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Chaotic Neural Networks

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Abstract

A neural network model composed of neurons with chaotic dynamics is proposed by considering some properties of real neurons. The model possesses not only complex dynamics with abundant spatio-temporal chaotic patterns implying applicability to neurocomputing but also simplicity enough to be easily implemented in an electronic curcuit.

1 INTRODUCTION

It is nowadays well recognized that chaotic phenomena are ubiquitous in many fields[1]. It is also reported in the field of neuroscience that there exists chaotic dynamics not only in neurons[2-5] but also in neural networks and brains[6-7]. Moreover, possible roles of chaos are discussed from the viewpoint of biological information processing[6,8-10].

In order to clarify significance of the chaos in neural information processing, it is an important approach to analyse dynamical characteristics of artificial neural networks composed of neurons with chaotic dynamics theoretically. We have proposed a simple mathematical model[11-12] of "chaotic neurons" from this viewpoint. In this paper, we review our framework of chaotic neural networks[12] and demonstrate the network dynamics.

2 CHAOTIC DYNAMICS IN REAL NERVE MEMBRANES

It has been clarified experimentally with squid giant axons that real nerve membranes in the resting state respond to stimulation of periodic pulses not only synchronously but also chaotically according to the values of amplitude and period of the stimulating pulses[5]. Fig. 1(a) is an example of the chaotic response in squid giant axons. The response characteristics of squid giant axons can be described quantitatively with the Hodgkin-Huxley equations[13] and qualitatively with the FitzHugh-Nagumo equations[14-15]. Fig. 1(b) and (c) show the corresponding chaotic responses of the nerve equations. Moreover, approximately 1-dimensional return mappings have been obtained with stroboscopically plotting of the chaotic responses as shown in Fig. 2.

3 MODELING CHAOTIC RESPONSES

The Hodgkin-Huxley equations and the FitzHugh-Nagumo equations are too complicated for analyses of artificial neurocomputing. In this section we explain a simple neuron model which can reproduce the chaotic responses of real nerve membranes qualitatively[12].

In 1971, Nagumo and Sato proposed an interesting neuron model[16] based upon the Caianiello's neuronic equation[17]. They assumed that the influence of the refractoriness due to a past firing decreases exponentially with time[16]. Eq.(1) shows the Nagumo-Sato model[16]:

$$x(t+1) = u(A(t) - \alpha \sum_{d=0}^{t} k^d x(t-d) - \theta) \tag{1}$$

where

$x(t+1)$: the output of the neuron at the descrete time $t+1$ which takes either 1 (firing) or 0 (non-firing),

u : the unit step function such that $u(y) = 1$ (for $y \geq 0$) and $= 0$ (for $y < 0$),

$A(t)$: the strength of the input at the discrete time t,

α : a positive parameter,

k : the damping factor of the refractoriness which takes a value between 0 and 1,

θ : the threshold for the all-or-none firing of the neuron.

By defining a new variable $y(t+1)$ corresponding to the internal state of the neuron as follows

$$y(t+1) = A(t) - \alpha \sum_{d=0}^{t} k^d x(t-d) - \theta, \tag{2}$$

eq. (1) can be simplified as eqs. (3) and (4)[16]:

$$y(t+1) = ky(t) - \alpha u(y(t)) + a(t) \tag{3}$$

$$x(t+1) = u(y(t+1)) \tag{4}$$

where

$$a(t) = A(t) - kA(t-1) - \theta(1-k). \tag{5}$$

In particular, when the input stimulation is composed of periodic pulses with the constant amplitude A, $a(t)$ of eq. (5) is temporally constant as follows

$$a = (A-\theta)(1-k). \tag{6}$$

Responses of eqs. (3) and (4) have been analysed in detail and clarified that almost all the responses of eqs. (3) and (4) are periodic, forming complete devil's staircases[16,18,19]; that is, the equations have chaotic solutions only at a self-similar Cantor set of the values of the bifurcation parameter a with zero Lebesgue measure. Fig. 3(a) shows an example of the response characteristic with changing the value of the bifurcation parameter a where the average firing rate, or the excitation number ρ, is defined as follows:

$$\rho = \lim_{n \to +\infty} \frac{1}{n} \sum_{t=0}^{n-1} x(t) \tag{7}$$

Although almost all the solutions of eqs. (3) and (4) are periodic, the chaotic responses of real giant axons of squid can be easily observed with the experiment that the nerve membrane is stimulated by periodic pulses with the constant amplitude[5] as demonstrated in Figs. 1 and 2. This desagreement between the model and the experiment requires a modification of eq. (1).

Physiological experiments on responses of nerve membranes to current stimulation are usually conducted under a space-clamp condition. The process of generating action potentials by a single pulse current does not obey the so-called all-or-none law under the space-clamp condition[14,20]. In other words, the stimulus-response property of the nerve membrane is described not by an discontinuous step function such as the function u in eq. (1) but by a continuously increasing function[14,20]. Moreover, the actual situation that action potentials are triggered at a limited portion of a real neuron, or an axon hillock is similar to the space-clamp condition. Accordingly we replace the unit step function u in eq. (1) by a continuous function f as follows

$$x(t+1) = f(A(t) - \alpha \sum_{d=0}^{t} k^d g(x(t-d)) - \theta) \tag{8}$$

where

$x(t+1)$: the output of the neuron, or a graded action potential generated at the time $t+1$, which takes an analog value between 0 and 1,

f : a continuous output function, which is the logistic function $f(y) = 1/(1 + \exp(-y/\varepsilon))$ with the steepness parameter ε in this paper,

g : a function describing the relationship between the analog output and the magnitude of the refractoriness to the following stimulation. The function g

is kept to be the identity function $g(x) = x$ for the sake of simplicity in this paper.

As is the case with the Nagumo-Sato model, defining the internal state $y(t+1)$ by

$$y(t+1) = A(t) - \alpha \sum_{d=0}^{t} k^d g(x(t-d)) - \theta \tag{9}$$

reduces eq. (8) to the following eqs. (10) and (11)[12],

$$y(t+1) = ky(t) - \alpha g(f(y(t))) + a \tag{10}$$

$$x(t+1) = f(y(t+1)). \tag{11}$$

Fig. 4 shows examples of periodic and chaotic solutions to eq. (10) with the graphs. Fig. 5 shows the response characteristics of eqs. (10) and (11) with the bifurcation parameter a. The excitation number ρ is defined here as follows

$$\rho = \lim_{n \to +\infty} \frac{1}{n} \sum_{t=0}^{n-1} h(x(t)) \tag{12}$$

where h is a function which describes waveform-shaping dynamics of the axon with a strict threshold for propagating action potentials and assumed to be $h(x) = 1$ (for $x \geq 0.5$) and $= 0$ (for $x < 0.5$). It should be noted that unlike the space-clamp condition, an all-or-none law holds for the propagation of action potentials along the axon if the length of the axon is sufficiently long[14,20-21]. The response characteristics in Fig. 5 qualitatively reproduce alternating periodic-chaotic sequences of responses experimentally observed in squid giant axons[4-5]. Fig. 6 shows classification of solutions to eq. (10) in the parameter space $a \times \varepsilon$. Shaded regions in Fig. 6 correspond to chaotic solutions.

4 A MODEL OF CHAOTIC NEURAL NETWORKS

The neuron model with chaotic dynamics explained above can be generalized as an element of neural networks which we call "chaotic neural networks"[12]. Generally speaking, we need to consider two kinds of inputs, namely feedback inputs from component neurons such as Hopfield networks[22] and externally applied inputs such as back-propagation networks[23], in order to design arbitrary architectures of artificial neural networks.

The dynamics of the ith chaotic neuron in a neural network composed of M chaotic neurons can be modeled as eq. (13)[24].

$$x_i(t+1) = f_i\Big(\sum_{j=1}^{M} V_{ij} \sum_{d=0}^{t} k_e^d A_j(t-d)$$

$$+ \sum_{j=1}^{N} W_{ij} \sum_{d=0}^{t} k_f^d h_j(x_j(t-d)) - \alpha \sum_{d=0}^{t} k_r^d g_i(x_i(t-d)) - \theta_i\Big) \tag{13}$$

where

$x_i(t+1)$: the output of the ith chaotic neuron at the discrete time t+1,

f_i : the continuous output function of the ith chaotic neuron,

M : the number of the externally applied inputs,

V_{ij} : the connection weight from the jth externally applied input to the ith chaotic neuron,

$A_j(t-d)$: the strength of the jth externally applied input at the time $t-d$,

N : the number of the chaotic neurons in the network,

W_{ij} : the connection weight from the jth chaotic neuron to the ith chaotic neuron,

h_j : the transfer function of the axon for the propagating action potentials in the jth chaotic neuron,

g_i : the refractory function of the ith chaotic neuron.

k_e, k_f and k_r are the decay parameters for the external inputs, the feedback inputs and the refractoriness, respectively.

Eq. (13) is the neuron model with the following three properties: (1) the continuous output function, (2) the relative refractoriness and (3) the spatio-temporal summation of both external inputs and feedback inputs.

We can deal with eq. (13) in a reduced form by letting the terms in the parentheses of function f_i be $\xi_i(t+1)+\eta_i(t+1)+\zeta_i(t+1)$ similar to the previous section as follows:

$$\xi_i(t+1) = \sum_{j=1}^{M} V_{ij}A_j(t) + k_e\xi_i(t) \tag{14}$$

$$\eta_i(t+1) = \sum_{j=1}^{N} W_{ij}h_j(x_j(t)) + k_f\eta_i(t) \tag{15}$$

$$\zeta_i(t+1) = -\alpha g_i(x_i(t)) + k_r\zeta_i(t) - \theta_i(1-k_r) \tag{16}$$

$$x_i(t+1) = f_i(\xi_i(t+1) + \eta_i(t+1) + \zeta_i(t+1)) \tag{17}$$

where ξ_i, η_i and ζ_i are defined as

$$\xi_i(t+1) = \sum_{j=1}^{M} V_{ij} \sum_{d=0}^{t} k_e^d A_j(t-d) \tag{18}$$

$$\eta_i(t+1) = \sum_{j=1}^{N} W_{ij} \sum_{d=0}^{t} k_f^d h_j(x_j(t-d)) \tag{19}$$

$$\zeta_i(t+1) = -\alpha \sum_{d=0}^{t} k_r^d g_i(x_i(t-d)) - \theta_i. \tag{20}$$

Equations (14)-(17) represent some of discrete-time neural network models, such as the McCulloch-Pitts model[25)] and the back-propagation network[23)]; i.e. our modeling of chaotic neurons is a natural extension of the former models for producing chaotic dynamics and is easy to adjust to these neuron models.

When $k_e = k_f = k_r \equiv k$, eqs. (14)-(17) are simplified to eqs. (21) and (22)[12)]

$$y_i(t+1) = ky_i(t)+\sum_{j=1}^{M} V_{ij}A_j(t)+\sum_{j=1}^{N} W_{ij}h_j(f_j(y_j(t)))-\alpha g_i(f_i(y_i(t)))-\theta_i(1-k) \tag{21}$$

$$x_i(t+1) = f_i(y_i(t+1)) \tag{22}$$

where $y_i(t+1) = \xi_i(t+1) + \eta_i(t+1) + \zeta_i(t+1)$.

Examples of dynamical behavior in simple chaotic neural networks are shown in Fig. 7 and 8 where all of g_i's and h_i's are assumed to be the identity functions.

Fig. 7 demonstrates a chaotic spatio-temporal pattern with positive Lyapunov exponents. Fig. 8 shows the dynamical behavior of the chaotic neural network composed of 100 neurons with feedback interconnections corresponding to superposed autocorrelation matrixes of the four patterns shown in Fig. 8(a)[24]. When the mutual interactions are stronger than the refractory effect, the network dynamics is similar to content addressable memory[22] as shown in Fig. 8(b). On the other hand, when the mutual interactions are weaker than the refractory effect, the network produces chaotic temporal sequences of patterns stored by the autocorrelation matrixes in advance as shown in Fig. 8(c) because the network can't stay around any equilibrium states due to the accumulating refractoriness. Similar memory dynamics has been reported in a neural network composed of stochastic neurons[10].

5 DISCUSSION

We have proposed a neural network model composed of the neurons with chaotic dynamics[12,24]. The neurons have the properties of the continuous output function, the relative refractoriness and the spatio-temporal summation of fan inputs. Since the chaotic neuron model is simple, it can be easily implemented by an electronic circuit[26]. Fig. 9 shows examples of strange attractors in the chaotic neural network electronically implemented.

Although it is still an open problem to explore applicability of chaotic dynamics in neurocomputing, our framework of the chaotic neural networks at least makes it possible to introduce functions of the deterministic chaos into artificial neural networks whenever necessary.

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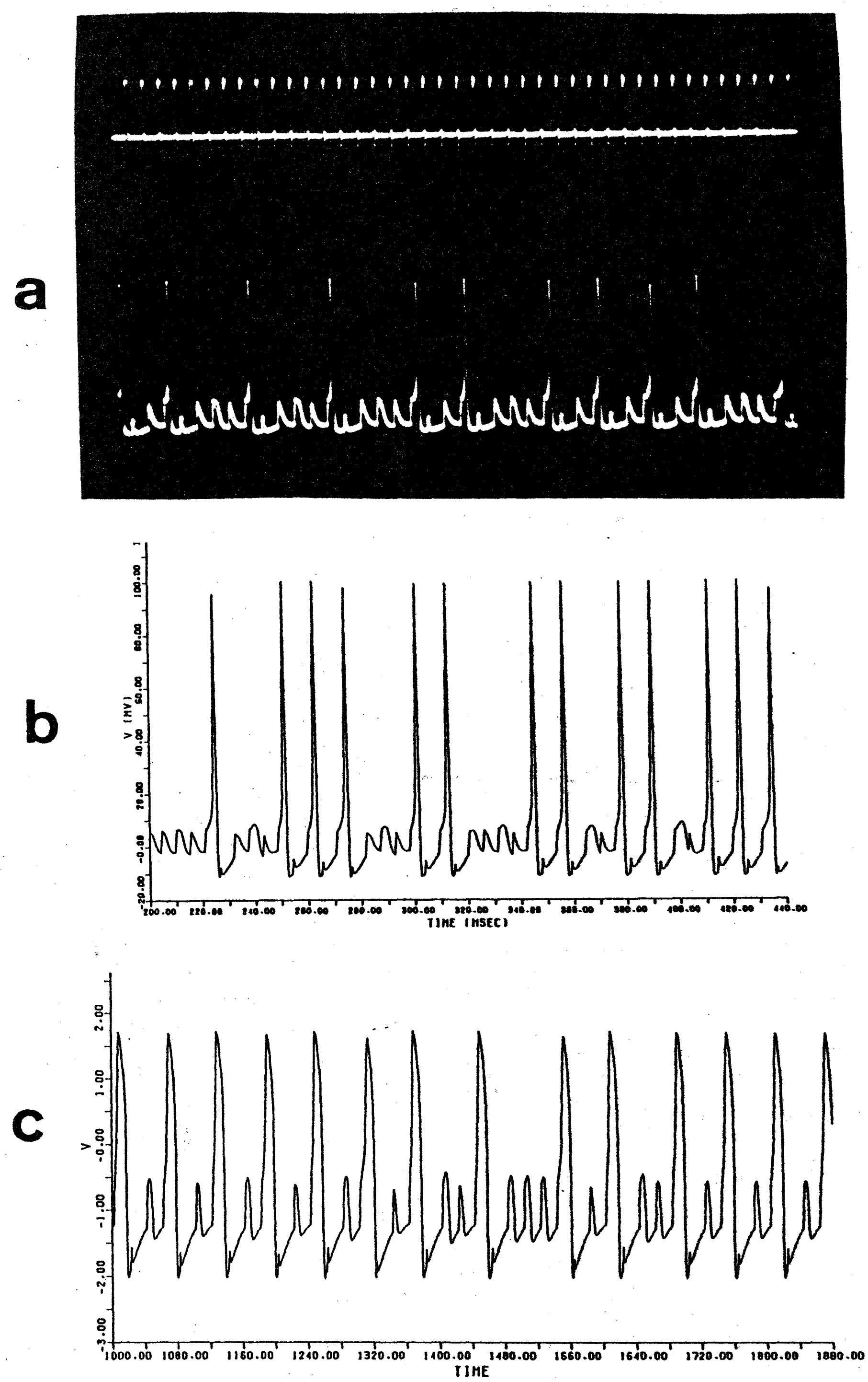


Fig. 1

Chaotic responses in a single neuron. (a) Squid giant axon, (b) the Hodgkin-Huxley eqs. and (c) the FitzHugh-Nagumo eq.

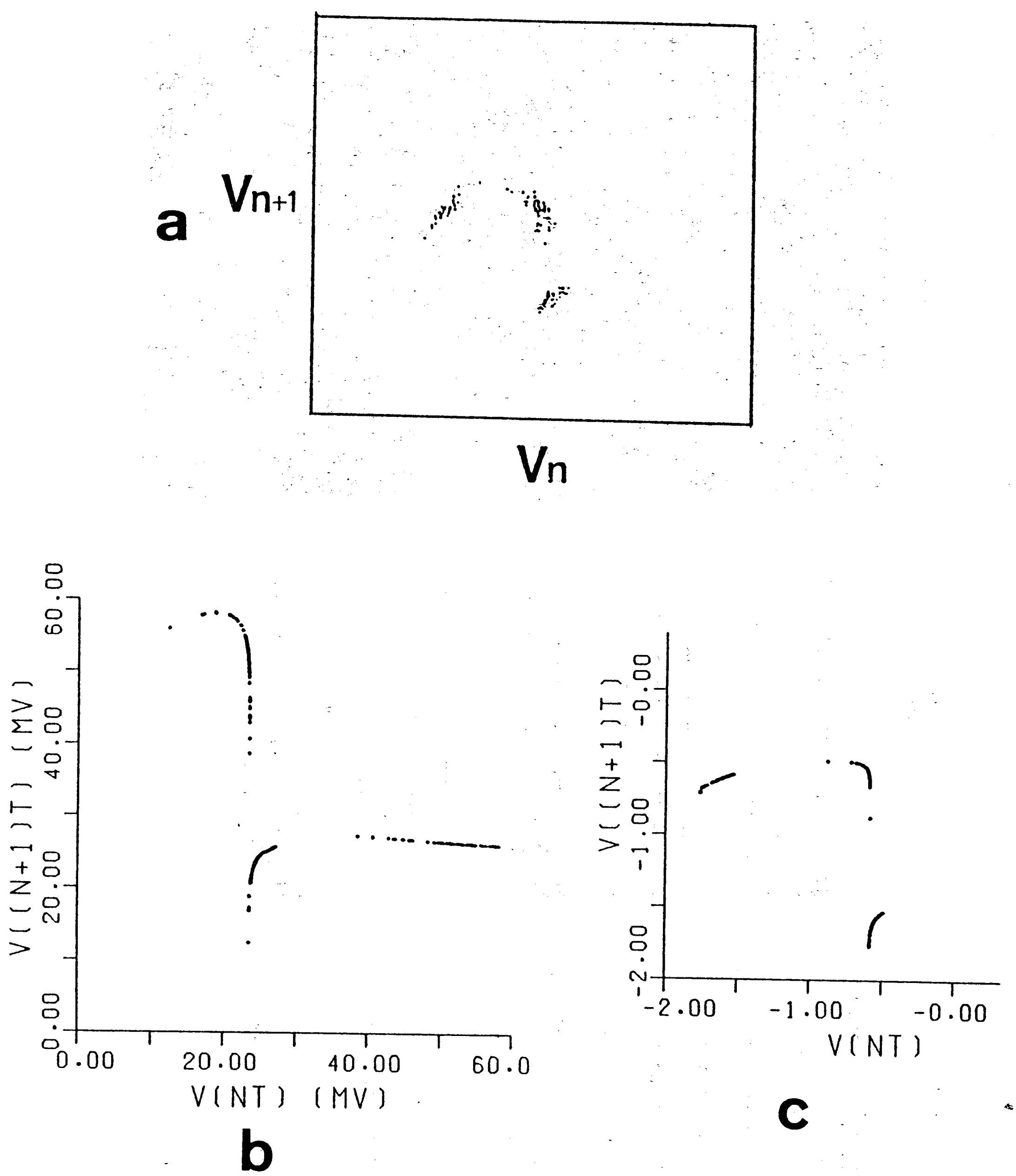


Fig. 2

Approximately 1-dimensional return mappings by stroboscopically plotting of the chaotic responses. (a) Squid giant axon, (b) the Hodgkin-Huxley eqs. and (c) the FitzHugh-Nagumo eqs.

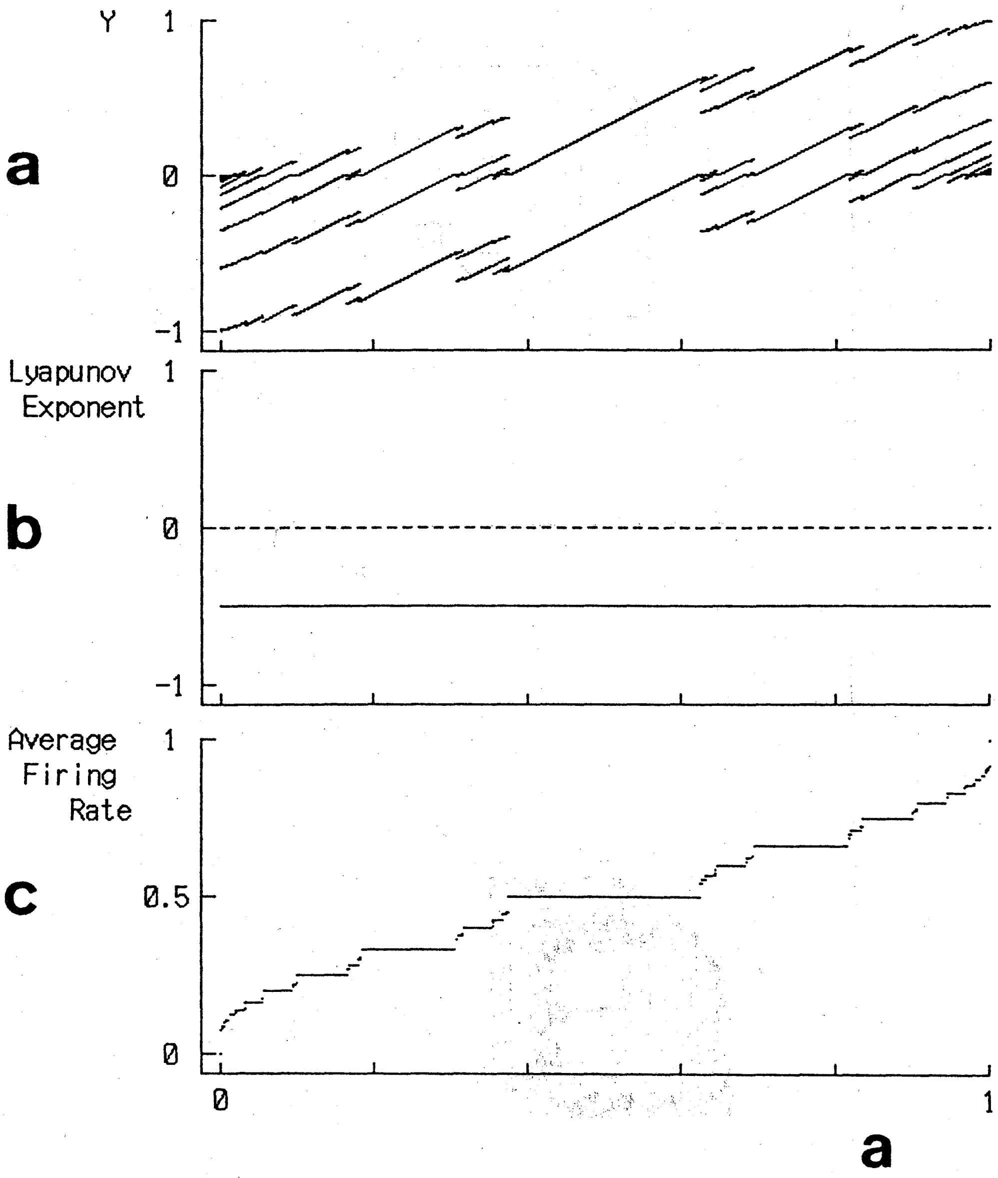
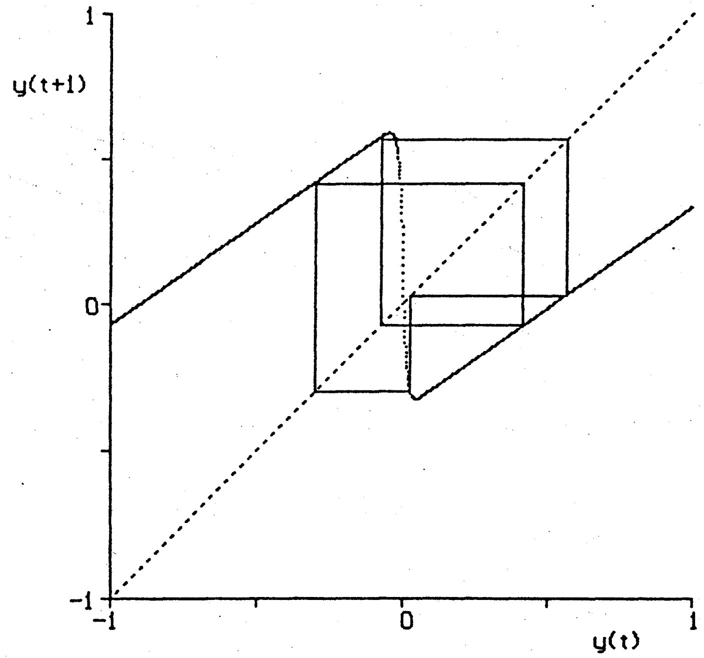


Fig. 3

Response characteristics of eqs. (3) and (4) with the bifurcation parameter a of eq. (6) where $k = 0.6$, $\alpha = 1.0$ and $y(0) = 0.1$: (a) the bifurcation diagram, (b) the Lyapunov exponent λ and (c) the average firing rate ρ.

a



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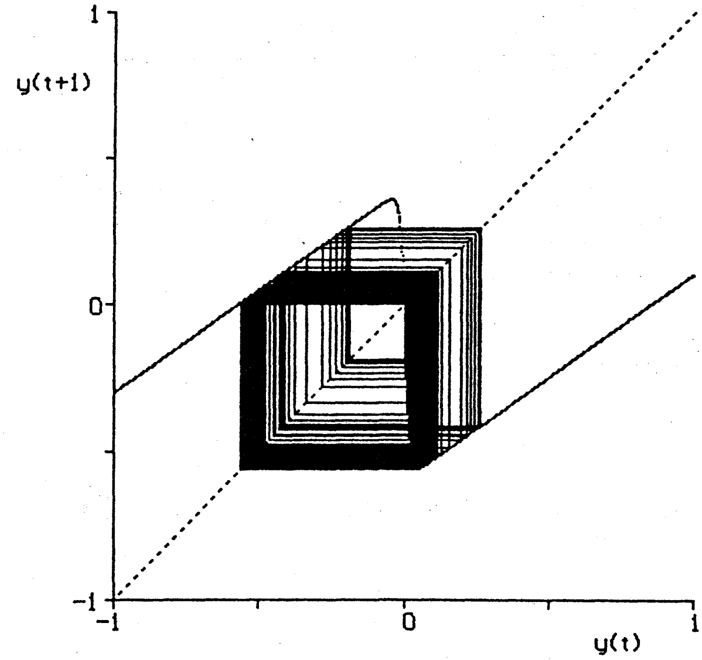

Fig. 4

(a) Periodic and (b) chaotic solutions to eq. (10) with the graphs, where $k = 0.7$, $a = 0.6288$ and $\varepsilon = 0.01$ in (a) and $k = 0.7$, $a = 0.3968$ and $\varepsilon = 0.01$ in (b).

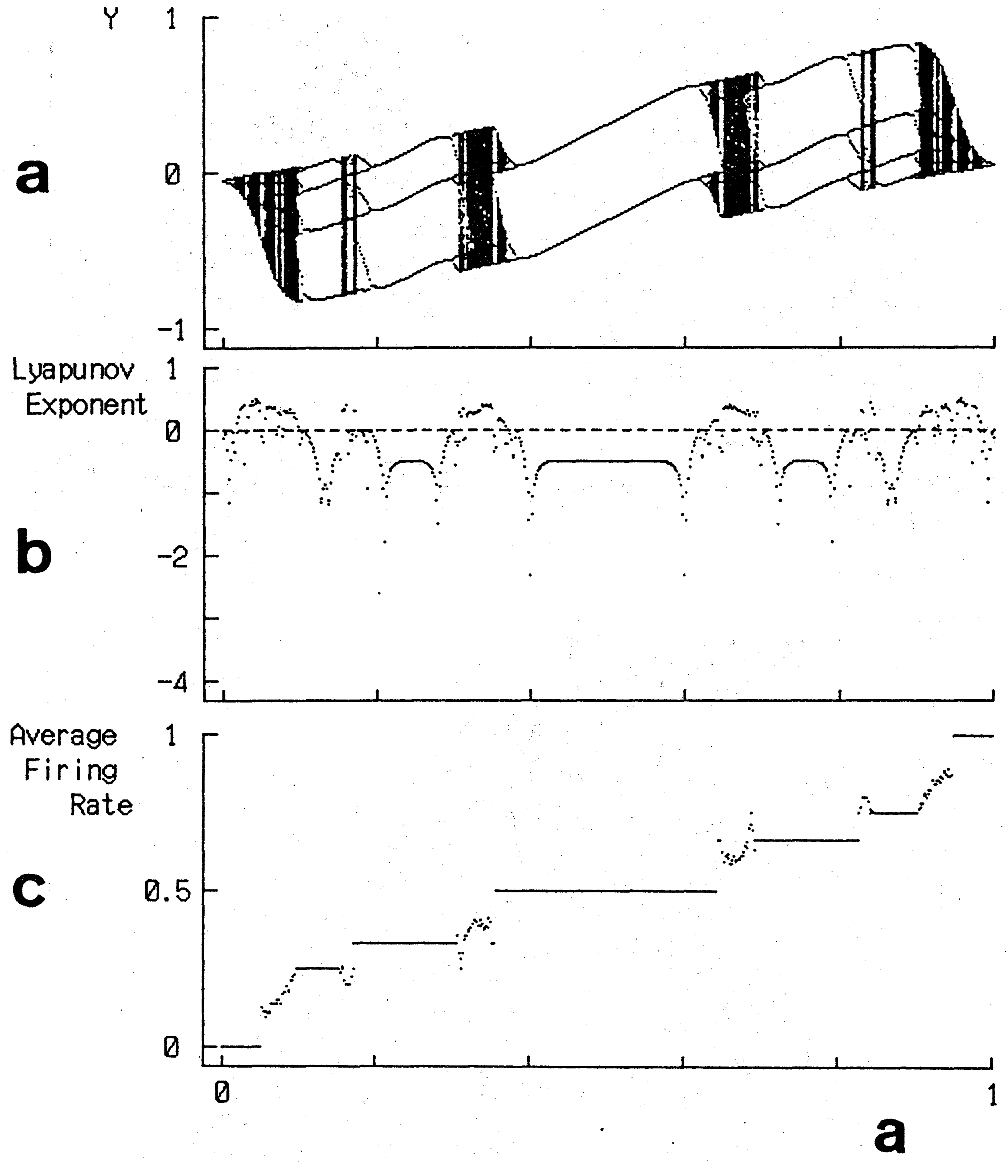

Fig. 5

Response characteristics of eqs. (10) and (11) with the bifurcation parameter a where $k = 0.6$, $\alpha = 1.0$, $f(y) = 1/(1 + \exp(-y/0.015))$, $y(0) = 0.1$ and g is the identity function. (a) the bifurcation diagram, (b) the Lyapunov exponent λ and (c) the average firing rate ρ.

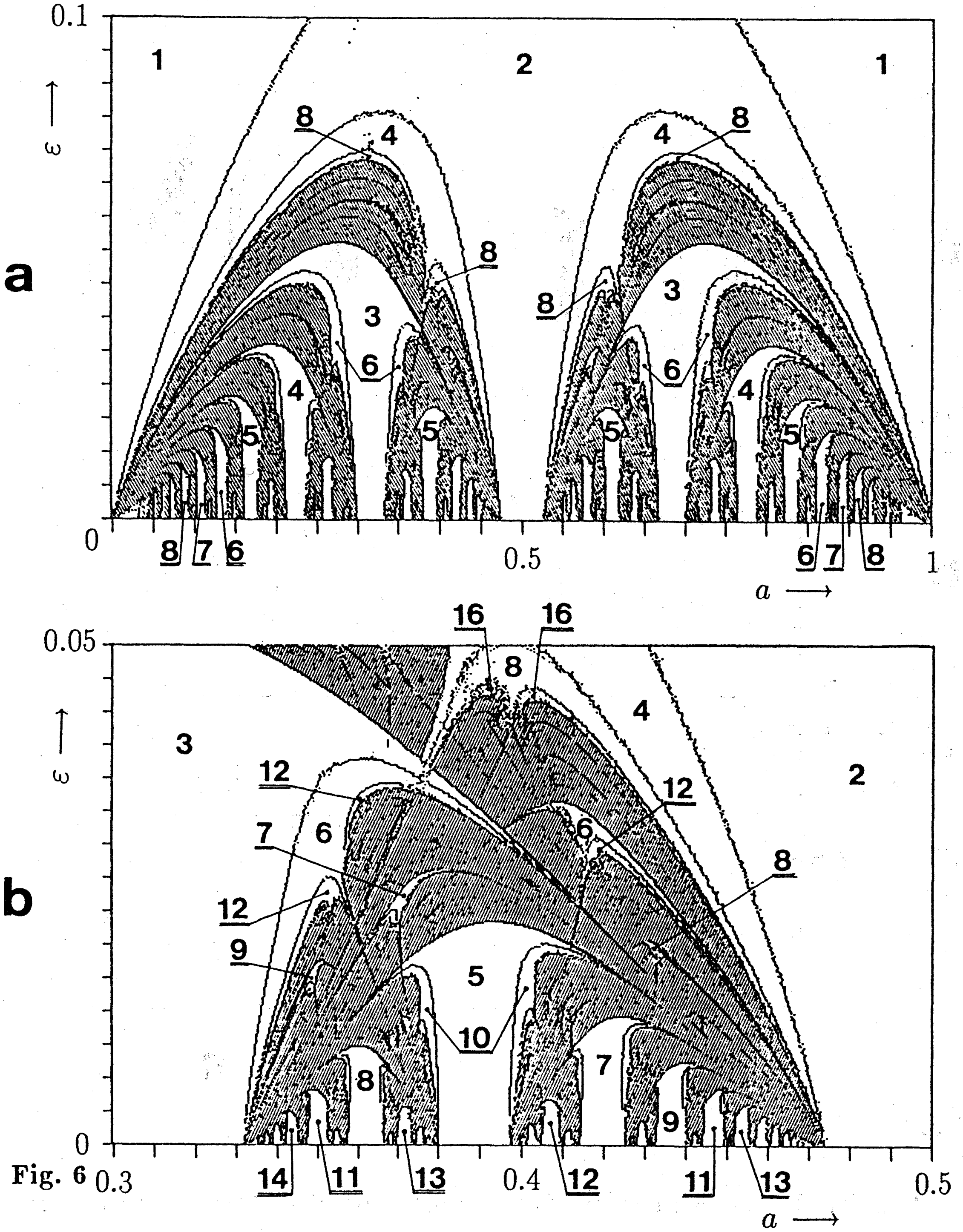

Fig. 6 Classification of solutions to eq. (10) in the parameter space $a \times \varepsilon$. Fig. 6(b) is an enlargement of a part of Fig. 6(a). While each number k designates a region of periodic solutions with the period k, shaded regions correspond to chaotic solutions.

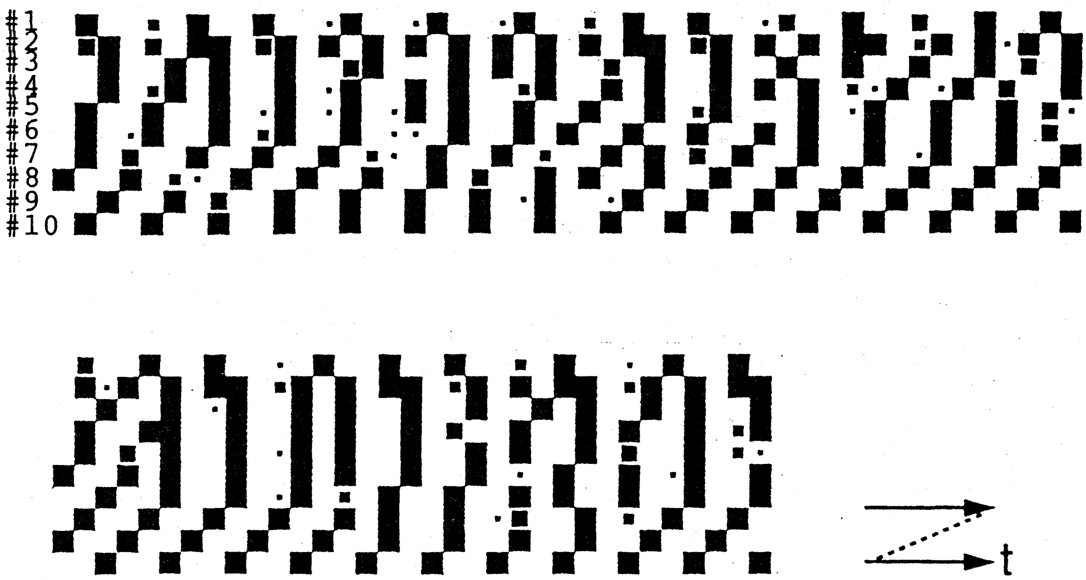

Fig. 7

An example of the dynamical behavior of the chaotic neural network composed of ten neurons. The size of each square is proportional to the strength of the output. The Lyapunov spectra are (0.38, 0.12, 0.02, –0.02, –0.06, –0.10, –0.16, –0.19, –0.26, –0.29).

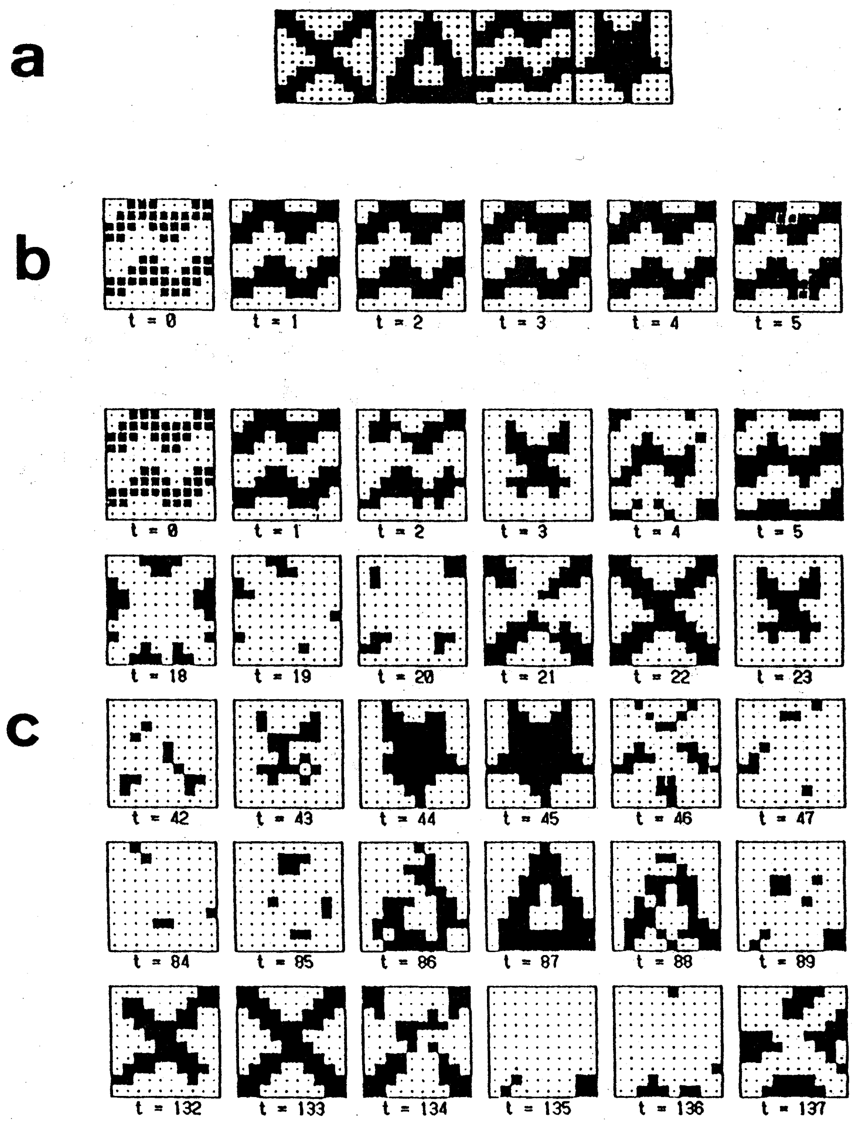

Fig. 8

An example of chaotic pattern dynamics. (a) Patterns stored with autocorrelated weight-matrix. (b) Dynamics with attraction to the nearest stored pattern(k_f = 0.5 and k_r = 0.6). (c) Chaotic wondering around the cross, triangle and star-like patterns stored by aurocorrelation weight-matrix(k_f = 0.2 and k_r = 0.9).

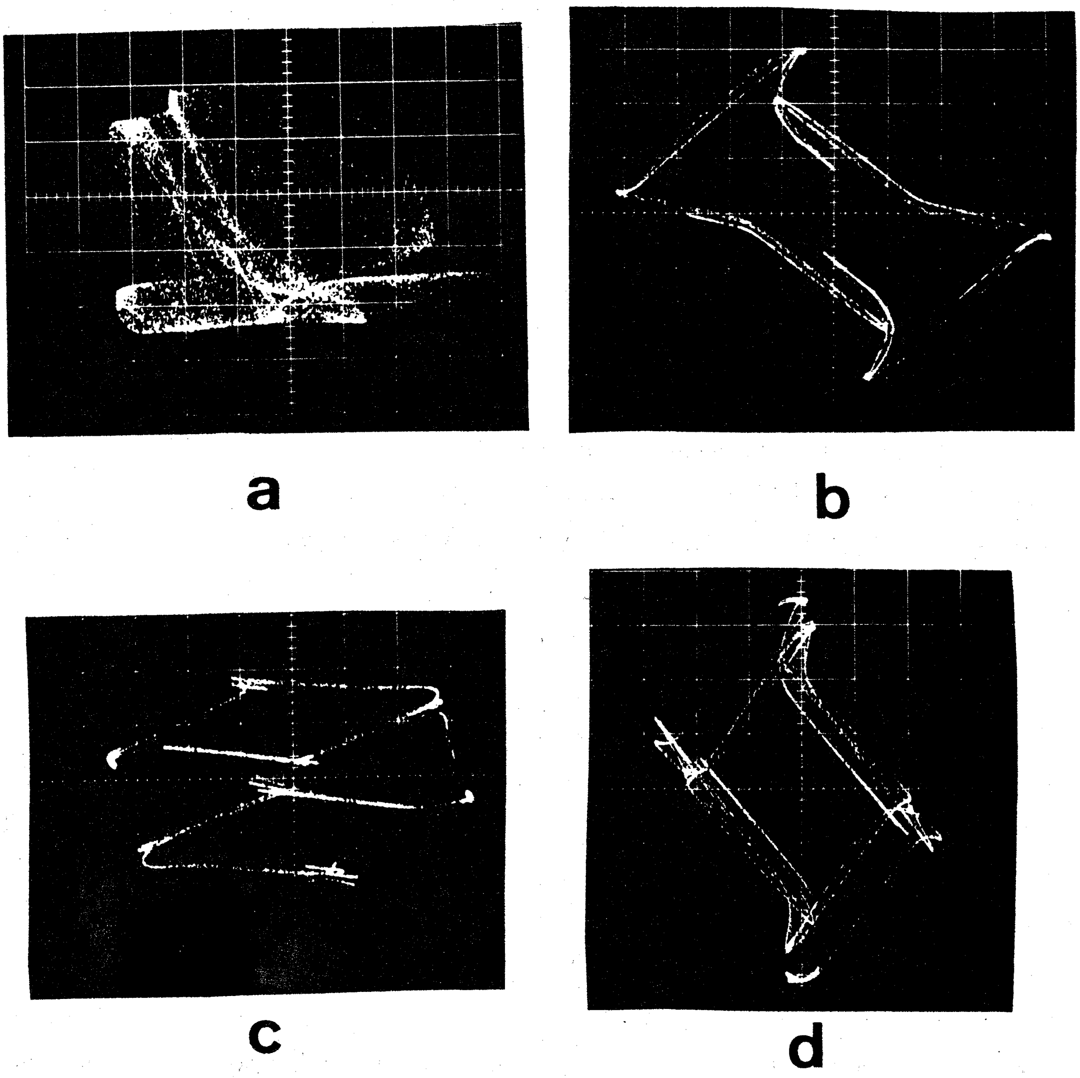

Fig. 9

Strange attractors in the chaotic neural network electronically implemented.