) I_k^{\,2} + (g' - \lambda C)V^2. \qquad (21)$$

We have already shown that all the solutions of the system(4),(5) not only remain bounded but also penetrate into the domain D defined previously. So in the following we restrict our discussion to the dynamics in D. Let λ be so chosen as

$$-\frac{1}{L_k}\frac{dh_k}{di_k} < \lambda < \frac{1}{C} g'(v_0) \qquad (22)$$

in the domain D, assuming

$$\frac{1}{L_k}\frac{dh_k}{di_k} + \frac{1}{C} g'(v_0) > 0 ,$$

the function S is bounded from below. Since

$$S = \frac{1}{2Cg'(v)} \left(g'(v) - \frac{3}{4}\lambda C \right)(CV)^2 + \frac{1}{2}\sum_{k=1}^{2} L_k I_k{}^2 + \lambda K(i,v)$$

$$\geq \lambda K(i,v) \geq \sum_{k=1}^{2} \int_0^{i_k} h_k di_k \geq 0 \; . \qquad (23)$$

On the other hand, eq.(21) shows that the time derivative vanishes only at the equilibrium points and satisfies the inequality

$$-dS/dt \geq 0 \; .$$

The equality holds only at equilibrium states $I_k(k=1,2)= 0$, $V = 0$. From the results so far obtained, we have

Theorem 3.

Under the condition

$$\frac{1}{L_k}\frac{dh_k}{di_k} + \frac{1}{C} g'(v_0) > 0 \quad (k=1,2)$$

all the solutions of (4),(5) tend to the steady state solutions.

4. UNSTABLE REGION ON THE PARAMETERS OF THE PARALLEL BLOWER SYSTEM

By theorem 1 the bistable system(4),(5) is bounded for all $t \geq 0$, whatever values of the parameters L_k and C take. Then if all the equilibrium points are hyperbolic and unstable, the system has attractors which are different from the equilibrium points [14].

The conditions on local instability of each equilibrium point of a system of differential equations are given by stable manifold theorem for an equilibrium point.

Stable manifold theorem for an equilibrium point [15] is summarized below: Suppose that

$$\frac{dx}{dt} = f(x) \; , \; f\colon C^1 \text{ class, } x \in R^n \qquad (24)$$

has a hyperbolic equilibrium point $\bar{x}$. Then there exist local stable and unstable manifolds $W^u{}_{loc}(\bar{x})$ and $W^s{}_{loc}(\bar{x})$ of the same dimension n_s and n_u respectively as those of the eigenspaces E^s and E^u of the linearized system of eq.(24):

$$\frac{d\xi}{dt} = Df(\bar{x})\xi \; , \; \xi \in R^n \quad (x = \bar{x} + \xi, \; |\xi| << 1) \; ,$$

where $Df=(\partial f_i/\partial x_j)$ is the Jacobian matrix of the first partial derivatives of f and tangent to E^s and E^u at $\bar{x}$. $W^s_{loc}(\bar{x})$ and $W^u_{loc}(\bar{x})$ are as smooth as f is.

The above theorem provides the following two lemmas.

Lemma 1

If one of the real parts of the eigenvalues of $Df(\bar{x})$ is positive, the equilibrium point $\bar{x}$ is unstable.

Lemma 2

If $\bar{x}$ is the hyperbolic equilibrium point of the above equation and trace $Df(\bar{x})>0$, then $\bar{x}$ is unstable.

The boundedness for all $t>0$ of the bistable system(4),(5) is already shown. If all the equilibrium points of the system are unstable, then attractors (periodic or non-periodic attractors) exist in the system. We apply lemma 2 to our system and have the unstable region on the parameters of the system.

Unstable region on the parameters of the system

$$\bigcap_{(i_1^*,i_2^*,v^*)\in E} \{(L_1,L_2,C)\,|\,-(dg/dv)|_{v^*}/C-\sum_{k=1}^{2}(dh_k/di_k)|_{i_k^*}/L_k>0\} \qquad (25)$$

E: the set of the equilibrium points of the system(4),(5)

5. FUNDAMENTAL ASPECTS OF THE DYNAMICS IN THE SYSTEM

To understand more intuitively,the following discussion is made in the (q,p) space in stead of the (i,v) space.We use the symbols x , y and z instead of q_1, q_2 and p respectively and the blower characteristics $f_k(k=1,2)$ are shown in the appendix. For the present system there exists the set of the system parameters

B_k (k=1,2) and G which makes all the equilibrium points (EPs) of the system unstable . The equilibrium point and determined by

$$f_1(x)-z=0 \ , \ f_2(y)-z=0$$
$$x + y - g(z) = 0$$

As shown in Fig.6 , we have these EPs when we chose the parameter G=23.5 . we designate them as E_k(k=1,2,3) (see Fig.6). Varying the parameters B_1 and B_2 we have the phase space portraits corresponding to the characteristic exponent(CE) of each EP as shown in Fig.7. Since the functions f_1 and f_2 have the same characteristics in the present case, the portrait maps of E_1 and E_3 are in mirror symmetry with regard to the line $B_1 = B_2$. So the map of E_3 is omitted.

The completely stable and unstable regions are illustrated in Fig.8, However the unstable region is not calculated using eq.(25) defining the unstable region but is determined by the portrait map of each EP in Fig.7. We have the parameter space corresponding to typical nonlinear behavior in Fig.9 and Fig.10 with the symbol expressing the stability or instability of each EP. Fig.10 shows the bifurcation process of attractors around E_1 (E_3). The attractors around E_1 (E_3) in A and D are non-periodic orbits because their Liapunov dimensions are non integer between 2 and 3 [16]. The attractors around E_1 (E_3) in B and C are three and six period orbits respectively. All the attractors around E_2 in from A to E are one period orbits. The dynamical behavior is classified into the static equilibrium states and periodic or non-periodic attractors.

In Fig.10 varying B_1 and B_2 form A to E, we find that attractors around E_1 and E_3 disappear in E. In order to see more closely this disappearance, we calculate the saddle type closed orbit which show the rough location of the separaterices and illustrate the relation among the closed orbits and attractors in Fig.11. The closed orbit approaches to the attractor around E_1 (E_3). From the above result it can be inferred that the collision between the attractor around E_1 (E_3) and the saddle type close orbit (the separaterix) causes the attractor disappearance mentioned above.

CONCLUSIONS

1) It is shown that the present parallel blower system is dual to the standard bistable circuit studied by Moser.

2) Sufficient conditions of the complete stability and the instability conditions on the parameters of a parallel blower system are provided analytically.

3) In the unstable region on the parameters of the present system, the periodic or non-periodic attractor and saddle type closed orbit are observed by numerical simulation.

APPENDIX

$$f_i(r) = a_{ik}(r - b_{ik})^2 + c_{ik} \qquad (i=1,2 , k=1,2,3)$$

$a_{i1}=180.0$, $b_{i1}=0.05$, $c_{i1}=1.0$, ($r<b_{i1}$, $i=1,2$)
$a_{i2}=6.198$, $b_{i2}=0.05$, $c_{i2}=1.0$, ($b_{i1}\leqq r\leqq b_{i3}$, $i=1,2$)
$a_{i3}=-47.34$, $b_{i3}=0.27$, $c_{i3}=1.3$, ($b_{i3}<r$, $i=1,2$)

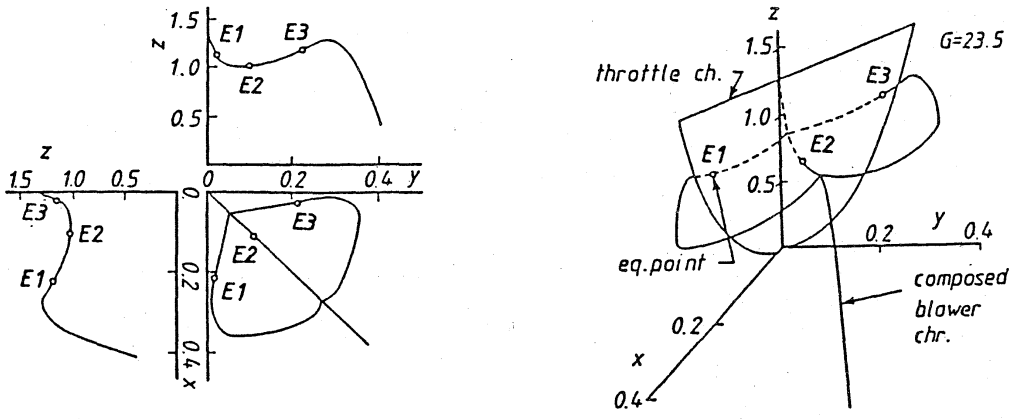


Fig.6 Composed blower characteristics and EPs

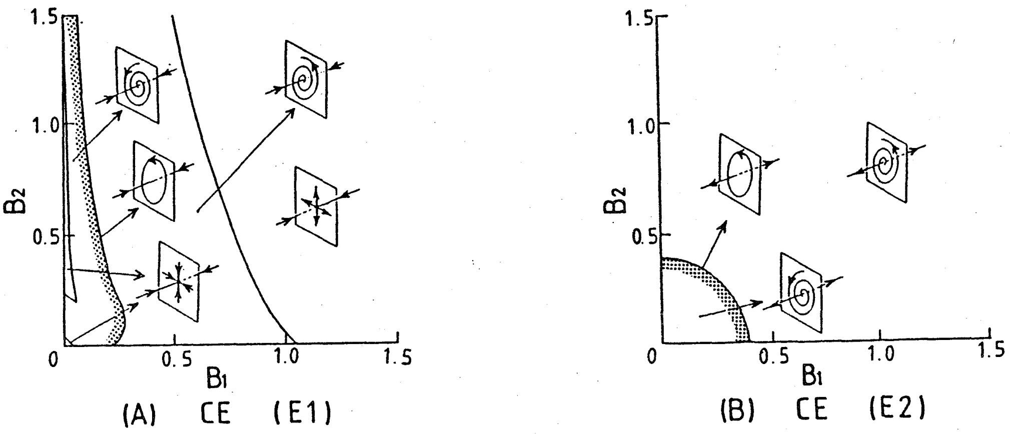


Fig.7 Properties of each EP

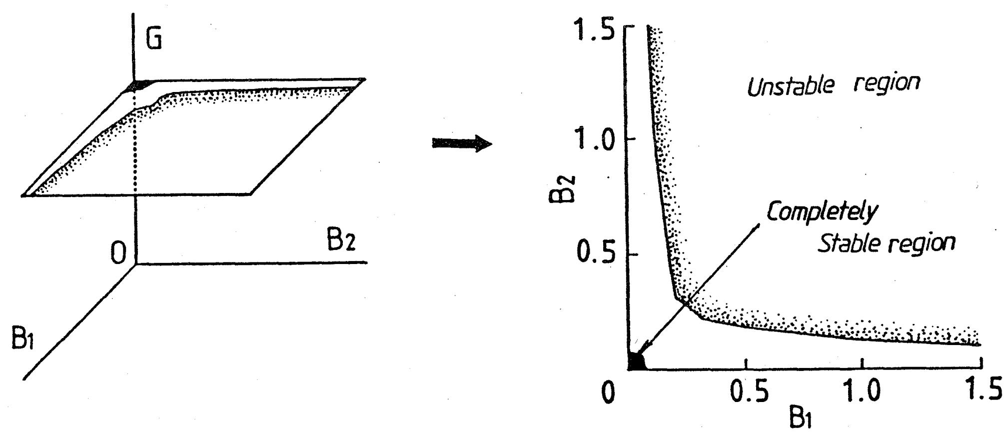


Fig.8 Completely stable and unstable region

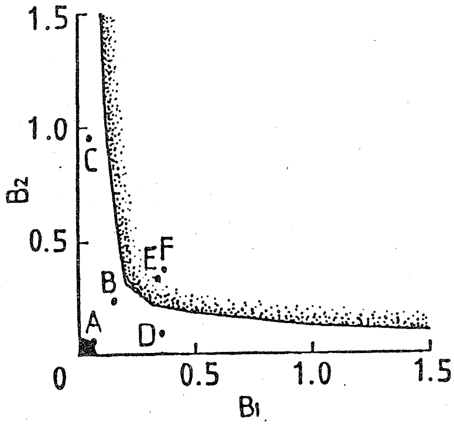
Symbol

(S U S)

E1 stable — E2 unstable — E3 stable

Type of the equilibrium point

(1) S U S
(2) U U S or S U U
(3) U U U



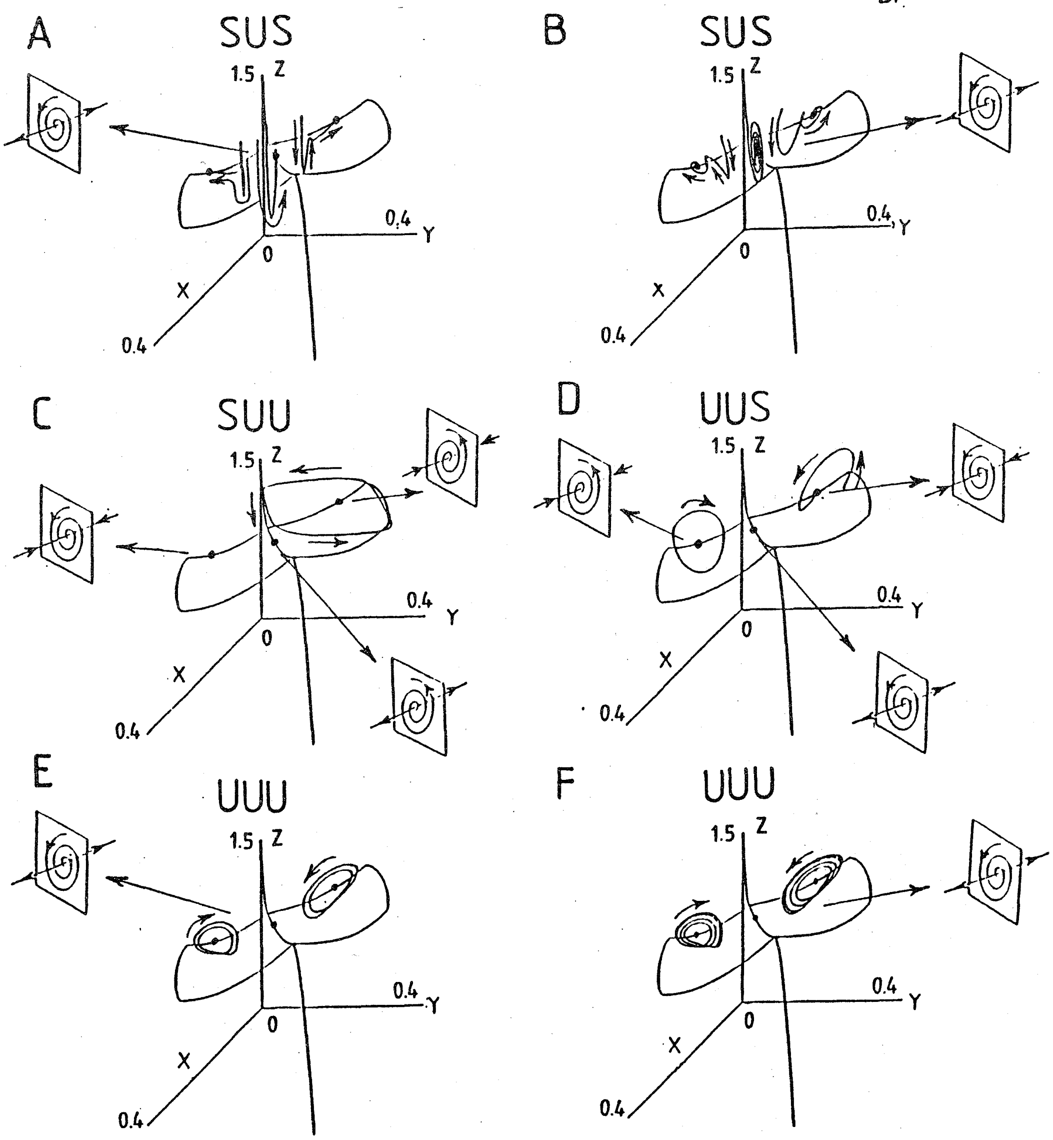


Fig.9 Typical nonlinear behavior in phase space

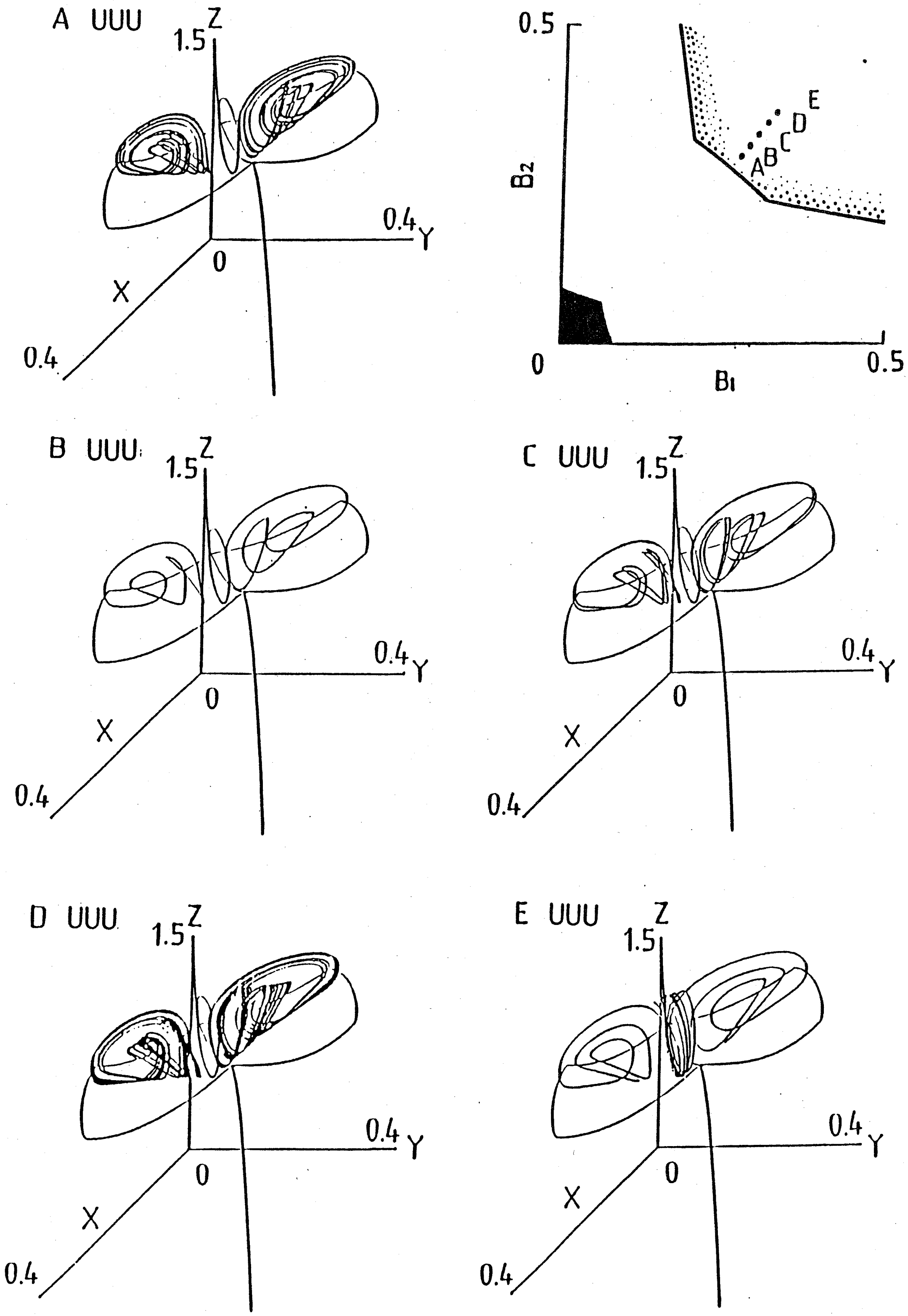


Fig.10 Bifurcation process

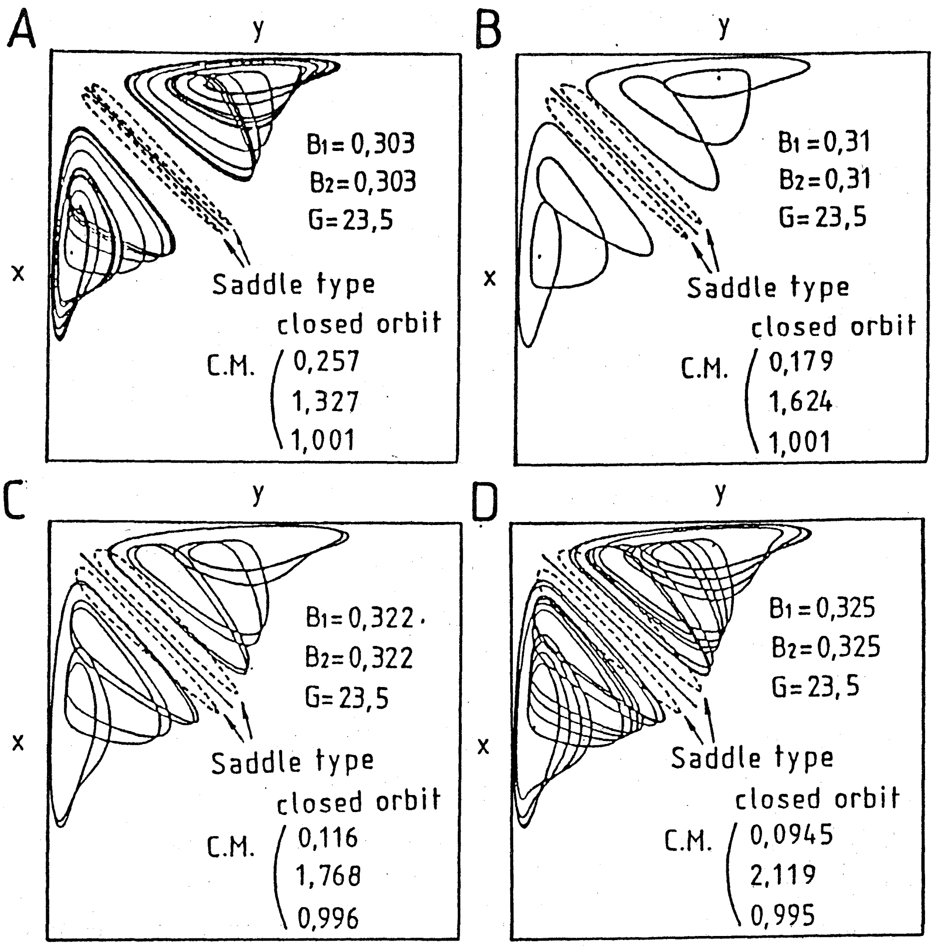


Fig.11 Saddle type closed orbits and attractors

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