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An Approach to Knowledge Representation using Multi-world Logic

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Abstract

Our knowledge of real world is partial by our finite ability of the observation. But, from such a partial knowledge, we not only infer the fact which we can make sure but the fact which we cannot. It means that our inference is not closed in the world which we can observe. To formalize such inference, MWL (Multi-world logic) is proposed in this paper. Each formula of MWL takes a closed set as truth value, accordingly, MWL can express vagueness of Knowledge.

1.Introduction

Partial world

Since our finite ability of the observation, we do not have guarantee to observe every part of real world. Thus, if our knowledge is the fact of real world, then our knowledge is correct within our observation of real world. Consequently, we are not able to mention about total real world, since there are unknown parts of real world for us yet. Let call the part of real world where we can observe as partial world.

Let consider the statement "bird can fly". It is necessary to see this statement as a correct fact of real world that we have already observed all of birds existed in real world flied. But, we can not do it. Accordingly, the statement "bird can fly"is the fact only within the partial world. Our knowledge is such a thing ? Of course, not. Because, if we recognize the statement "bird can fly" as the knowledge of real world, then we apply it to the bird which we did not observe yet, and we infer that it can fly.

It means that our knowledge is not the description of the fact occurred in real world, but it is the description of the explanation of real world which is the fact in the partial world.

In this paper,the knowledge of real world is recognized as the explanation of real world. And MWL (Multi-world Logic) is proposed as logic of such explanation of the world.

Multi-world Logic

MWL aims at the theory explains our inference that can infer facts in real-world from facts in the partial world of the real world. MWL supposes that, for the concerning of the inference facts on the total state from the facts in the partial state, the truth values and inference have properties as follow:

① There are two kinds of truth value. One is reflected of our observation of partial world. ,and the other is reflected of our guess about real world.

② We formalize both truth value as set ,and give the relation between them as the truth value reflected our guess is closure of the truth value reflected our observation.

③ And, inference progresses with the validity of our guess, in other words, truth values of the sentence of MWL are formalized as closed set.

2.Truth value space

Let $(P,\leqq)$ be a partiality ordered set, and we call element p of P as path . Mapping $f:X \to Y$ is a continuous mapping if and only if f satisfies conditions as following:

① $f(a\in X)=a\in Y$,

② for any $A\subset X$, $f(CL_X(A)) \subseteq CL_Y(f(A))$,

We denote that there is a continuous mapping from topological space(X,CL_X) to (Y,CL_Y) as topological space (X,CL_X) is contained by (Y,CL_Y), and express as $(X,CL_X) \subseteq (Y,CL_Y)$.

Definition:Truth value space

Let X_P be a set of the topological spaces indexed by P. Let (X_p,CL_p) be a topological space which is an element of X_P called truth value space of path p. We suppose that X_P satisfies the condition as

for any p,q of P, if $p\leqq q$,then $(X_p, CL_p) \subseteq (X_q, CL_q)$.

3.Formal language L

Let us define the formal language L of MWL. Let C_v and C_c be two enumerable sets of symbols called a variable and an individual constant (or constant). First we define a term of MWL , according to the following rules:

① Every variables and constants are terms, ② If $t_1,\ldots,t_n$ are variable symbols or constant symbols, then $(t_1,\ldots,t_n)$ is a term,

③ Only those defined by a finite number of applications of the above rules are terms.

The variable which is not bounded any quantifier $\forall$, $\exists$ is called a free variable, and which is bounded is called a bounded variable. Let C_r be enumerable sets of symbols called a predicate symbol. Next we define the formula of MWL, according to the following rules:

Definition (formula)

① If $t_1,\ldots,t_n$ are variables or constants and α is a n-variables predicate symbol, where $n \geqq 0$, then $\alpha(t_1,\ldots,t_n)$ is a formula,

② If α, αi, where $i \in I$, and β are formulas, then $\neg\alpha$, $\alpha\rightarrow\beta$, $\bigwedge_{i\in I}\alpha_i$, $\bigvee_{i\in I}\alpha_i$, $\Box\alpha$, and $\Diamond\alpha$ are formulas,

③ If $\alpha(t)$ is a formula and x is a bounded variable which not occurs within $\alpha(t)$, then $\forall x\, \alpha(x)$ and $\exists x\, \alpha(x)$ are formulas,

④ Only those defined by a finite number of applications of the above rules are formulas.

The term which has no free variable is called closed term. And the formula which has no free variable is called sentence (or closed formula).

Multi-world model

Multi-world model M is a triple $<P,d,V',V>$. P is a set of paths. d is a function assigning individual domain to each path of P. V' is a set of the function V'_p which assigns the truth value reflected of observation to each atomic symbols. And, V is the set of the function V_p which assigns the truth value reflected of guess to each sentence. We discuss their in following sections.

Assignment function

Let X and Y be two sets, we denote a set of all the functions from X to Y as $(X\rightarrow Y)$. If f is an element of $(A\rightarrow B)$, then the function $f|_{A'}$, called restriction of f within A', is defined as

$$f|_{A'}: A' \rightarrow B,$$
$$f|_{A'}(x) = f(x) \quad \text{-if } x \in A \cap A',$$
$$f|_{A'}(x) = \perp \quad \text{-if } x \in A' - A.$$

If B is a family of sets, then let $\perp$ be an empty set ϕ. By $(A\rightarrow B)|_{A'}$, we denote $\{f|_{A'} | f: A\rightarrow B\}$.

Let V'_p be a partial function assigning some individual and truth value respectively to term and predicate symbol of L. V'_p is defined according to the following rules:

①If t is a variable or constant, then $V'_p(t) \in D_p$,

②If R is a n-variable predicate symbol, then $V'_p(R) \in (D_p{}^n \rightarrow 2^X)) | D_p{}^n$.

Let d be a function assigning a nonempty set d(p)to each path p of P. We denote d(p)as D_p, and call D_p as scope of path p. We suppose that there is the relation between paths of P as

for any p,q of P, if $p \leqq q$,then $D_p \subseteq D_q$.

Properties of V'_p

We denote the domain of g as dom(g). The partial order $\subseteq$ between function f and g is defined as

for any x of dom(g), if $g(x) \subseteq f(x)$, then $g \subseteq f$.

Now we assume that V'_p satisfies the condition as

for any path p and q of P, if $p \leqq q$, then $V'_p \subseteq V'_q$

Now we define V_p which is a partial function assigning some truth value in extended world to each term and predicate symbol of L. First we define the assignment of closed term as following rules:

①If t is a constant, then $V_p(t) = V'_p(t)$,
②If t is a closed term $(t_1,...,t_n)$, then $V_p(t) = \langle V_p(t_1),...,V_p(t_n)\rangle \in D_p{}^n$,

Let C_p be a smallest family which includes all the closed sets of X_p. Note we define X_p as the union of all truth values given to formula by V'_p. We define the relation R of the path p as

$R \in (D_p{}^n \rightarrow C_p) |_{D_p{}^n}$.

The relation R is the function assigning a closed set of C_p to each closed terms . We define the function V_p according to the following rules:

①If α is a n-variables predicate symbol β, then
$V_p(\alpha) \in (D_p{}^n \rightarrow C_p) |_{D_p{}^n}$,

②If α is β(t), then
$V_p(\alpha) = V_p(\beta)(V_p(t))$,

③If α is ¬β, then

$V_p(\alpha)= CL_p(X_p - V_p(\beta))$,

④If α is $\bigwedge_{i \in I} \beta_i$,then
$V_p(\alpha) = \bigcap_{i \in I} V_p(\beta_i)$,

⑤If α is $\bigvee_{i \in I} \beta_i$,then
$V_p(\alpha)= CL_p(\bigcup_{i \in I} V_p(\beta_i))$,

⑥If α is $\Diamond\beta$, then
$V_p(\alpha)= CL_q(\bigcup_{p \leqq q} V_q(\beta))$,

⑦If α is $\Box\beta$, then
$V_p(\alpha) = \bigcap_{p \leqq q} V_q(\beta)$,

⑧If α is $\beta \rightarrow \gamma$, then
$V_p(\alpha)= CL_p((X_p - V_p(\beta)) \cup V_p(\gamma))$,

⑨If α is $\forall x\ \beta(x)$, where β is the n-variable predicate , then
$V_p(\alpha) = \bigcap V_p(x) \in D_p{}^n\ V_p(\beta(a))$,

⑩If α is $\exists x\ \beta(x)$, where β is the n-variable predicate , then
$V_p(\alpha)= CL_p(\bigcup V_p(x) \in D_p{}^n\ V_p(\beta(a)))$

Let us assume that V'_p and V_p satisfy the following :
for any $\alpha \in C_r$, $V_p(\alpha(t)) = CL_p(V'_p(\alpha(t)))$.

True and False

" $M \models_p \alpha$($M \dashv_p \alpha$)", where α is a formula, M is a model of MWL, and p is a path of the model M, is used to express that the formula α is true(false)at the path p of the model M. We define $M \models_p \alpha$($M \dashv_p \alpha$)as the following;

$M \models_p \alpha$ iff $V_p(\alpha) = X_p$ at the path p of the model M,
$M \dashv_p \alpha$ iff $V_p(\alpha) = \phi$ at the path p of the model M.

" $M \models \alpha$($M \dashv \alpha$)" is used to express that α is true(false) at any path of model M, furthermore, "$\models \alpha(\dashv \alpha)$" is used to express that α is true(false) at any path of any model . In particular, if the formula α holds $\models \alpha(\dashv \alpha)$, then we call α as axiom (contradiction) of MWL.

Proposition

Let M be a model of MWL , p a path of M, and α a formula of L.

It is not the case that α hold both $M \models_p \alpha$ and $M \dashv_p \alpha$.

Accordingly, when $M \models_p \alpha$ and $M \dashv_p \alpha$ are both held, we say that the formula α is contradict.

Negation

In MWL, according to Definition of negation of V_p,

for any formula α, if $\dashv\alpha(\models\alpha)$,then $\models\neg\alpha(\dashv\neg\alpha)$

is held, but

for any formula α, if not $\models\alpha(\dashv\alpha)$,then $\models\neg\alpha(\dashv\neg\alpha)$

is not held. Since the truth value of formula is a closed set, thus

for any formula α, $\models\alpha$ or $\dashv\alpha$

is not held. Accordingly,

for any formula α, if not $\models\alpha(\dashv\alpha)$,then $\dashv\alpha(\models\alpha)$

is not held.

3. Inference

We now consider inference rules of MWL by Gentzen's formalization.

Definition:sequent

Let consider the formula which is provable logically . When $A_1,\dots,A_n,B_1,\dots,B_m$ are formulas, where n,m $\geqq 0$,then we say the expression of the form

$$A_1,\dots,A_n \Rightarrow B_1,\dots,B_m$$

is sequent. The sequent $A_1,\dots,A_n\Rightarrow B_1,\dots,B_m$ has the same interpretation as the formula $A_1\wedge\dots\wedge A_n\rightarrow B_1\vee\dots\vee B_m$. When the part of succedent (the part of $B_1,\dots,B_m$) is empty sequence as

$$A_1,\dots,A_n \Rightarrow$$

, then we say that contradiction is occurred from $A_1,\dots,A_n$. And when the part of antecedent (the part of $A_1,\dots,A_n$)is empty sequence as

$$\Rightarrow B_1,\dots,B_m$$

, then we say "any of $B_1,\dots,B_m$ is hold ". In particular, when the sequent is

$$\Rightarrow$$

,then we say that contradiction is occurred without any assumption . We defined the sequent $\Gamma\Rightarrow\Delta$ is interpreted as

$$V_p(\Gamma)\subseteq V_p(\Delta),$$

Furthermore, we define $V_p(\Gamma)$ and $V_p(\Delta)$ as $\cap_{A\in\Gamma}V_p(A)$ and $CL_p(\cup_{B\in\Delta}V_p(B))$. In particular, when Γ and Δ are empty sequences, we define $V_p(\Gamma)$ and $V_p(\Delta)$ as X_p and ϕ.

We define the sequent of MWL as the sequent which appears at most one formula in the part of antecedent.

The truth value given to each formula by V_p implies the truth value given by V'_p. It means that interpretation of the formula in MWL may be vague against real world. On the other hand, the part of antecedent expresses assumption of the sequent , thus to appear several formulas in the part of antecedent effects as increasing vagueness of assumption. This is the reason why in MWL the sequent is restricted as before. For example,the sequent $\neg\alpha\wedge\alpha$ is not always false in MWL .

Inference rules of MWL

Now we consider the inference rules of MWL. Let α, β be formulas, and Δ, Δ_i ($i\in I$), Π are sequences of zero or more formulas . In particular, let Γ be a sequence of at most one formula. We say formula α, β appears in the upper sequent of inference rules as sub-formula . And we say formulas which contain sub-formula α, β and appears in the lower sequent of inference rules as chief-formula. The whole difference between MWL's inference rules and LK (classical predicate logic)'s inference rules is that there is at most one formula in the part of ancident of sequent in MWL's inference rules. Now inference rules of MWL show as the following:

w⇒ $\dfrac{\Rightarrow\Delta}{\alpha\Rightarrow\Delta.}$ ⇒w $\dfrac{\Gamma\Rightarrow\Delta}{\Gamma\Rightarrow\alpha,\Delta.}$

e⇒ — ⇒e $\dfrac{\Gamma\Rightarrow\Delta,\alpha,\beta,\Pi}{\Gamma\Rightarrow\Delta,\beta,\alpha,\Pi.}$

c⇒ — ⇒c $\dfrac{\Gamma\Rightarrow\Delta,\alpha,\alpha,\Pi}{\Gamma\Rightarrow\Delta,\alpha,\Pi.}$

syllogism $\dfrac{\Gamma\Rightarrow\Delta i,\alpha\ (i\in I)\quad \alpha\Rightarrow\Pi}{\Gamma\Rightarrow\ldots,\Delta i,\ldots,\Pi.}$

¬⇒ $\dfrac{\Rightarrow\Delta,\alpha}{}$ ⇒¬ $\dfrac{\alpha\Rightarrow\Delta}{}$

$\neg\alpha\Rightarrow\Delta.$ $\qquad\qquad$ $\Rightarrow\Delta, \neg\alpha.$

$\wedge\Rightarrow$ $\quad \dfrac{\alpha\Rightarrow\Delta}{\alpha\wedge\beta\Rightarrow\Delta},$ and $\dfrac{\beta\Rightarrow\Delta}{\beta\wedge\alpha\Rightarrow\Delta}.$ $\qquad \Rightarrow\wedge$ $\quad \dfrac{\Gamma\Rightarrow\Delta, \alpha_i\ (i\in I)}{\Rightarrow\Delta, \bigwedge_{i\in I}\alpha_i}.$

$\vee\Rightarrow$ $\quad \dfrac{\alpha_i\Rightarrow\Delta\ (i\in I)}{\bigvee_{i\in I}\alpha_i\Rightarrow\Delta}.$ $\qquad \Rightarrow\vee$ $\quad \dfrac{\Gamma\Rightarrow\Delta, \alpha}{\Gamma\Rightarrow\Delta, \alpha\vee\beta},$ and $\dfrac{\Gamma\Rightarrow\Delta, \beta}{\Gamma\Rightarrow\Delta, \alpha\vee\beta}.$

$\rightarrow\Rightarrow$ $\quad \dfrac{\Rightarrow\Delta, \alpha \quad \beta\Rightarrow\Lambda}{\alpha\rightarrow\beta\Rightarrow\Delta, \Lambda}.$ $\qquad \Rightarrow\rightarrow$ $\quad \dfrac{\alpha\Rightarrow\Delta, \beta}{\Rightarrow\Delta, \alpha\rightarrow\beta}.$

$\forall\Rightarrow$ $\quad \dfrac{\alpha(t)\Rightarrow\Delta}{\forall x\, \alpha(x)\Rightarrow\Delta},$ where t is an arbitrary term. $\qquad \Rightarrow\forall$ $\quad \dfrac{\Gamma\Rightarrow\Delta, \alpha(a)}{\Gamma\Rightarrow\Delta, \forall x\, \alpha(x)},$ where the free variable a does not occur in the lower sequent.

$\exists\Rightarrow$ $\quad \dfrac{\alpha(a)\Rightarrow\Delta}{\exists x\, \alpha(x)\Rightarrow\Delta},$ where the free variable a does not occurring the lower sequent. $\qquad \Rightarrow\exists$ $\quad \dfrac{\Gamma\Rightarrow\Delta, \alpha(t)}{\Gamma\Rightarrow\Delta, \exists x\, \alpha(x)},$ where t is an arbitrary term.

$\Box\Rightarrow$ $\quad \dfrac{\alpha\Rightarrow\Delta}{\Box\alpha\Rightarrow\Delta}.$ $\qquad \Rightarrow\Box$ $\quad \dfrac{\Box\Gamma\Rightarrow\alpha}{\Box\Gamma\Rightarrow\Box\alpha}.$

$\Diamond\Rightarrow$ $\quad \dfrac{\alpha\Rightarrow\Diamond\Delta}{\Diamond\alpha\Rightarrow\Diamond\Delta}.$ $\qquad \Rightarrow\Diamond$ $\quad \dfrac{\Gamma\Rightarrow\Delta, \alpha}{\Gamma\Rightarrow\Delta, \Diamond\alpha}.$

Theorem :Normal form theorem of MWL

If the sequent S is provable in MWL, then there is a proof P' of S without syllogism .

(Proof)omit

The whole difference between MWL's inference rules and LK's inference rules is that there is at most one formula in the part of ancident of sequent in MWL's inference rules. Hence the proof of normal form theorem of MWL is the proof of normal form theorem of LK under the restriction on the sequent in the proof, restriction which there is at most one formula in the part of ancident of each sequent.

Theorem :Consistency of MWL

MWL is consistent.

(Proof)

MWL is consistent , if and only if the sequent $\Rightarrow$ is not provable in MWL. By normal form theorem of MWL, if the sequent $\Rightarrow$ is provable in MWL, then there is a proof of the sequent $\Rightarrow$ without using syllogism . The inference rule whose lower sequent does not contains the chief-formula is only syllogism. Thus, there is no proof of the sequent $\Rightarrow$ without using syllogism. Consequently, the sequent $\Rightarrow$ is not provable in MWL.

Theorem :Soundness of MWL

If the sequent S is provable,then $\models$S .

(Proof)omit

Theorem

Let α and β be an arbitrary formula of MWL.

① $\neg\alpha\wedge\alpha\Rightarrow$ is not provable in MWL,
② $\Rightarrow\neg\alpha\vee\alpha$ is provable in MWL,
③ $\neg\neg\alpha\Rightarrow\alpha$ is provable in MWL,
④ $\alpha\Rightarrow\neg\neg\alpha$ is not provable in MWL,
⑤ $\alpha\wedge\beta\Rightarrow\alpha$(or $\beta\wedge\alpha\Rightarrow\alpha$) is provable in MWL,
⑥ $\alpha\Rightarrow\alpha\vee\beta$(or $\alpha\Rightarrow\beta\vee\alpha$) is provable in MWL,
⑦ $\neg\alpha\vee\neg\beta\Leftrightarrow\neg(\alpha\wedge\beta)$ is provable in MWL,
⑧ $\alpha\rightarrow\beta\Rightarrow\neg\alpha\vee\beta$ is provable in MWL,
⑨ $\neg\alpha\vee\beta\Rightarrow\alpha\rightarrow\beta$ is not provable in MWL,
⑩ $\alpha\Rightarrow\beta\rightarrow\alpha$ is not provable in MWL,

Let α be an arbitrary formula, and t a arbitrary term.

①' $\forall x\ \alpha(x)\Rightarrow\exists x\ \alpha(x)$ is provable in MWL, ②' $\forall x\ \neg\alpha(x) \Rightarrow \neg\exists x\ \alpha(x)$ is not provable in MWL,

③' $\neg\exists x\ \alpha(x) \Rightarrow \forall x\ \neg\alpha(x)$ is provable in MWL,

④' $\neg\forall x\ \alpha(x) \Rightarrow \exists x\ \neg\alpha(x)$ is provable in MWL,

⑤' $\exists x\ \neg\alpha(x) \Rightarrow \neg\forall x\ \alpha(x)$ is provable in MWL,

⑥' $\neg\forall x\ \neg\alpha(x) \Rightarrow \exists x\ \alpha(x)$ is provable in MWL,

⑥" $\exists x\ \alpha(x) \Rightarrow \neg\forall x\ \neg\alpha(x)$ is not provable in MWL,

⑦' $\neg\exists x\ \neg\alpha(x) \Rightarrow \forall x\ \alpha(x)$ is provable in MWL,

⑦" $\forall x\ \alpha(x) \Rightarrow \neg\exists x\ \neg\alpha(x)$ is not provable in MWL,

⑧' $\forall x\ \alpha(x) \Rightarrow \alpha(t)$ is provable in MWL,

⑧" $\alpha(t) \Rightarrow \exists x\ \alpha(x)$ is provable in MWL,

(Proof)

We consider proofs of several theorem .

①Let suppose that the sequent $\neg\alpha\wedge\alpha\Rightarrow$ is provable in MWL. Let P be the proof of $\neg\alpha\wedge\alpha\Rightarrow$ without using syllogism and let I be the last inference rule of P. Since the lower sequent of I is $\neg\alpha\wedge\alpha\Rightarrow$, I must be one of the following cases :

① $\dfrac{\Rightarrow}{\neg\alpha\wedge\alpha\Rightarrow}$, ② $\dfrac{\neg\alpha\Rightarrow}{\neg\alpha\wedge\alpha\Rightarrow}$, ③ $\dfrac{\alpha\Rightarrow}{\neg\alpha\wedge\alpha\Rightarrow}$.

Case①:If I is ①, then the sequent $\Rightarrow$ is provable. Thus, this case conflicts with consistency of MWL.

Case②, ③:The upper sequent of the inference rule, of which lower sequent is $\neg\alpha\Rightarrow$,is either $\Rightarrow$ or $\Rightarrow\alpha$. On the other hand, at least any sequent of MWL satisfies that the logical symbol occurs in it or antecedent and succedent are not empty sequence of formula. Axiom $\alpha\Rightarrow\alpha$ satisfies this condition. For any inference rule except syllogism, if the upper sequent satisfies this condition, then the lower sequent satisfies. Thus, any provable sequent of MWL satisfies this condition. Thus, the sequent $\Rightarrow\alpha$ and $\alpha\Rightarrow$ are not provable in MWL. Consequently, $\neg\alpha\wedge\alpha\Rightarrow$ is not provable in MWL.

① $\forall x\ \neg\alpha(x) \Rightarrow \neg\exists x\ \alpha(x)$ is not provable in MWL.

Suppose that the sequent $\forall x\ \neg\alpha(x) \Rightarrow \neg\exists x\ \alpha(x)$ is provable in MWL ,and we denote its proof without using syllogism as P.

(A) The formula $\neg\exists x\ \alpha(x)$ does not appear in the antecedent of any sequent of P. Thus, the succedent of the initial sequent is not $\neg\exists x\ \alpha(x)$, since initial sequent is the expression of the form $\Gamma\Rightarrow\Gamma$. Thus, there is a inference rule I_1 of which the succedent of the upper sequent is not $\neg\exists x\ \alpha(x)$, and the succedent of the lower sequent is $\neg\exists x\ \alpha(x)$. Such inference rule I_1 is $\Rightarrow\neg$ or $\Rightarrow$w .

(B) If I_1 is $\Rightarrow\neg$,and the antecedent of the upper sequent of I_1 is $\exists x\ \alpha(x)$, then there is a inference rule I_2 of which the succedent of the upper sequent is not $\exists x\ \alpha(x)$, and the succedent of the lower sequent contains $\exists x\ \alpha(x)$. Such inference rule I_2 is $\exists\Rightarrow$ or $w\Rightarrow$.

(C) If I_2 is $\exists\Rightarrow$,then the antecedent of the upper sequent of I_2 contains $\alpha(a)$. Thus, there is a inference rule I_3 of which the antecedent of the lower sequent contains the formula $\alpha(a)$. Such inference rule I_3 is $w\Rightarrow$.

(D) If the initial sequent is $\alpha(a)\Rightarrow\alpha(a)$, then there is a inference rule I_4 of which the antecedent of the lower sequent is not $\alpha(a)$, and the antecedent of the upper sequent is $\alpha(a)$. Such inference rule I_4 is $\exists\Rightarrow$, and I_4 appears upper than I_2. Thus, the antecedent of the lower sequent of I_4 contains the formula $\alpha(a)$. But, it does not satisfies the condition that the lower sequent of $\exists\Rightarrow$does not contain free variable a.

(E) If I_1,I_2,and I_3 are the weaken rule (i.e. $w\Rightarrow$ or $\Rightarrow w$), then ,since the sequent $\forall x\ \neg\alpha(x)$ is provable, it conflicts. Consequently, the sequent $\forall x\ \neg\alpha(x)\Rightarrow\neg\exists x\ \alpha(x)$ is not provable in MWL.

Conclusion

We discuss our inference that can infer facts which consist in total world from facts which consist in partial world, and , in this paper, we formalize it as the inference of MWL.

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