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A CHARACTERIZATION OF $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_\alpha + 2v_\beta; t,q\}$-MIN·HYPERS IN PG(t,q) ($t \geq 2$, $q \geq 5$ and $0 \leq \alpha < \beta < t$) AND ITS APPLICATIONS TO ERROR-CORRECTING CODES

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1. Introduction

Let F be a set of f points in a finite projective geometry PG(t,q) of t dimensions where $t \geq 2$, $f \geq 1$ and q is a prime power. If (a) $|F \cap H| \geq m$ for any hyperplane H in PG(t,q) and (b) $|F \cap H| = m$ for some hyperplane H in PG(t,q), then F is said to be an {f,m;t,q}-min·hyper (or an {f,m;t,q}-minihyper) where $m \geq 0$ and |A| denotes the number of points in the set A. The concept of a min·hyper (or a max·hyper) has been introduced by Hamada and Tamari [17]. In the special case $t = 2$ and $m \geq 2$, an {f,m;2,q}-min·hyper F is called an m-blocking set if F contains no 1-flat in PG(2,q).

For example, let F be a μ-flat in PG(t,q) where $0 \leq \mu < t$. Then $|F| = v_{\mu+1}$ and $|F \cap H| = v_\mu$ or $v_{\mu+1}$ for any hyperplane H in PG(t,q) according as $F \not\subset H$ or $F \subset H$ where $v_\ell = (q^\ell-1)/(q-1)$ for any integer $\ell \geq 0$. Hence F is a $\{v_{\mu+1}, v_\mu; t,q\}$-min·hyper. Tamari [27,29] shows that the converse holds, i.e., if F is a $\{v_{\mu+1}, v_\mu; t,q\}$-min·hyper, then F is a μ-flat in PG(t,q).

Let V(n;q) be an n-dimensional vector space consisting of row vectors over a Galois field GF(q) of order q where n is a positive integer. A k-dimensional subspace C of V(n;q) is said to be an (n,k,d;q)-code (or a q-ary linear code with length n, dimension k, and minimum distance d) if the minimum (Hamming) distance of the code C is equal to d where $n > k \geq 3$ and

$d \geqq 1$ (cf. McWilliams and Sloane [24]). It is well known that if there exists an $(n,k,d;q)$-code for given integers k, d and q, then

$$n \geqq \sum_{\ell=0}^{k-1} \left\lceil \frac{d}{q^{\ell}} \right\rceil \qquad (1.1)$$

where $\lceil x \rceil$ denotes the smallest integer $\geqq x$. In what follows, we shall confine ourself to the case $k \geqq 3$ and $1 \leqq d < q^{k-1}$. In this case, d can be expressed as follows:

$$d = q^{k-1} - \sum_{\alpha=0}^{k-2} \varepsilon_\alpha q^\alpha \qquad (1.2)$$

using some integers k, q and ε_α $(\alpha = 0,1,\cdots,k-2)$ and the Griesmer bound (1.1) can be expressed as follows:

$$n \geqq v_k - \sum_{\alpha=0}^{k-2} \varepsilon_\alpha v_{\alpha+1} \qquad (1.3)$$

where $0 \leqq \varepsilon_\alpha \leqq q-1$ for $\alpha = 0,1,\cdots,k-2$. Recently, Hamada [5,10] showed that in the case $k \geqq 3$ and $d = q^{k-1} - \sum_{\alpha=0}^{k-2} \varepsilon_\alpha q^\alpha$, there is a one-to-one correspondence between the set of all nonequivalent $(n,k,d;q)$-codes meeting the Griesmer bound (1.3) and the set of all $\{\sum_{\alpha=0}^{k-2} \varepsilon_\alpha v_{\alpha+1}, \sum_{\alpha=1}^{k-2} \varepsilon_\alpha v_\alpha; k-1,q\}$-min·hypers if we introduce some equivalence relation among $(n,k,d;q)$-codes. Hence in order to obtain a necessary and sufficient condition for integers k, d and q that there exists an $(n,k,d;q)$-code meeting the Griesmer bound (1.3) in the case $1 \leqq d < q^{k-1}$ and to characterize all $(n,k,d;q)$-codes meeting the Griesmer bound (1.3) in the case $1 \leqq d < q^{k-1}$, it is sufficient to solve the following problem.

Problem A. (1) Find a necessary and sufficient condition for integers t, q and ε_α ($\alpha = 0,1,\cdots,t-1$) that there exists a $\{\sum_{\alpha=0}^{t-1} \varepsilon_\alpha v_{\alpha+1}, \sum_{\alpha=0}^{t-1} \varepsilon_\alpha v_\alpha; t,q\}$-min·hyper.

(2) Characterize all $\{\sum_{\alpha=0}^{t-1} \varepsilon_\alpha v_{\alpha+1}, \sum_{\alpha=0}^{t-1} \varepsilon_\alpha v_\alpha; t,q\}$-min·hypers in the case where there exist such min·hypers.

Since all $(n,k,d;q)$-codes meeting the Griesmer bound (1.3) have been characterized by Helleseth [20] and Tilborg [30] in the special case $q = 2$, $k \geqq 3$ and $1 \leqq d < 2^{k-1}$, we shall confine ourself to the case $q \geqq 3$, $k \geqq 3$ and $1 \leqq d < q^{k-1}$ in what follows.

In the case $\sum_{\alpha=0}^{k-2} \varepsilon_\alpha = 1$ (i.e., $\varepsilon_\alpha = 1$ for some integer α), it is shown by Tamari [27,29] that F is a $\{v_{\alpha+1}, v_\alpha; k-1,q\}$-min·hyper if and only if F is an α-flat in PG(k-1,q). In the case $\sum_{\alpha=0}^{k-2} \varepsilon_\alpha = 2$, it is shown by Hamada [5,6,7] that F is a $\{v_{\alpha+1} + v_{\beta+1}, v_\alpha + v_\beta; k-1,q\}$-min·hyper if and only if F is the union of an α-flat and a β-flat in PG(k-1,q) which are mutually disjoint where $0 \leqq \alpha \leqq \beta < k-1$. In the case $\sum_{\alpha=0}^{k-2} \varepsilon_\alpha = 3$, it is shown by Hamada [5, 6,7,8,9] and Hamada and Deza [14] that F is a $\{v_{\alpha+1} + v_{\beta+1} + v_{\gamma+1}, v_\alpha + v_\beta$ $v_\gamma; k-1,q\}$-min·hyper if and only if F is the union of an α-flat, a β-flat and a γ-flat in PG(k-1,q) which are mutually disjoint where $q \geqq 5$ and either $0 \leqq \alpha \leqq \beta < \gamma < k-1$ or $0 \leqq \alpha < \beta \leqq \gamma < k-1$.

In the case $k \geqq 3$, $q \geqq 3$ and $\varepsilon_\alpha = 0$ or 1 for $\alpha = 0,1,\cdots,k-2$, it is shown by Hamada [5] that F is a $\{\sum_{\alpha=0}^{k-2} \varepsilon_\alpha v_{\alpha+1}, \sum_{\alpha=0}^{k-2} \varepsilon_\alpha v_\alpha; k-1,q\}$-min·hyper if and only if F is the union of ε_0 0-flats, ε_1 1-flats, $\cdots$, ε_{k-2} (k-2)-flats in PG(k-1,q) which are mutually disjoint. Hence in the case $k \geqq 5$, $q \geqq 3$ and

$0 \leqq \alpha < \beta < \gamma < \delta < k-1$, F is a $\{v_{\alpha+1} + v_{\beta+1} + v_{\gamma+1} + v_{\delta+1}, v_\alpha + v_\beta + v_\gamma + v_\delta; k-1,q\}$-min·hyper if and only if F is the union of an α-flat, a β-flat, a γ-flat and a δ-flat in PG(k-1,q) which are mutually disjoint. Recently, it has been shown by Hamada [8] and Hamada and Deza [12] that (1) in the case $k = 3$, $q \geqq 5$, $\alpha = \beta = 0$ and $\gamma = \delta = 1$, there is no $\{2v_1 + 2v_2, 2v_0 + 2v_1; 2,q\}$-min·hyper and (2) in the case $k \geqq 4$, $q \geqq 5$, $\alpha = \beta = 0$ and $\gamma = \delta = 1$, F is a $\{2v_1 + 2v_2, 2v_0 + 2v_1; k-1,q\}$-min·hyper if and only if F is the union of two 0-flats and two 1-flats in PG(k-1,q) which are mutually disjoint. The purpose of this paper is to extend the above results, i.e., to prove the following theorem (cf. Reference [13] in detail).

<u>Theorem 1.1.</u> Let t and q be any integer $\geqq 2$ and any prime power $\geqq 5$, respectively, and let α and β be any integers such that $0 \leqq \alpha < \beta < t$.

(1) In the case $t > 2\beta$, F is a $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_\alpha + 2v_\beta; t,q\}$-min·hyper if and only if F is the union of two α-flats and two β-flats in PG(t,q) which are mutually disjoint.

(2) In the case $t \leqq 2\beta$, there is no $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_\alpha + 2v_\beta; t,q\}$-min·hyper.

From Theorem 1.1 and Theorem 5.2 in Hamada [10], we have the following

<u>Corollary 1.1.</u> Let k and q be any integer $\geqq 3$ and any prime power $\geqq 5$, respectively. Let $d = q^{k-1} - 2q^\alpha - 2q^\beta$ and $n = v_k - 2v_{\alpha+1} - 2v_{\beta+1}$ where $0 \leqq \alpha < \beta < k-1$.

(1) In the case $k > 2\beta+1$, C is an (n,k,d;q)-code meeting the Griesmer bound if and only if C is an (n,k,d;q)-code constructed by using two α-flats and two β-flats in PG(k-1,q) which are mutually disjoint.

(2) In the case $k \leqq 2\beta+1$, there is no (n,k,d;q)-code meeting (1.1).

2. Propositions for the proof of Theorem 1.1

Let $\mathcal{F}_U(\varepsilon,\mu_1,\mu_2;t,q)$ denote a family of all unions of ε points, a μ_1-flat and a μ_2-flat in PG(t,q) which are mutually disjoint where $0 \leq \varepsilon \leq q-1$ and $1 \leqq \mu_1 \leqq \mu_2 < t$. Let $\mathcal{F}(\nu_1,\nu_2,\cdots,\nu_h;t,q)$ denote a family of all unions of a ν_1-flat, a ν_2-flat, $\cdots$, a ν_h-flat in PG(t,q) which are mutually disjoint where $h \geqq 2$ and $0 \leq \nu_1 \leqq \nu_2 \leqq \cdots \leqq \nu_h < t$.

In order to prove Theorem 1.1, we shall prepare the following propositions.

Proposition 2.1. (Hamada [5,10])

Let t and q be any integer $\geqq 3$ and any prime power $\geqq 3$, respectively, and let α and β be any integers such that $0 \leqq \alpha < \beta < t/2$. If $F \in \mathcal{F}(\alpha,\alpha,\beta,\beta;t,q)$, then F is a $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_\alpha + 2v_\beta;t,q\}$-min·hyper.

Proposition 2.2. (Hamada [5,10])

Let t and q be any integer $\geqq 2$ and any prime power $\geqq 3$, respectively. If there exists a $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_\alpha + 2v_\beta;t,q\}$-min·hyper F for some integers α and β such that $0 \leq \alpha < \beta < t$, there exists at least one (t-2)-flat G in PG(t,q) such that $|F \cap G| = 2v_{\alpha-1} + 2v_{\beta-1}$ where $v_{-1} = 0$ and $v_\ell = (q^\ell-1)/(q-1)$ for any integer $\ell \geqq 0$. Let H_i $(i = 1,2,\cdots,q+1)$ be q+1 hyperplanes in PG(t,q) which contain G.

(1) In the case $\alpha = 0$, $F \cap H_i$ is a $\{\delta_i + 2v_\beta, 2v_{\beta-1};t,q\}$-min·hyper in H_i for $i = 1,2,\cdots,q+1$ where δ_i's are some nonnegative integers such that $\sum_{i=1}^{q+1} \delta_i = 2$.

(2) In the case $\alpha \geqq 1$, $F \cap H_i$ is a $\{2v_\alpha + 2v_\beta, 2v_{\alpha-1} + 2v_{\beta-1};t,q\}$-min·hyper in H_i for $i = 1,2,\cdots,q+1$.

Proposition 2.3. (Hamada [5,10])

Let t and q be any integer $\geqq 4$ and any prime power $\geqq 3$, respectively.

(1) Let ε, β and δ_i $(i = 1,2,\cdots,q+1)$ be any nonnegative integers such that $0 \leqq \varepsilon \leqq q-1$, $2 \leqq \beta \leqq t/2$ and $\sum_{i=1}^{q+1} \delta_i = \varepsilon$. If F is a $\{\varepsilon v_1 + 2v_{\beta+1}, \varepsilon v_0 + 2v_\beta; t,q\}$-min·hyper such that (a) $|F \cap G| = 2v_{\beta-1}$ for some (t-2)-flat G in PG(t,q) and (b) $F \cap H_i \in \mathcal{F}_U(\delta_i,\beta-1,\beta-1;t,q)$ for any hyperplane H_i $(1 \leqq i \leqq q+1)$ which contain G, then $F \in \mathcal{F}_U(\varepsilon,\beta,\beta;t,q)$.

(2) Let α and β be any integers such that $2 \leqq \alpha < \beta \leqq t/2$. If F is a $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_\alpha + 2v_\beta; t,q\}$-min·hyper such that (a) $|F \cap G| = 2v_{\alpha-1} + 2v_{\beta-1}$ for some (t-2)-flat G in PG(t,q) and (b) $F \cap H_i \in \mathcal{F}(\alpha-1,\alpha-1,\beta-1,\beta-1;t,q)$ for any hyperplane H_i $(1 \leqq i \leqq q+1)$ which contain G, then $F \in \mathcal{F}(\alpha,\alpha,\beta,\beta;t,q)$.

Proposition 2.4. (Hamada and Deza [13])

Let t and q be any integer $\geqq 4$ and any prime power $\geqq 5$, respectively, and let β be any integer such that $2 \leqq \beta \leqq t/2$. If F is a $\{2v_2 + 2v_{\beta+1}, 2v_1 + 2v_\beta; t,q\}$-min·hyper such that (a) $|F \cap G| = 2v_{\beta-1}$ for some (t-2)-flat G in PG(t,q) and (b) $F \cap H_i \in \mathcal{F}(0,0,\beta-1,\beta-1;t,q)$ for any hyperplane H_i $(1 \leqq i \leqq q+1)$ which contain G, then $F \in \mathcal{F}(1,1,\beta,\beta;t,q)$.

Proposition 2.5. (Hamada [6,7,10])

Let t and q be any integer $\geqq 2$ and any prime power $\geqq 3$, respectively, and let β be an integer such that $0 \leqq \beta < t$.

(1) In the case $t > 2\beta$, F is a $\{2v_{\beta+1}, 2v_\beta; t,q\}$-min·hyper if and only if $F \in \mathcal{F}(\beta,\beta;t,q)$.

(2) In the case $t \leqq 2\beta$, there is no $\{2v_{\beta+1}, 2v_\beta; t,q\}$-min·hyper.

Proposition 2.6. (Hamada [6,7] and Hamada and Deza [14])

Let t and q be any integer $\geqq 2$ and any prime power $\geqq 5$, respectively, and let α and β be integers such that $0 \leqq \alpha < \beta < t$.

(1) In the case $t > 2\beta$, F is a $\{v_{\alpha+1} + 2v_{\beta+1}, v_{\alpha} + 2v_{\beta}; t,q\}$-min·hyper if and only if $F \in \mathfrak{F}(\alpha,\beta,\beta;t,q)$.

(2) In the case $t \leqq 2\beta$, there is no $\{v_{\alpha+1} + 2v_{\beta+1}, v_{\alpha} + 2v_{\beta}; t,q\}$-min·hyper.

Proposition 2.7. (Hamada [8] and Hamada and Deza [12])

(1) In the case $t = 2$ and $q \geqq 5$, there is no $\{2v_1 + 2v_2, 2v_0 + 2v_1; t,q\}$-min·hyper.

(2) In the case $t \geqq 3$ and $q \geqq 5$, F is a $\{2v_1 + 2v_2, 2v_0 + 2v_1; t,q\}$-min·hyper if and only if $F \in \mathfrak{F}(0,0,1,1;t,q)$.

Proposition 2.8. (Hamada and Tamari [19])

Let t and q be any integer $\geqq 2$ and any prime power $\geqq 3$, respectively, and let α, β, γ and δ be any integers such that $0 \leqq \alpha \leqq \beta < \gamma \leqq \delta < t$. Then $\mathfrak{F}(\alpha,\beta,\gamma,\delta;t,q) \neq \emptyset$ if and only if $\gamma + \delta \leqq t-1$.

3. The proof of Theorem 1.1

It follows from Proposition 2.1 that if $F \in \mathfrak{F}(\alpha,\alpha,\beta,\beta;t,q)$, then F is a $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_{\alpha} + 2v_{\beta}; t,q\}$-min·hyper where $0 \leqq \alpha < \beta < t/2$.

Suppose there exists a $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_{\alpha} + 2v_{\beta}; t,q\}$-min·hyper F for some integer α and β such that $0 \leqq \alpha < \beta < t$. Then it follows from Proposition 2.2 that there exists at least one (t-2)-flat G in PG(t,q) such that $|F \cap G| = 2v_{\alpha-1} + 2v_{\beta-1}$. Let H_i $(i = 1,2,\cdots,q+1)$ be q+1 hyperplanes in PG(t,q) which contain G. Then it follows from Proposition 2.2 that (1) in the case $\alpha = 0$, $F \cap H_i$ is a $\{\delta_i + 2v_{\beta}, 2v_{\beta-1}; t,q\}$-min·hyper in H_i for $i = 1,2,\cdots,q+1$ and (2) in the case $\alpha \geqq 1$, $F \cap H_i$ is a $\{2v_{\alpha} + 2v_{\beta}, 2v_{\alpha-1} + 2v_{\beta-1}; t,q\}$-min·hyper in H_i for $i = 1,2,\cdots,q+1$ where δ_i's are some nonnegative integers such that $\sum_{i=1}^{q+1} \delta_i = 2$. We shall prove Theorem 1.1 by induction on α and β.

<u>Case I</u> : $\alpha = 0$ and $\beta = 1$. It follows from Proposition 2.7 that Theorem 1.1 holds.

<u>Case II</u> : $\alpha = 0$ and $\beta \geqq 2$ (i.e., $\beta = \theta+1$ and $\theta \geqq 1$). Suppose Theorem 1.1 holds in the case $\alpha = 0$ and $\beta = \theta$, i.e., suppose that (1) in the case $t > 2\theta$, F is a $\{2v_1 + 2v_{\theta+1}, 2v_0 + 2v_\theta; t,q\}$-min.hyper if and only if $F \in \mathfrak{F}(0,0,\theta,\theta;t,q)$ and (2) in the case $t \leqq 2\theta$, there is no $\{2v_1 + 2v_{\theta+1}, 2v_0 + 2v_\theta; t,q\}$-min.hyper F.

In the case $\beta = \theta+1$, it follows from induction on β and Propositions 2.5 and 2.6 that (1) in the case $t-1 > 2\theta$ (i.e., $t > 2\beta-1$), $F \cap H_i$ is a $\{\delta_i v_1 + 2v_{\theta+1}, \delta_i v_0 + 2v_\theta; t,q\}$-min.hyper in the (t-1)-flat H_i if and only if $F \cap H_i$ is the union of δ_i 0-flats (i.e., δ_i points) and two θ-flats in H_i which are mutually disjoint (i.e., $F \cap H_i \in \mathfrak{F}_U(\delta_i, \beta-1, \beta-1; t,q)$) and (2) in the case $t-1 \leqq 2\theta$ (i.e., $t \leqq 2\beta-1$), there is no $\{\delta_i + 2v_{\theta+1}, 2v_\theta; t,q\}$-min.hyper in H_i. Hence it follows from Propositions 2.2 and 2.3 that (1) in the case $t > 2\beta-1$, $F \in \mathfrak{F}(0,0,\beta,\beta;t,q)$ and (2) in the case $t \leqq 2\beta-1$, there is no $\{2v_1 + 2v_{\beta+1}, 2v_0 + 2v_\beta; t,q\}$-min.hyper F. Since it follows from Proposition 2.8 that $\mathfrak{F}(0,0,\beta,\beta;t,q) = \emptyset$ in the case $t = 2\beta$, there is no $\{2v_1 + 2v_{\beta+1}, 2v_0 + 2v_\beta; t,q\}$-min.hyper F in the case $t = 2\beta$. Hence Theorem 1.1 holds in Case II.

<u>Case III</u> : $\alpha = 1$ and $\beta \geqq 2$. It follows from Cases I and II that (1) in the case $t-1 > 2(\beta-1)$ (i.e., $t > 2\beta-1$), $F \cap H_i$ is a $\{2v_1 + 2v_\beta, 2v_0 + 2v_{\beta-1}; t,q\}$-min.hyper in the (t-1)-flat H_i if and only if $F \cap H_i \in \mathfrak{F}(0,0,\beta-1,\beta-1;t,q)$ and (2) in the case $t-1 \leqq 2(\beta-1)$ (i.e., $t \leqq 2\beta-1$), there is no $\{2v_1 + 2v_\beta, 2v_0 + 2v_{\beta-1}; t,q\}$-min.hyper in H_i. Hence it follows from Proposition 2.4 that (1) in the case $t > 2\beta-1$, $F \in \mathfrak{F}(1,1,\beta,\beta;t,q)$ and (2) in the case $t \leqq 2\beta-1$, there is no $\{2v_2 + 2v_{\beta+1}, 2v_1 + 2v_\beta; t,q\}$-min.hyper F. Since it follows from Proposition 2.8 that $\mathfrak{F}(1,1,\beta,\beta;t,q) = \emptyset$ in the case $t = 2\beta$, there is no $\{2v_2 + 2v_{\beta+1}, 2v_1 + 2v_\beta; t,q\}$-min.hyper in the case $t = 2\beta$.

Hence Theorem 1.1 holds in Case III.

Case IV : $2 \leq \alpha < \beta < t$. It follows from Propositions 2.2, 2.3 and induction on α and β that Theorem 1.1 holds. This completes the proof.

Remark 3.1. In the case $t = 2$, $\alpha = 0$, $\beta = 1$ and $q = 3$ or 4, it is shown by Hamada [8,11] that there exists a $\{2v_1 + 2v_2, 2v_0 + 2v_1; 2, q\}$-min·hyper F in PG(2,q) such that $F \notin \mathcal{F}(0,0,1,1;2,q)$. Hence Theorem 1.1 does not hold in the case $q = 3$ or 4.

Remark 3.2. In the case $t \geq 3$ and $q \geq 5$, we can characterize all $\{2v_{\alpha+1} + v_{\beta+1} + v_{\gamma+1}, 2v_\alpha + v_\beta + v_\gamma; t, q\}$-min·hypers for any distinct integers α, β and γ in $\{0,1,\cdots,t-1\}$ using a method similar to the proof of Theorem 1.1.

Remark 3.3. In order to solve Problem A, completely, for the case $q \geq 5$ and $\sum_{\alpha=0}^{t-1} \varepsilon_\alpha = 3$ or 4, it is necessary to solve the following open problem.

Problem B. Let t and q be any integer ≥ 2 and any prime power ≥ 5, respectively.

(1) Characterize all $\{3v_{\alpha+1}, 3v_\alpha; t, q\}$-min·hypers and all $\{4v_{\alpha+1}, 4v_\alpha; t, q\}$-min·hypers for any integer α in $\{1,2,\cdots,t-1\}$.

(2) Characterize all $\{3v_{\alpha+1} + v_{\beta+1}, 3v_\alpha + v_\beta; t, q\}$-min·hypers for any distinct integers α and β in $\{0,1,\cdots,t-1\}$.

References

1. R. D. Carmichael, Introduction to the Theory of Groups of Finite Order, Dover Publications, New York, 1956.
2. P. Dembowski, Finite Geometries, Springer-Verlag, New York, 1968.
3. P. G. Farrell, Linear binary anticode, Electron. Lett. 6 (1970), 419-421.

4. J. H. Griesmer, A bound for error-correcting codes, IBM J. Res. Develop. 4 (1960), 532-542.

5. N. Hamada, Characterization resp. nonexistence of certain q-ary linear codes attaining the Griesmer bound, Bull. Osaka Women's Univ. 22 (1985), 1-47.

6. N. Hamada, Characterization of $\{2(q+1),2;t,q\}$-min·hypers and $\{2(q+1)+1, 2;t,q\}$-min·hypers in a finite projective geometry, to appear in Bull. Osaka Women's Univ. 26 (1989).

7. N. Hamada, Characterization of $\{2v_{\mu+1},2v_{\mu};t,q\}$-min·hypers and $\{2v_{\mu+1} + v_{\mu}, 2v_{\mu} + v_{\mu-1};t,q\}$-min·hypers and its applications to error-correcting codes, to appear in Bull. Osaka Women's Univ. 26 (1989).

8. N. Hamada, Characterization of $\{(q+1)+2,1;t,q\}$-min·hypers and $\{2(q+1)+2, 2;2,q\}$-min·hypers in a finite projective geometry, Grapha and Combinatorics 5 (1989). (in press)

9. N. Hamada, Characterization of $\{v_{\mu+1} + 2v_{\mu}, v_{\mu} + 2v_{\mu-1};t,q\}$-min·hypers and its applications to error-correcting codes, Graphs and Combinatorics 5 (1989). (in press)

10. N. Hamada, Characterization of min·hypers in a finite projective geometry and its applications to error-correcting codes, Bull. Osaka Women's Univ. 24 (1987), 1-24.

11. N. Hamada, Characterization of $\{12,2;2,4\}$-min·hypers in a finite projective geometry PG(2,4), Bull. Osaka Women's Univ. 24 (1987), 25-31.

12. N. Hamada and M. Deza, Characterization of $\{2(q+1)+2,2;t,q\}$-min·hypers in PG(t,q) ($t \geqq 3$, $q \geqq 5$) and its applications to error-correcting codes, Discrete Mathematics 63 (1988). (in press)

13. N. Hamada and M. Deza, Characterization of $\{2v_{\alpha+1} + 2v_{\beta+1}, 2v_{\alpha} + 2v_{\beta}; t,q\}$-min·hypers in PG(t,q) ($t \geqq 2$, $q \geqq 5$ and $0 \leqq \alpha < \beta < t$) and its applications to error-correcting codes, submitted for publication.

14. N. Hamada and M. Deza, A characterization of some (n,k,d;q)-codes meeting the Griesmer bound for given integers $k \geqq 3$, $q \geqq 5$ and $d = q^{k-1} - q^{\alpha} - q^{\beta} - q^{\gamma}$ ($0 \leqq \alpha \leqq \beta < \gamma < k-1$ or $0 \leqq \alpha < \beta \leqq \gamma < k-1$), Bull. Inst. Math. Academia Sinica 16 (1988). (in press)

15. N. Hamada and M. Deza, A characterization of $\{v_{\mu+1} + \varepsilon, v_{\mu};t,q\}$-min·hypers and its applications to error-correcting codes and factorial designs, J. Statistical Planning and Inference 19 (1988). (in press)

16. N. Hamada and M. Deza, A survey of recent works with respect to a

characterization of an (n,k,d;q)-code meeting the Griesmer bound using a min·hyper in a finite projective geometry, to appear in Annals of Discrete Mathematics (1989).

17. N. Hamada and F. Tamari, On a geometrical method of construction of maximal t-linearly independent sets, J. Combinatorial Theory (A) 25 (1978), 14-28.

18. N. Hamada and F. Tamari, Construction of optimal codes and optimal fractional factorial designs using linear programming, Annals of Discrete Mathematics 6 (1980), 175-188.

19. N. Hamada and F. Tamari, Construction of optimal linear codes using flats and spreads in a finite projective geometry, European J. Combinatorics 3 (1982), 129-141.

20. T. Helleseth, A characterization of codes meeting the Griesmer bound, Information and Control 50 (1981), 128-159.

21. T. Helleseth, New construction of codes meeting the Griesmer bound, IEEE Trans. Information Theory IT-29 (1983), 434-439.

22. T. Helleseth and H. C. A. van Tilborg, A new class of codes meeting the Griesmer bound, IEEE Trans. Information Theory IT-27 (1981), 548-555.

23. D. R. Hughes and F. C. Piper, Projective Planes, Springer, New York(1973).

24. F. J. MacWilliams and N. J. A. Sloane, The Theory of Error-Correcting Codes, North-Holland Mathematical Library Vol. 16 (1977), Amsterdam.

25. N. L. Manev, A characterization up to isomorphism of some classes of codes meeting the Griesmer bound, Comptes rendus de l'Academie bulgare des Sciences 37 (1984), 481-483.

26. G. Solomon and J. J. Stiffler, Algebraically punctured cyclic codes, Information and Control 8 (1965), 170-179.

27. F. Tamari, A note on the construction of optimal linear codes, J. Statistical Planning and Inference 5 (1981), 405-411.

28. F. Tamari, On an {f,m;t,s}-max.hyper and a {k,m;t,s}-min·hyper in a finite projective geometry PG(t,s), Bull. Fukuoka University of Education 31 (III) (1981), 35-43.

29. F. Tamari, On linear codes which attain the Solomon-Stiffler bound, Discrete Mathematics 49 (1984), 179-191.

30. H. C. A. van Tilborg, On the uniqueness resp. nonexistence of certain codes meeting the Griesmer bound, Information and Control 44 (1980), 16-35.