

Eberlein-compact 空間とその仲間

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Eberlein-compact 空間の研究の流れから Talagrand-compact, Gul'ko-compact, Corson-compact 等の空間が出てきた。これらの空間の特徴付けを整理しながら未解決の問題を紹介する。

以下空間は Tychonoff とする。空間 X 上の実数値連続関数全体を $C(X)$ で表す。 $C(X)$ 上に各点収束位相を導入した空間を $C_p(X)$ で表す。

定義。空間 X をコンパクト空間とする。

空間 X があるバナッハ空間 E の弱コンパクト部分空間と位相同型となる時、この空間 X を Eberlein-compact (略して、E-C) と呼ぶ [AL]。

空間 X に対して、 $C_p(X)$ が \mathcal{K} -analytic となる時、即ち $C_p(X)$ が \mathcal{K}_{cs} -空間の連続像となる時、空間 X を Talagrand-compact (T-C) と呼ぶ。

空間 X に対して、 $C_p(X)$ が Lindelof Σ -空間となる時、空間 X を Gul'ko-compact (G-C) と呼ぶ。

空間 X が Σ -product of real lines の部分空間と位相同型となる時、空間 X を Corson-compact (C-C) と呼ぶ。

$$E - C \begin{matrix} \xrightarrow{[T1]} \\ \not\Leftarrow [N] \end{matrix} T - C \begin{matrix} \xrightarrow{[T2]} \\ \not\Leftarrow [T3] \end{matrix} G - C \begin{matrix} \xrightarrow{[Gu]} \\ \not\Leftarrow [N] \end{matrix} C - C$$

特徴付けを表にすると以下のようなになる。

『？』の記号が付いている所は未解決である。

X compact space	E-C	T-C	G-C	C-C
$C_p(X)$ P:irrationals K:compact P' \subset P	cont.image of a sep.metric space X compact space. [A]	cont.image of a closed subspace $F \subset P \times K$.	cont.image of a closed subspace $F \subset P' \times K$	cont.image of a closed subspace $F \subset L(T)^\omega$ [P]
$X \subset \Sigma(T)$ $\Sigma(T)$: Σ -product of real lines	$X \subset \Sigma(T)$: Σ -product [AL]	$X \subset \Sigma(T)$ such that (a) [S]	$X \subset \Sigma(T)$ such that (b) [S]	$X \subset \Sigma(T)$: Σ -product
T_0 -separating coll. of open F_σ -sets	σ -point-finite [R]	(a') [S]	weakly σ -point finite [S]	point-countable [MR]
X^2	hereditarily σ -metacompact [G1]	?	hereditarily weakly σ -metacompact ? [G3]	hereditarily metaLindelof [G1]
$X^2 - \Delta$	σ -metacompact [G1]	?	weakly σ -metacompact ? [G3]	metaLindelof [G1]
game $G(\Delta, X^2)$	0 has Markov winning strategy [G2]	?	?	W-set [G1]

- ①. The space $L(T)$ is a union of a discrete space T and a point at infinity ∞ whose nbds are of the form $\{\infty\} \cup (T - C)$, where C is countable.
- ②. (a), (a') and (b) ; see Sokolov [S] Theorem 9, Theorem 8.

- ③. A collection \mathcal{U} of subsets of X is weakly σ -point-finite if $\mathcal{U} = \bigcup \{ \mathcal{U}_n : n \in \omega \}$ so that, for each $x \in X$, $\mathcal{U} = \bigcup \{ \mathcal{U}_n : \text{ord}(x, \mathcal{U}_n) < \omega \}$.
- ④. A space X is weakly σ -metacompact if every open cover has a weakly σ -point-finite open refinement.

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