数理解析研究所講究録
第 659 巻 1988 年 1-24

Notes on cohomological dimension modulo p - nonmetrizable version

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1. Introduction. In 1978, R. D. Edwards [4] announced the strong result as follws;

Edwards-Walsh Theorem. *Every compact metric space* X *of cohomological dimension* $\dim_{\mathbb{Z}} X \leq n$ *(integer coefficients) is the image of a cell-like mapping* $f: Z \longrightarrow X$ *of a compact metric space* Z *with* $\dim Z \leq n$.

At that time, while we knew the famous **Alexandorff problem**: *is there an infinte-dimensional compact metric space whose cohomological dimension is finite ?*, our geometric topologists had (even now have) much interest in the **CE-problem**: *does every cell-like mapping preserve the (covering) dimension?** However the result and the classical Vietoris-Begle theorem showed the equivalence of both interesting problems, and therefore gave a big motivation for attacking the problems. A proof of the result was given by J. J. Walsh [10]. Then he used an interesrting characterization of cohomological dimension. Namely,

The first characterization of cohomological dimension (Edwards-Walsh). *Let* X *be a comapct metric space and let* $(X_i, p_{i,i+1})$ *be an inverse sequence of compact polyhedra whose limit is* X. *Then* $\dim_{\mathbb{Z}} X \leq n$, $n \geq 1$, *if and only if for every integer* i *and* $\varepsilon > 0$, *there is an integer* $j > i$ *and a triangulation* T_i *of* X_i *such that for every triangulation* T_j *of* X_j, *there is a mapping* $h: |T_j^{(n+1)}| \longrightarrow |T_i^{(n)}|$ *such that* $d(h, p_{i,j}||T_j^{(n+1)}|) \leq \varepsilon$.

Recently, L. R. Rubin and P.J. Schapiro [9] generalized the Edwards-Walsh Theorem to the case of metrizable spaces X and Z. Moreover, S. Mardešić and L. R.

*) Recently A. N. Dranishnikov announced that he had obtained a negative answer of the Alexandorff problem. However we do not know the detail.

Rubin [8] succeeded to genenarize their theorem to the case of compact Hausdorff spaces X and Z. In the latter generalization, they used the following Mardešić's characetrization of cohomological dimension [6], which is an an useful version of Edwards-Walsh's one. Moreover, S. Mardešić [6] showed the factorization theorem of cohomological dimension $\dim_Z$.

The second charactrization of cohomological dimension (Mardešić). *A compact Hausdorff space* X *has cohomological dimension* $\dim_Z X \leq n$, $n \geq 1$, *if and only if for every polyhedron* P, *every mapping* $f: X \longrightarrow P$, *and every* $\varepsilon > 0$, *there is a polyhedron* Q *and there are mappings* $g: X \longrightarrow Q$, $p: Q \longrightarrow P$ *satisfying the following two conditions:*

(1) $d(pg, f) \leq \varepsilon$, *and*

(2) *for every triangulation* M *of* Q, *there is a mapping* $p': |M^{(n+1)}| \longrightarrow P$ *such that*

$$d(p', p||M^{(n+1)}|) \leq \varepsilon \text{ and } \dim \operatorname{Im} p' \leq n.$$

Here, if a mapping p satisfies the condition (2), p is called (n,ε)-*approximable*, and p' is called an (n,ε)-*approximation* of p.

The factorization theorem on cohomological dimension (Mardešić). *Let* X *be a compact metric space with* $\dim_Z X \leq n$, $n \geq 1$. *Let* Y *be a compact metric space and let* $f: X \longrightarrow Y$ *be a mapping. Then there is a compact metric space* Z *with* $\dim_Z Z \leq n$ *and mappings* $g: X \longrightarrow Z$ *and* $h: Z \longrightarrow Y$ *such that* $f = hg$.

On the other hand, modifying their theorem, A. Dranishnikov [1] has characterized cohomological dimension with the coefficient group Z_p from the view point of Edwards-Walsh.

The first characterization of cohomological dimension modulo p (Dranishnikov). *Let* X *be a compact metric space and let* $(X_i, p_{i,i+1})$ *be an inverse sequence of compact polyhedra whose limit is* X. *Then* $\dim_{Z_p} X \leq n$, $n \geq$

2, *if and only if for every integer* i *and* $\varepsilon > 0$, *there is an integer* $j > i$ *and a triangulation* T_i *of* X_i *such that for every triangulation* T_j *of* X_j, *there is a mapping* $h: |T_j^{(n)}| \longrightarrow |T_i^{(n)}|$ *such that* $d(h, p_{i,j}||T_j^{(n)}|) \leq \varepsilon$, *and* $[h|\partial\sigma] \in p\cdot\pi_n(|T_i^{(n)}|)$ *for every (n+1)-simplex* σ *of* T_j.

Using the characterization, Dranishnikov showed two interesitng theorems. One is the Edwards-Walsh-type theorem modulo p, and another is the negative answer of the Alexandorff problem modulo p. Namely,

The first Dranishnikov theorem . *Every compact metric space* X *with* $\dim_{Z_p} X \leq n$, $n \geq 2$, *is the image of a mapping* $f: Z \longrightarrow X$ *of a compact metric space* Z *of* $\dim Z \leq n$ *whose fibers are acyclic modulo* Z_p.

The second Dranishnikov Theorem. *For each prime number* p *and each* n $= 2,3,4,\ldots,\infty$, *there exists a compapct metric space* X(n,p) *such that* $\dim X(n,p) = n$ *and* $\dim_{Z_p} X(n,p) \leq 2$.

Motivated by the above development of cohomological dimension theory, in this note, we will show the second characterization of cohomological dimension modulo p. And as applicatiopns of the characterization, we will obtain the Mardesic-type factorization theorem modulo p and the generalization of the first Dranishnikov theorem for compact Hausdorff spaces. Our proof is essentially due to [1], [6] and [8].

In this note we mean the definition of cohomological dimension as follows: the *cohomological dimension of a space* X *with a coefficient group* G *is less than* n, denoted by $\dim_G X \leq n$, provided that every mapping $f: A \longrightarrow K(G,n)$ of a closed subset A of X into an Eilenberg-MacLane complex K(G,n) of type (G,n) admits a continuous extension over X (c.f. [5]).

2. Approximate (inverse) systems. We will use the new notion, aprroximate (inverse) systems and their limits, instead of usual inverse systems and inverse limits. They were introduced by S. Mardešić and L. R. Rubin [7], and took an important role in [8]. Now we quote their basic definitions [7].

Definition 1. An *approximate (inverse) system* of metric compacta $\mathbf{X}$ = $(X_a,\varepsilon_a,p_{aa'},A)$ consists of the following: A directed ordered set $(A,<)$ with no maximal element; for each $a\in A$, a compact metric space X_a with a metric $d = d_a$ and a real number $\varepsilon_a > 0$; for each pair $a \leq a'$ from A, a mapping $p_{aa'}: X_{a'} \longrightarrow X_a$, satisfying the following conditions:

(A1) $d(p_{a_1a_2}p_{a_2a_3}, p_{a_1a_3}) \leq \varepsilon_{a_1}$, $a_1 \leq a_2 \leq a_3$; $p_{aa} = id$,

(A2) for every $a \in A$ and every $\eta > 0$, there exists $a' \geq a$ such that

$$d(p_{aa_1}p_{a_1a_2}, p_{aa_2}) \leq \eta,$$

(A3) for every $a\in A$ and $\eta > 0$, there exists $a' \geq a$ such that for every $a'' \geq a'$ and every pair of points x,x' of $X_{a''}$, if $d(x,x') \leq \varepsilon_{a''}$, then

$$d(p_{aa''}(x),p_{aa''}(x')) \leq \eta.$$

If $\pi_a: \Pi X_a \longrightarrow X_a$, $a\in A$, denote the projections, we define the limit space X = lim $\mathbf{X}$ and the natural projections $p_a: X \longrightarrow X_a$ as follows:

Definitioin 2. A point $\mathbf{x} = (x_a)\in \Pi X_a$ belongs to X = lim $\mathbf{X}$ provided that for every $a\in A$,

$$x_a = \lim\ p_{aa_1}(x_{a_1}).$$

The natural projection $p_a = \pi_a|X: X \longrightarrow X_a$.

Next we quote results from [7] and [8] needed in this note. The proofs may be found in [7] and [8].

Proposition 1. *Let* $\mathbf{X} = (X_a,\varepsilon_a,p_{aa'},A)$ *be an approximate system. Then we have the following properties:*

(1) *if evevry* X_a *is nonempty, then* X = lim $\mathbf{X}$ *is a nonempty compact Hausdorff space,*

(2) *for each* $a \in A$, $\lim d(p_a, p_{aa_1}p_{a_1}) = 0$, *where* $d(f,g) =$ *sup* $\{d(f(x),g(x)) | x \in X\}$,

(3) *for each open covering* $\mathcal{U}$ *of* $X = \lim \mathbf{X}$, *there is* $a \in A$ *such that for any* $a_1 \geq a$, *there exists an open covering* $\mathcal{V}$ *of* X_{a_1} *for which* $(p_{a_1})^{-1}(\mathcal{V})$ *refines* $\mathcal{U}$,

(3)' *if* $\dim X_a \leq n$ *for all* $a \in A$, *then* $\dim X \leq n$,

(4) *for every* $\varepsilon > 0$, *every compact* ANR P, *and every mapping* $h: X \longrightarrow P$, *there is* $a \in A$ *such that for any* $a' \geq a$, *there is a mapping* $f: X_{a'} \longrightarrow P$ *which satisfies* $d(fp_{a'}, h) \leq 2\varepsilon$.

Propositon 2. *Let* $\mathbf{X} = (X_a, \varepsilon_a, p_{aa'}, A)$ *be an approximate system. If for every* $a_1 \in A$ *and every* ANR P *and every mapping* $h: X_{a_1} \longrightarrow P$, *there is* $a_1' \geq a_1$ *such that for every* $a_2 \geq a_1'$, *there is* $a_2' \geq a_2$ *such that for any* $a_3 \geq a_2'$,

$$hp_{a_1a_2}p_{a_2a_3} \simeq 0,$$

then every map from $X = \lim \mathbf{X}$ *to* P *is inessential.*

Namely, under the above condintion, the set $[X,P]$ *of all homotopy classes of mappings from* X *to* P *is trivial.*

Proposition 3. *Let* X *be a compact Hausdorff space with* $\dim_G X \leq n \geq 1$. *Then there exists an approximate system* $\mathbf{X} = (X_a, \varepsilon_a, p_{aa'}, A)$ *with* $\lim \mathbf{X} = X$ *such that*

(i) X_a *is a polyhedron with a metric* $d = d_a \leq 1$,

(ii) $\dim X_a \geq n$,

(iii) $p_{aa'}: X_{a'} \longrightarrow X_a$ *is a surjective* PL-*mapping,*

(iv) *card*$(A) \leq \omega(X)$.

3. The second characterization of cohomological dimension modulo p. In this section we consider a fixed but arbitrary prime number p. First, we will intruduce the Z_p-version of (n,ε)-approximation.

Definition 3. A mapping $\psi: Q \longrightarrow P$ is called *(p,n,ε)-approximable,* where $n \geq 1$, if there exists a triangulation L of P such that for any triangulation M of Q, there is a mapping $\psi': |M^{(n)}| \longrightarrow |L^{(n)}|$ satisfying the following conditions:

(1) $d(\psi', \psi||M^{(n)}|) \leq \varepsilon$,

(2) for every $(n+1)$-simplex σ of M, $[\psi'||\partial\sigma|] \in p\cdot\pi_n(|L^{(n)}|)$.

Theorem 1. *A compact Hausdorff space* X has *cohomological dimension modulo* p, $\dim_{Z_p} X \leq n$, $n > 1$, *if and onnly if for every polyhedron* P, *every map* f: $X \longrightarrow P$, *and every* $\varepsilon > 0$, *there is a polyhedron* Q *and there are mappings* ϕ: $X \longrightarrow Q$, ψ: $Q \longrightarrow P$ *such that*

(3) $d(f, \psi\phi) \leq \varepsilon$,

(4) ψ *is (p,n,ε)-approximable.*

Proof of Sufficiency. Let a closed subset A of X and $h: A \longrightarrow K(Z_p,n)$. Now we have a finite subcomplex $K \subseteq K(Z_p,n)$ containing $h(A)$ and a contractible polyhedron P such that $K \subseteq P$. Then there are a continuous extension $f: X \longrightarrow P$ of h, and a closed polyhedral neighborhood N of K in P and a retraction $r: N \longrightarrow K$. Moreover, take $\delta > 0$ such that

(5) $O_\delta(K) \subseteq N$, where $O_\delta(K)$ is the δ-neighborhood around K,

(6) any two δ-near mappings into N are homotopic in N.

Then by the assumption of Theorem 1, there is a polyhedron Q and there are mappings $\phi: X \longrightarrow Q$, $\psi: Q \longrightarrow P$ such that

(7) $d(f, \psi\phi) \leq \delta/3$,

(8) ψ is $(p,n,\delta/3)$-approximable.

By (7) and (5), we have a closed polyhedral neighborhood G of $\phi(A)$ in Q such that

(9) $\psi(G) \subseteq O_{\delta/2}(f(A)) \subseteq N$.

Let take a triangulation M of Q such that G is the carrier of a subcomplex M_1 of

M. Then by (8), we have a triangulation L of P and a mapping $\psi': |M^{(n)}| \longrightarrow |L^{(n)}|$ satisfying the following conditions:

(10) $d(\psi', \psi||M^{(n)}|) \leq \delta/3$,

(11) for any (n+1)-simplex σ of M, $[\psi'|\partial\sigma] \in p\cdot\pi_n(|L^{(n)}|)$.

Then by (10) and the definition of δ, $\psi'(|M_1 \cap M^{(n)}|) \subseteq O_{\delta/2}(\psi(|M_1{}^{(n)}|) \subseteq N$. Hence by (6) and (10),

(12) $\psi|G \cap |M^{(n)}| \simeq \psi'|G \cap |M^{(n)}|$.

Since $\psi|G \cap |M^{(n)}|$ has an extension $\psi|G: G \longrightarrow N$, by (12), $\psi'|G \cap |M^{(n)}|$ also has an extension $\psi^*: G \cup |M^{(n)}| \longrightarrow N \cup |L^{(n)}| \subseteq P$ such that

(13) $\psi^*|G \simeq \psi|G$ in N.

If we consider the retraction r as a mapping into $K(Z_p,n)$, then we have an extension $r^*: N \cup |L^{(n)}| \longrightarrow K(Z_p,n)$ of r. Then for any (n+1)-simplex σ of M, by (11), $[r^*\psi^*|\partial\sigma] = 0$ in $\pi_n(K(Z_p,n))$. It follws that we have an extension $\psi^{**}: G \cup |M^{(n+1)}| \longrightarrow K(Z_p,n)$ of $r^*\psi^*$. Therefore $r^*\psi^*$ admits an extension $\theta: Q \longrightarrow K(Z_p,n)$. Then we define the mapping $h': X \longrightarrow K(Z_p,n)$ by $\theta\phi$. By (13), (7) and (6), we have that

(14) $h'|A \simeq h|A$.

Therefore h admits an extension over X. It completes the proof.

In oder to show the necessity, we introduce the *Edwards'* n*-modification of a complex* L *modulo* p, where n > 1. Let L be a finite complex, and we write

$L = L^{(n)} \cup \sigma_1 \cup \sigma_2 \cup .... \cup \sigma_s$, where $n+1 \leq \dim \sigma_1 \leq \dim \sigma_2 \leq .... \leq \dim \sigma_s$.

For any simplex σ with $\dim \sigma \geq n+1$, r_σ is the rank of $\pi_n(\sigma^{(n)}) \cong H_n(\sigma^{(n)})$, and we define

$$K(\sigma) = K(\bigoplus_{i=1}^{r_\sigma} Z_p, n).$$

Then by the induction on $s \geq 1$, we can define a CW-complex

$\hat{L} = L^{(n)} \cup K(\sigma_1) \cup K(\sigma_2) \cup \dots \cup K(\sigma_s)$

satisfying the following conditions:

(a) $\hat{L}^{(n)} = L^{(n)}$ and $L^{(n)} \cap K(\sigma_i) = \sigma_i^{(n)}$, $i = 1,2,\dots,s$,

(b) $\hat{L}^{(n+1)}$ is obtained from $L^{(n)}$ by attaching to each (n+1)-simplex of L by a mapping of degree p,

(c) $K(\sigma_i) \cap K(\sigma_j) = \begin{cases} \sigma_i \cap \sigma_j & \text{if } \dim(\sigma_i \cap \sigma_j) \leq n, \\ K(\sigma_i \cap \sigma_j) & \text{if } \dim(\sigma_i \cap \sigma_j) \geq n+1. \end{cases}$

Remark 1. If dim σ = n+1, the construction of $K(\sigma)$ starts from attaching an (n+1)-cell on $\partial\sigma \simeq S^n$ by a mapping of degree p. Namely,

$K(\sigma)^{(n)} = \partial\sigma$, and $K(\sigma)^{(n+1)} = \partial\sigma \cup_\phi B^{n+1}$, where $\phi: S^n \longrightarrow \partial\sigma$ is a mapping of degree p.

If dim $\sigma \geq n+1$, by the condition (c), then $K(\sigma) = K_1(\sigma) \cup K_2(\sigma) \cup \dots$ such that

(d) $K_1(\sigma) = \bigcup_{\tau < \sigma} K(\tau)$, where the union is taken over all proper faces τ of σ,

(e) for $i = 2,3,\dots$, $K_i(\sigma)$ is obtained from $K_{i-1}(\sigma)$ by attaching to $K_{i-1}(\sigma)^{(n+i-1)}$ a collection of (n+i)-cells killing the (n+i-1)-th homotopy group.

Namely,

$K_i(\sigma)^{(n+i-1)} = K_{i-1}(\sigma)^{(n+i-1)}$ and $\pi_{n+i-1}(K_i(\sigma)) = 0$.

Hence

$K(\sigma)^{(n+i)} = K_i(\sigma)^{(n+i)}$, $i = 1,2,\dots$.

Remark 2. Since $K(\sigma) = K(\bigoplus_{i=1}^{r_\sigma} Z_p, n) \simeq \prod_{i=1}^{r_\sigma} K(Z_p, n)$, every mapping $f: A \longrightarrow K(\sigma)$ of a closed subset of a compact Hausdorff space with $\dim_{Z_p} X \leq n$ admits an extension $f^*: X \longrightarrow K(\sigma)$.

Proof of Necessity. Assume that $\dim_{Z_p} X \leq n$. Let take a polyhedron P, a mapping $f: X \longrightarrow P$ and $\varepsilon > 0$. Then choose a triangulation L of P such that

(15) $mesh(L) \leq \varepsilon/4$,

and let consider the Edwards' n-modification $\hat{L}$ of L modulo p. Define an open covering $\hat{\mathcal{U}}$ of $\hat{L}$ consisting of all sets of the form

(16) $\hat{U}(\sigma) = \hat{L} - (\bigcup_{\sigma\cap\tau=\emptyset} \hat{L}(\tau))$, where $\hat{L}(\tau) = \tau$ if $\dim \tau \leq n$.

Then we note that

(17) $\hat{L}(\sigma) \subseteq \hat{U}(\sigma)$ for each $\sigma \in L$.

Claim 1. *There is a mapping* $\hat{f}: X \longrightarrow |\hat{L}|$ *such that*

(18) $\hat{f}|f^{-1}(|L^{(n)}|) = f|f^{-1}(|L^{(n)}|)$,

(19) $\hat{f}(f^{-1}(\sigma)) \subseteq K(\sigma)$ *for every simplex* σ *of* L *with* $\dim \sigma \geq n+1$.

Proof of Claim 1. Write L as the form

$L = L^{(n)} \cup \sigma_1 \cup \sigma_2 \cup \dots \cup \sigma_s$, where $n+1 \leq \dim \sigma_1 \leq \dots \leq \dim \sigma_s$.

First, we define the mapping $f_0 = f|f^{-1}(|L^{(n)}|): f^{-1}(|L^{(n)}|) \longrightarrow |L^{(n)}| = |\hat{L}^{(n)}| \subseteq |\hat{L}|$.

Since $\dim_{Z_p} f^{-1}(\sigma_1) \leq \dim_{Z_p} X \leq n$, the mapping $f_0|f^{-1}(\partial\sigma_1): f^{-1}(\partial\sigma_1) \longrightarrow \partial\sigma_1 \subseteq K(\sigma_1)$ has an extension $f_{\sigma 1}: f^{-1}(\sigma_1) \longrightarrow K(\sigma_1)$. Hence we can define the mapping $f_1: f^{-1}(|L^{(n)}|) \cup f^{-1}(\sigma_1) \longrightarrow |L^{(n)}| \cup K(\sigma_1) \subseteq |\hat{L}|$ by

(20) $f_1|f^{-1}(|L^{(n)}|) = f_0$ and $f_1|f^{-1}(\sigma_1) = f_{\sigma 1}$.

Then clearly $f_1(f^{-1}(\sigma_1)) \subseteq K(\sigma_1)$. For each $i \geq 2$, since $\partial\sigma_i \subseteq |L^{(n)}| \cup \sigma_1 \cup \dots \cup \sigma_{i-1}$, we can similarly obtain the mapping $f_i: f^{-1}(|L^{(n)}|) \cup f^{-1}(\sigma_1) \cup \dots \cup f^{-1}(\sigma_i) \longrightarrow |L^{(n)}| \cup K(\sigma_1) \cup \dots \cup K(\sigma_i)$ such that

(21) $f_i|f^{-1}(|L^{(n)}|) \cup f^{-1}(\sigma_1) \cup \dots \cup f^{-1}(\sigma_{i-1}) = f_{i-1}$ and $f_i(f^{-1}(\sigma_i)) \subseteq K(\sigma_i)$.

Therefore the mapping f_s is the desired one.

Next we consider the mapping $h = f \times \hat{f}: X \longrightarrow |L| \times |\hat{L}|$ and the two projectons $\psi: |L| \times |\hat{L}| \longrightarrow |L|$ and $\hat{\psi}: |L| \times |\hat{L}| \longrightarrow |\hat{L}|$. Then h(X) is contained in a finite subcomplex of $|L| \times |\hat{L}|$, which can be embedded in a polyhedron. Hence the mappings ψ and $\hat{\psi}$ can be extended to a closed polyhedral neighborhood K of h(X). Now we consider h as a mapping $h: X \longrightarrow K$ and ψ, $\hat{\psi}$ as mappings $\psi: K \longrightarrow |L|$, $\hat{\psi}: K \longrightarrow |\hat{L}|$. Clearly

(21) $\psi h = f$ and $\hat{\psi} h = \hat{f}$.

We choose $\eta > 0$, which is less than the Lebesgues numbers of both $\psi^{-1}(\mathcal{U})$ and $\hat{\psi}^{-1}(\hat{\mathcal{U}})$, where $\mathcal{U}$ is the open star covering of L. Moreover, we may assume that

(22) if $d(z,z') \leq \eta$, $z,z' \in K$, then $d(\psi(z),\psi(z')) \leq \varepsilon$.

Then by the same way as in [6], we can have a mapping $\phi: X \longrightarrow K$ such that

(23) $d(\phi,h) \leq \eta$,

(24) $\phi(X)$ is a subpolyhedron Q of K.

Hence as we consider ϕ as a surjective mapping $\phi: X \longrightarrow Q$ and ψ as a mapping $\psi: Q \longrightarrow |L|$, by (22) and (23),

(25) $d(\psi\phi,f) \leq \varepsilon$.

Therefore it suffices to show the following.

Claim 2. *The mapping* $\psi: Q \longrightarrow |L|$ *is* (p,n,ε)-*approximable.*

Proof of Claim 2. Take a triangulation M of Q. First, we show the existance of a mapping $\theta: |M^{(n+1)}| \longrightarrow |\hat{L}^{(n+1)}|$ such that

(26) $\theta|\hat{\psi}^{-1}(|\hat{L}^{(n+1)}|) = \hat{\psi}|\hat{\psi}^{-1}(|\hat{L}^{(n+1)}|)$

(27) $\theta(\hat{\psi}^{-1}(K(\sigma)) \subseteq K(\sigma)^{(n+1)}$ for every simplex σ of L with $\dim \sigma \geq n+1$.

Since $|M^{(n+1)}|$ is compact, there exists a finite collection of cells $\{\tau_1,\tau_2,\ldots,\tau_k\}$, $\dim \tau_1 \geq \dim \tau_2 \geq \ldots \geq \dim \tau_k \geq n+2$, such that

(28) $\psi(|M^{(n+1)}|) \cap \tau_i \neq \emptyset$ for each $i = 1,\ldots,k$,

(29) $\psi(|M^{(n+1)}|) \subseteq |\hat{L}^{(n+1)}| \cup \tau_1 \cup \ldots \cup \tau_k$.

We take a small ball $B \subseteq \tau_1 - \partial\tau_1$ such that $\dim B = \dim \tau_1$, and consider the mapping $\hat{\psi}|\hat{\psi}^{-1}(\partial B)\cap|M^{(n+1)}|: \psi^{-1}(\partial B)\cap|M^{(n+1)}| \longrightarrow \partial B$. Since $\dim \psi^{-1}(B)\cap|M^{(n+1)}| \leq n+1 < \dim B$, there exists an extension $\psi_1: \hat{\psi}^{-1}(B)\cap|M^{(n+1)}| \longrightarrow \partial B$ of $\hat{\psi}|\hat{\psi}^{-1}(\partial B)\cap|M^{(n+1)}|$. Considering a retraction from $|\hat{L}^{(n+1)}|\cup(\tau_1 - \mathrm{Int}B)\cup\tau_2\cup\ldots\cup\tau_k$ onto $|L^{(n+1)}|\cup\tau_2\cup\ldots\cup\tau_k$, we have a mapping $\theta_1: |M^{(n+1)}| \longrightarrow |\hat{L}^{(n+1)}|\cup\tau_2\cup\ldots\cup\tau_k$ such that

(30) $\theta_1||\hat{L}^{(n+1)}|\cup\tau_2\cup\ldots\cup\tau_k = \hat{\psi}||\hat{L}^{(n+1)}|\cup\tau_2\cup\ldots\cup\tau_k$,

(31) $\theta_1(\hat{\psi}^{-1}(\tau_1))\subseteq\partial\tau_1$.

By the inductive step, we obtain the desired mapping $\theta_k = \theta$. Moreover taking a siutable subdivisions, we may assume that θ is simplicial.

Now we choose a point $z_\tau\in\tau - [\partial\tau\cup\theta(|M^{(n)}|)]$ for each $(n+1)$-simplex τ of L, and take the retraction $r: |L^{(n+1)}| - \{z_\tau \mid \tau\in L \text{ and } \dim\tau = n+1\} \longrightarrow |\hat{L}^{(n)}| = |L^{(n)}|$ given by the radial projection on each $\tau - \{z_\tau\}$. Then we define a mapping $\psi': |M^{(n)}| \longrightarrow |L^{(n)}|$ by $r\theta||M^{(n)}|$. Then by the same way as in [6], we have that

(1) $d(\psi', \psi||M^{(n)}|) \leq \varepsilon$.

Let σ be a $(n+1)$-simplex of M. If $\theta(\sigma)\subseteq|L^{(n)}|$, then $\psi'|\partial\sigma = \theta|\partial\sigma \simeq 0$ in $|L^{(n)}|$. Otherwise, there exists finite $(n+1)$-balls $B_1,\ldots,B_m$ in $\sigma - \partial\sigma$ such that

(32) $\bigcup_{i=1}^{m}\mathrm{Int}B_i \supseteq \theta^{-1}(\{z_\tau \mid \tau\in\hat{L}^{(n+1)}$ and $\dim\tau = n+1\})\cap\sigma$,

(33) $\theta(B_i)\subseteq\tau - \partial\tau$ for some $\tau\in L^{(n+1)}$ such that $\dim\tau = n+1$.

Then we have that

(34) $[\psi'|\partial\sigma] = [r\theta|\partial B_1] + \ldots + [r\theta|\partial B_k]$ in $\pi_n(|L^{(n)}|)$.

Since each $r\theta|\partial B_i$ can be factorized through the attaching $(n+1)$-cell of $L^{(n+1)}$ coontaining $\theta(B_i) - \{z_\tau\}$, $[r\theta|\partial B_i]\in p\cdot\pi_n(|L^{(n)}|)$ for each $i = 1,2,\ldots,m$. Hence by (34), $[\psi'|\partial\sigma]\in p\cdot\pi_n(|L^{(n)})$. That is, in the both cases, the condition (2) is satisfied.

Therefore ψ is (p,n,ε)-approximable. It completes the proof of Claim 2.

Using Theorem 1 instead of [6], Theorem 1, we similarly have two corollaries which corresponds to Corollaries 1 and 2 in [6].

Corollary 1. *Let* $\mathbf{Q} = (Q_i, q_{ii+1})$ *be an inverse sequence of polyhedra with the inverse limit* $Z = \lim \mathbf{Q}$ *and projections* $q_i: Z \longrightarrow Q_i$. *Let* $\varepsilon_i > 0$ *be numbers such that*

(*) *if for* $w, w' \in Q_j$, $d(w,w') \leq \varepsilon_j$, then $d(p_{ij}(w), p_{ij}(w')) \leq 1/2^{j-i}$, $j > i$,

and let each of the mapping p_{ii+1} *be* (p,n,ε_i)-*approximable, where* $n \geq 2$. *Then* $\dim_{Z_p} Z \leq n$.

Corollary 2. *Let* X *be a comapct Hausdorff space with* $\dim_{Z_p} X \leq n$, $n \geq 2$. *Let* $P_1, \ldots, P_k$ *be polyhedra,* $f_1: X \longrightarrow P_1, \ldots, f_k: X \longrightarrow P_k$, *and* $\varepsilon_1 > 0, \ldots, \varepsilon_k > 0$ *be arbitrary positive numbers. Then there is a polyhedron* Q *and mappings* $f: X \longrightarrow Q$, $\psi_1: Q \longrightarrow P_1, \ldots, \psi_k: Q \longrightarrow P_k$ *such that*

(i) $f(X) = Q$,

(ii) $d(\psi_i \phi, f_i) \leq \varepsilon_i$,

(iii) ψ_i *is* (p,n,ε_i)-*approximable.*

Next we have a criterion of cohomological dimension modulo p, $\dim_{Z_p} X \leq n$, when X is the limit of an approximate system of polyhedra. A proof can be given by the similar way as in [8], if we apply Therom 1 intested of Theorem 1 in [6]. Hence we omit the proof here.

Theoem 2. *Let* $\mathbf{X} = (X_a, \varepsilon_a, p_{aa'}, A)$ *be an approximate inverse system of polyhedra with limit* $X = \lim \mathbf{X}$. *Then* $\dim_{Z_p} X \leq n$ *if and only if for every* $a \in A$ *and every* $\varepsilon > 0$, *there is* $a' \geq a$ *such that for every* $a'' \geq a'$, *the mapping* $p_{aa''}$ *is* (p,n,ε)-*approximable.*

4. The factorization theorem on cohomological dimension modulo p. We state our main theorem in this section.

Theorem 3. *Let* X *be a compact Hausdorff space with* $\dim_{Z_p} X \leq n$. *Let* Y *be a compact metric space and let* $f: X \longrightarrow Y$ *be a mapping. Then there exists a compact metric space* Z *with* $\dim_{Z_p} \leq n$ *and there exist mappings* $g: X \longrightarrow Z$, $h: Z \longrightarrow Y$ *such that* $f = hg$.

Proof. Let take an inverse sequence $\mathbf{Y} = (Y_i, r_{ii+1})$ of polyhefra with limit Y and projections $r_i: Y \longrightarrow Y_i$. Then we note that there is a sequence $\{\eta_i\}$ of positive numbers such that for $y, y' \in Y$,

(1) if $d(r_i(y), r_i(y')) \leq \eta_i$ for all $i \geq 1$, then $y = y'$.

Moreover, by the uniform continuity of bonding mappings, there is a sequence {di} of positive numbers such that

(2) $3\delta_i \leq \eta_i$

(3) if for $u, u' \in Y_j$, $d(u, u') \leq 2\delta_j$, then $d(r_{ij}(u), r_{ij}(u')) \leq \delta_i/2^{j-i}$, $i \leq j$.

Then by using Corollaries 1 and 2 instead of [6], Corollaries 1 and 2, we sinilarly have positive numbers $0 < \varepsilon_i < 1$, polyhedra Qi, and mappings $g_i: X \longrightarrow Q_i$, $h_i: Q_i \longrightarrow Y_i$, $q_{ij}: Q_j \longrightarrow Q_i$, $i < j$, satisfying the follwing conditions:

(4) $g_i(X) = Q_i$,

(5) $d(g_i, q_{ii+1} g_{i+1}) \leq \varepsilon_i/2$,

(6) $d(r_i f, h_i g_i) \leq \delta_i/2$,

(7) if for $u, u' \in Q_i$, $d(u, u') \leq \varepsilon_i$, then $d(h_i(u), h_i(u')) \leq \delta_i/2$.

(8) if for $u, u' \in Q_j$, $d(u, u') \leq \varepsilon_j$, then $d(q_{ij}(u), q_{ij}(u')) \leq \varepsilon_i/2^{j-i}$, $i < j$,

(9) q_{ii+1} is (p, n, ε_i)-approximable.

Now we may assume that the sequence $\{\varepsilon_i\}$ is decreasing and converges to 0.

Applying (5) and (8) by the induction on $j-i \geq 0$, we have

(10) $d(g_i, q_{ij}g_j) \leq \varepsilon_i, \; i \leq j.$

Hence, by (8) and (10),

(11) $d(q_{ij}g_j, q_{ik}g_k) \leq \varepsilon_i/2^{j-i}, \; i \leq j \leq k.$

Thus, the sequence $\{q_{ij}g_j\}_{j\geq i}$ is a Cauchy sequence of mappings of X to Q_i. Hence the sequence induces a mapping $g^i: X \longrightarrow Q_i$ by

(12) $g^i = \lim q_{ij}g_j.$

Then by (10),

(13) $d(g_i, g^i) \leq \varepsilon_i.$

Moreover, by the definition, it is clearly hold that

(14) $q_{ij}g^j = g^i, \; i \leq j.$

Namely, putting a compact metric space Z as the inverse limit of an inverse sequence (Q_i, q_{ii+1}), the sequence $\{g_i\}$ induces a mapping $g: X \longrightarrow Z$ by

(15) $q_i g = g^i$ for each $i \geq 1$,

where $q_i: Z \longrightarrow Q_i$, $i \geq 1$, are the natural projections. Then since g(X) is dense in Z, g(X) = Z. And by (9) and Theorem 2,

(16) $\dim_{Z_p} Z \leq n.$

On the other hand, by (13), (7) nad (6),

(17) $d(r_i f, h_i g^i) \leq d(r_i f, h_i g_i) + d(h_i g_i, h_i g^i) \leq \delta_i/2 + \delta_i/2 = \delta_i.$

Hence by (17) and (3),

$$d(h_i g^i, r_{ii+1}h_{i+1}g^{i+1}) \leq d(h_i g^i, r_i f) + d(r_{ii+1}r_{i+1}f, r_{ii+1}h_{i+1}g^{i+1}) \leq 3\delta_i/2 \leq 2\delta_i. \tag{18}$$

Therefore by (3) and (18) and the induction on $j-i \geq 0$, we have

(19) $d(h_i g^i, r_{ij}h_j g^j) \leq 2\delta_i, \; i \leq j.$

Moreover by(3) and (19),

(20) $d(r_{ij}h_jg^j, r_{ik}h_kg^k) \leq \delta_i/2^{j-i}$, $i \leq j \leq k$.

Note that by (15), $g^k = g_kg$ and $g^j = g_jg$ and therefore, since g is surjective,

(21) $d(r_{ik}h_kq_k, r_{ij}h_jq_j) \leq \delta_i/2^{j-i}$, $i \leq j \leq k$.

It follows that $\{r_{ij}h_jq_j\}$ is a Cauchy sequence of mappings of Z to Y_i. Hence we have a mapping $h^i: Z \longrightarrow Y$ given by

(22) $h^i = \lim r_{ij}h_jq_j$.

Then, clearly,

(23) $r_{ij}h^j = h^i$, $i \leq j$.

Hence a mapping $h: Z \longrightarrow Y$ is given by the formula

(24) $r_ih = h^i$ for each $i \geq 1$.

Now for each $i \geq 1$, by the definitions of g^j and h^j and (19),

(25) $d(h^ig, h_ig^i) \leq 2\delta_i$.

Hence by (17), (25) and (6),

(26) $d(r_if, r_ihg) \leq d(r_if, h_ig^i) + d(h_ig^i, h^ig) \leq 3\delta_i \leq \eta_i$.

Therefore, by the condition of $\{\eta i\}$, (1), we have that

(27) $f = hg$.

That completes the proof of Theorem 3.

By the standard techniques, Theorem 3 induces the following corollaries. Their proofs are omitted here.

Corollary 3. *Let* X *be a compact Hausdorff space with* $\dim_{Z_p} X \leq n$. *Let* Y *be a compact Hausdorff space and let* $f: X \longrightarrow Y$ *be a mapping. Then there exists a compact Hausdorff space* Z *with* $\dim_{Z_p} Z \leq n$ *and* $\omega(Z) \leq \omega(Y)$ *and there are mappings* $g: X \longrightarrow Z$, $h: Z \longrightarrow Y$ *such that* $f = hg$.

Corollary 4. *Let* X *be a compact Hausdorff space with* $\dim_{Z_p} X \leq n$. *Then* X *has an inverse system* $\mathbf{Q} = (Q_b, q_{bb'}, B)$ *of metric compacta* Q_b *with* $\dim_{Z_p} Q_b \leq n$ *and* *card*(B) $\leq \omega$(X) *whose inverse limit is* X.

Corollary 5. *Let* X *be a compact Hausdorff space with* $\dim_{Z_p} X \leq n$. *Then there exists a compactification* Z *of* X *such that* $\dim_{Z_p} Z \leq n$.

Especially, if X *is a separable metric space with* $\dim_{Z_p} X \leq n$, *then there is a metric compactification* Z *of* X *such that* $\dim_{Z_p} Z \leq n$.

Corollary 6. *Let* X *be a separable metric space with* $\dim_{Z_p} X \leq n$. *Then there is a separable metric space* Z *with* $\dim Z \leq n$ *and a proper cell-like mapping* $f: Z \longrightarrow X$.

We note that Corollary 6 is a generalization of the first Dranishnikov Theorem to the case of separable metric spaces X and Z. In the nexr section we will show another generalization to compact Hausdorff spaces.

5. A resolution on a compact Hausdorff space X with $\dim_{Z_p} X \leq n$.

In this section we will show the generalization of the first Dranishnikov Theorem to the case of compact Hausdorff spaces X and Z. Our proof essentially depends on Mardešić-Rubin's way [8].

First we quote the notion of the n-dimensional core Z_K and the stacked n-dimensional core $Z_K{}^*$ of a complex K from [8]. The detail is omitted here.

Let take a finite complex K and an integer $n \geq 0$. Let K, K', K'',....,K^k,.... be the iterated subdivisions of K. For each $k \geq 0$, choose a simplicial approximation $q_{kk+1}: K^{k+1} \longrightarrow K^k$ of the identity $1_K: |K| = |K^{k+1}| \longrightarrow |K^k|$, and let $q_{kk+j} = q_{kk+1} \cdots q_{k+j-1\,k+j}: K^{k+j} \longrightarrow K^k$. Then q_{kk+j} is also a simplicial approximation of 1_K. Hence we have

(1) $d(q_{kk+j}, 1_K) \leq mesh(K^k)$, $j \geq 1$,

(2) $q_{kk+j}((K^{k+j})^{(n)}) \subseteq (K^k)^{(n)}$, $j \geq 1$.

Hence we have an inverse sequence of polyhedra

$$\mathbf{K} = (|(K^k)^{(n)}|, q_{kk+1}).$$

The n-*dimensional core* of K is defined as the inverse limit

(3) $Z_K = \lim \mathbf{K}$.

Clearly,

(4) $\dim Z_K \leq n$.

Let $q_k: Z_K \longrightarrow |(K^k)^{(n)}|$ be the projections. Then by the Sperner's lemma, each q_{kk+1} is surjective, all of q_{kk+j} and q_k are surjective. Moreover, by (1),

(5) $d(q_k, q_{k+j}) \leq mesh(K^k)$, $j \geq 1$.

Hence $\{q_k\}$ is a Cauchy sequence of mappings from Z_K to $|K|$, because of $\lim mesh(K^k) = 0$. Therefore we have the mapping $f_K: Z_K \longrightarrow |K|$ given by

(6) $f_K = \lim q_k$.

Then by (3),

(7) $d(f_K, q_k) \leq mesh(K^k)$.

Moreover, q_k is surjective and $\lim mesh(K^k) = 0$. Hence $f_K(Z)$ is dense in $|K|$, and therefore f_K is surjective.

Next, in order to describe the stacked n-dimensional core of K, we define a new inverse sequence as follows;for each k = 0,1,2,....,

(8) $K^{*k} = K^{(n)} \oplus (K')^{(n)} \oplus \ldots \oplus (K^k)^{(n)}$.

Hence

(9) $|K^{*k+1}| = |K^{*k}| \oplus |(K^{k+1})^{(n)}|$.

The bonding mappings $q_{kk+1}{}^*: |K^{*k+1}| \longrightarrow |K^{*k}|$ are defined by

(10) $q_{kk+1}{}^*||(K^{*k})| = 1_{|K^{*k}|}$,

(11) $q_{kk+1}{}^*||(K^{k+1})^{(n)}| = q_{kk+1}$.

We define the *stacked* n-*dimensional core* $Z_K{}^*$ as the inverse limit of the inverse sequence $\mathbf{K}^* = (|K^{*k}|, q_{kk+1}{}^*)$,

(12) $Z_K{}^* = \lim \mathbf{K}^* = (\bigoplus_{k\geq 0} |(K^k)^{(n)}|) \cup Z_K$,

and denote the natural projections by $q_k{}^*: Z_K{}^* \longrightarrow |K^{*k}|$. Then we have

(13) $\dim Z_K{}^* \leq n$.

Moreover we note the following properties;

(14) $Z_K \subseteq Z_K{}^*$ and $|K^{*k}| \subseteq Z_K{}^*$ for every $k \geq 0$,

(15) $q_k{}^*||(K^{k+j})^{(n)}| = q_{kk+j}$, $j \geq 1$,

(16) $q_k{}^*|Z_K = q_k$.

By (16), (5) and the definition of $q_{kk+1}{}^*$,

(17) $d(q_k{}^*, q_{k+j}{}^*) \leq$ *mesh* (K^k), $j \geq 0$.

Hence $\{q_k{}^*\}$ is a Cauchy sequence of mappings from $Z_K{}^*$ to $|K|$, and therefore we have the mapping $f_K{}^*: Z_K{}^* \longrightarrow |K|$ defined by

(18) $f_K{}^* = \lim q_k{}^*$.

Then we know that

(19) $d(f_K{}^*, q_k{}^*) \leq$ *mesh* (K^k),

(20) $f_K{}^*||(K^k)^{(n)}|$ is the inclusion of $|(K^k)^{(n)}|$ into $|K|$,

(21) $f_K{}^*|Z_K = f_K$.

We note that if we have a metric d on $|K|$ such that *diam* $(|K|) \leq 1$, then we can choose metrics d^* on $Z_K{}^*$ and d^k on $|K^{*k}|$ such that *diam* $(Z_K{}^*) \leq 1$, *diam* $(|K^{*k}|) \leq 1$ and

(22) $d^k(q_k{}^*(x), q_k{}^*(x')) \leq d^*(x,x')$, $x,x' \in Z_K{}^*$, $k \geq 0$.

We state our main theorem in this section.

Theorem 4. *Let* X *be a compact Hausdorff space whose cohomological dimension modulo* p, $\dim_{Z_p} X \leq n$, $n \geq 2$. *Then there exists a comapct Hausdorff space* Z *with* $\dim Z \leq n$ *and* $\omega(Z) \leq \omega(X)$ *and a surjective mapping* $f: Z \longrightarrow X$ *whose fibers are acyclic modulo* p.

Proof. For a compact Hausdorff space X with $\dim_{Z_p} X \leq n$, by Proposition 3, we have an approximate system $\mathbf{X} = (X_a, \varepsilon_a, p_{aa'}, A)$ with the limit $\lim \mathbf{X} = X$ which satisfies the conditions (i) - (iv) in Proposition 2. Moreover, for each $a \in A$, we may choose a triangulation K_a of X_a such that

(v) $6\, mesh(K_a) \leq \varepsilon_a$.

As the proof as in [8], we will define a new ordering <' in A. We consider the following three conditions for $a_1 < a_2$ and any integer $k \geq 0$:

(1) $d(p_{a_1a'}p_{a'a''}, p_{a_1a''}) \leq \mathrm{mesh}(K_{a_1}{}^k)$ for $a_2 \leq a' \leq a''$,

(2) if $d(x,x') \leq \varepsilon_{a''}$, for $x,x'' \in X_{a''}$, then $d(p_{a_1a''}(x), p_{a_1a''}(x')) \leq mesh(K_{a_1}{}^k)$ for $a_2 \leq a''$,

(3) $p_{a_1a''}: X_{a''} \longrightarrow X_{a_1}$ is $(p,n, mesh(K_{a_1}{}^k))$-approximable for $a_2 \leq a''$.

Now we put $a_1 <' a_2$ provided that $a_1 < a_2$ and the conditions (1) - (3) hold for k = 0. Then the odering <' on A satisfies the following conditions:

(4) if $a_1 <' a_2$, then $a_1 < a_2$,

(5) if $a_1 <' a_2$ and $a_2 \leq a_3$, then $a_1 <' a_3$,

(6) for every $a \in A$, there is $a' \in A$ such that $a <' a'$,

and therefore $A' = (A, <')$ is a directed set with no maximal element. We note that for any $a_1 \in A$ and integer $k \geq 0$, there exists $a_2 '> a_1$ such that the conditions (1) - (3) hold. Moreover

(7) if $a_1 <' a_2$, then the set of all integers $k \geq 0$, which satisfy the condition (2), is finite.

Hence, for each pair $a_1 <' a_2$, by (7), there is a maximal integer $k \geq 0$ such that the conditions (1) - (3) hold. We denote the integer by $k(a_1,a_2)$. Clearly we have the following properties:

(8) if $a_1 <' a_2$, then for $a' \geq a_2$, $d(p_{a_1a'}p_{a'}, p_{a_1}) \leq mesh(K_{a_1}{}^{k(a_1,a_2)})$,

(9) if $a_1 <' a_2$ and $a_2 \leq a_3$, then $k(a_1,a_2) \leq k(a_1,a_3)$,

(10) for any $a_1 \in A$ and integer $k \geq 0$, there is $a_2 \in A$ such that $a_1 <' a_2$ and $k \leq k(a_1,a_2)$.

For each pair $a_1 <' a_2$, by (6) and the definition of $k(a_1,a_2)$, we have a PL-mapping $g_{a_1a_2}: |K_{a_2}{}^{(n)}| \longrightarrow |(K_{a_1}{}^k)|^{(n)}|$, where $k = k(a_1,a_2)$, such that

(11) $d(g_{a_1a_2}, p_{a_1a_2}||K_{a_2}{}^{(n)}|) \leq 2\, mesh(K_{a_1}{}^k)$,

(12) $[g_{a_1a_2}|\partial\sigma] \in p\cdot\pi_n(|(K_{a_1}{}^k)^{(n)}|)$ for every $(n+1)$-simplex σ of K_{a_1}.

Now, for each $a \in A'$, we define

(13) $Z_a{}^* = Z_{K_a}{}^*$.

For $a_1 <' a_2$, we define the mapping $r_{a_1a_2}: Z_{a_2}{}^* \longrightarrow Z_{a_1}{}^*$ by

(14) $r_{a_1a_2} = g_{a_1a_2}q_{0a_2}{}^*$,

here $q_{0a_2}{}^*: Z_{a_2}{}^* \longrightarrow |K_{a_2}{}^{(n)}|$ is the mapping $q_0{}^*: Z_{K_{a_2}}{}^* \longrightarrow |K_{a_2}{}^{(n)}|$. Then note that

(15) $r_{a_1a_2}(Z_{a_2}{}^*) \subset |(K_{a_1}{}^k)^{(n)}|$, $k = k(a_1,a_2)$.

By the same way as in [8], Lemma 7, we have

(16) $\mathbf{Z} = (Z_a{}^*, \varepsilon_a, r_{aa'}, A')$ is an approximate system of nonempty metric compacta $Z_a{}^*$ with $\dim Z_a{}^* \leq n$.

Therefore, by Proposition 1,(1) and (3)', the limit $Z = \lim \mathbf{Z}$ is a nonempty compact Hausdorff space with $\dim Z \leq n$ and $\omega(Z) \leq card(A') = card(A) \leq \omega(X)$. Let $r_a: Z \longrightarrow Z_a{}^*$ be the projections.

For each $a \in A'$, by $f_a{}^*$, we denote the mapping $f_{K_a}{}^*: Z_a{}^* = Z_{K_a}{}^* \longrightarrow |K_a| = X_a$. Then by the same way as in [8], we can have the mapping $f: Z \longrightarrow X$ given by

(17) $f_a{}^*r_a = p_a f$ for each $a \in A$.

Now we will show that the mapping f satisfied the required condition. Let take a given point x of X. For each $a \in A$, put

(18) $x_a = p_a(x)$,

(19) $N_a = N_a(x) = \{x' \in X_a \mid d(x_a, x') \leq \varepsilon_a\}$,

(20) $M_a = M_a(x) = f_a{}^{*-1}(N_a)$.

Then by [8] Lemmas 12 and 14,

(21) $\mathbf{N}(x) = (N_a, \varepsilon_a, p_{aa'}, A')$ is an approximate system of nonempty compact Hausdorff spaces whose limit is $\{x\}$,

(22) $\mathbf{M}(x) = (M_a, \varepsilon_a, r_{aa'}, A')$ is an approximate system of nonempty compact Hausdorff spaces whose limit is $f^{-1}(x)$.

Hence by Proposition 1(2) and (22), $f^{-1}(x)$ is nonempty. Namely,

(23) f is surjective (see [8], Theorem 15).

Therefore it sufficies to show that $f^{-1}(x)$ is acyclic modulo p.

Claim 1. f *is a* UV^{n-1}*-mapping.*

Proof of Claim 1. For any $a_1 \in A'$, let take $a_2 \in A'$ such that $a_1 <' a_2$. Since N_{a_2} is a neighborhood of x_{a_2} in the polyhedron X_{a_2}, there exists a closed polyhedral neighborhood U of x_{a_2} in N_{a_2} such that

(24) U is contractible.

Now we may assume that

(25) $U = |L|$, where L is a subcomplex of the j-th barycentric subdivision $K_{a_2}{}^j$ for some sufficiently large j.

Then by the proof of [8], Lemma 17, there is $a_3 '> a_2$ such that

(26) $r_{a_2a_3}(M_{a_3}) \subseteq |L|$.

By (10), taking a sufficiently large a_3 if necessary, we may assume that for some $i \geq 0$, the i-th barycentric subdivision L^i of L is a subcomplex of $K_{a_2}{}^{k(a_2,a_3)}$.

Note that

(27) $|L^i| \cap |(K_{a_2}{}^{k(a_2,a_3)})^{(m)}| = |(L^i)^{(m)}|$ for every integer $m \geq 0$.

Moreover, by (24) and (25),

(28) $\pi_m(|(L^i)^{(n)}|) = \pi_m(|L|) = 0$ if $m < n$.

For any $1 \leq m < n$ and a mapping $\alpha: S^m \longrightarrow M_{a_3}$, by (26), (15) and (27),

(29) $r_{a_2a_3}(\alpha(S^m)) \subset |L| \cap |(K_{a_2}{}^{k(a_2,a_3)})^{(n)}| = |(L^i)^{(n)}| \subset |L| \subseteq N_{a_2}$.

Hence by (28),

(30) $r_{a_2a_3}\alpha \simeq 0$ in $|(L^i)^{(n)}|$.

Considering $|(L^i)^{(n)}| \subset |(K_{a_2}{}^{k(a_2,a_3)})^{(n)}| \subseteq Z_{a_2}{}^*$, by [8], Lemma 17,

(31) $r_{a_1a_2}(|(L^i)^{(n)}|) \subseteq M_{a_1}$.

By (30) and (31),

(32) $r_{a_1a_2}r_{a_2a_3}\alpha \simeq 0$ in M_{a_1}.

It follows that $f^{-1}(x)$ is UV^m-connected for every $m \leq n-1$. We complete the proof of Claim 1.

Claim 2. $\check{H}^n(f^{-1}(x):Z_p) = 0$ *for every* $x \in X$.

Proof of Claim 2. By Proposition 3, it sufficies to show that for every $a_1 \in A'$ and every mapping $\alpha: M_{a_1} \longrightarrow K(Z_p,n)$,

(33) $\alpha r_{a_1a_2}r_{a_2a_3} \simeq 0$,

here we use the same notation as in Claim 1, so the indexes a_2 and a_3 are the one taken in the proof of Claim 1 (see [8], Lemma 17).

Let σ be a (n+1)-simplex of L^i. Since $q_{0a_2}{}^*||(K_{a_2}{}^{k(a_2,a_3)})^{(n)}|$ is a restriction of the simplicial approximation $q_{0k(a_2,a_3)}: K_{a_2}{}^{k(a_2,a_3)} \longrightarrow K_{a_2}$ of 1_K, by (27), $q_{0a_2}(\sigma) = \tau$ is at most (n+1)-dimensional simplex of K_{a_2}. In the case of dim $\tau \leq n$, it is clear that $\alpha r_{a_1a_2}|\partial\sigma = \alpha g_{a_1a_2}q_{0a_2}{}^*|\partial\sigma$ has the extension $\alpha g_{a_1a_2}q_{0k(a_1,a_2)}$ over σ.

If dim $\tau = n+1$, by (12), $[g_{a_1a_2}|\partial\tau] \in p\cdot\pi_n(|(K_{a_1}{}^{k(a_1,a_2)})^{(n)}|)$. Hence

(34) $\alpha g_{a_1a_2}|\partial\tau \simeq 0$ in $K(Z_p,n)$.

Therefore we have an extension $h_\sigma: \sigma \longrightarrow K(Z_p,n)$ of $\alpha r_{a_1a_2}|\partial\sigma$. It follows that $\alpha r_{a_1a_2}||(L^i)^{(n)}|$ has an extension $h^*: |(L^i)^{(n+1)}| \longrightarrow K(Z_p,n)$. Since $|L^i|$ is contractible, $|(L^i)^{(n)}|$ is contractible in $|(L^i)^{(n+1)}|$. Hence

(35) $\alpha r_{a_1a_2} = h^*||(L^i)^{(n)}| \simeq 0$ in $K(Z_p,n)$.

Threrefore, by (26), (15) and (35), we have (33). It completes the proof of Claim 2.

Since dim $f^{-1}(x) \leq$ dim $Z \leq n$, by Claims 1 and 2, we have that $f^{-1}(x)$ is acyclic modulo p. We complete the proof of Theorem 3.

Some generalizations of Theorem 3 to the case of noncompact spaces will be obtained by the same way as in [8] as follows. However the proof is omitted here.

Corollary 7. *Let* $\mathbf{C}$ *be a class of paracompact spaces with the following two properties:*

(i) *if* $g: Z \longrightarrow X$ *is a proper mapping,* Z *is Hausdorff and* $X \in \mathbf{C}$, *then also* $Z \in \mathbf{C}$,

(ii) *if* Y *is a normal space and* $Z \in \mathbf{C}$ *is a subspace of* Y, *then* dim $Z \leq$ dim Y.

Then every space $X \in \mathbf{C}$ *with* $\dim_{Z_p} X \leq n$, $n \geq 2$, *is the image of a mapping* $f: Z \longrightarrow X$ *of a space* $Z \in \mathbf{C}$ *with* dim $Z \leq n$ *and* $\omega(Z) \leq \omega(X)$ *whose fibers are acyclic modulo* p.

Note that as a such a class of paracompact Hausdorff spaces, we know the followings; *paracompact locally strongly paracompact spaces, strongly paracompact spaces, paracompact locally compact spaces.*

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