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Refinable maps and cohomological dimension

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The notion of refinable maps was introduced by Jo Ford and Rogers [2] as follows: A map $r: X \longrightarrow Y$ between compacta is *refinable* provided that for every $\varepsilon > 0$, there exists an ε-map $f_\varepsilon: X \longrightarrow Y$ such that $d(r,f) < \varepsilon$. It is known that it plays an interesting part in dimension theory. Namely, let $r: X \longrightarrow Y$ be a refinable map between compacta. Then we have:

(1) $\dim X = \dim Y$ (see [4] and [6]),

(2) *if* X *is weakly infinite-dimensional, then so is* Y (see [5]),

(3) *if* X *has the property* C, *then* Y *also has the property* C (see [1] and [3]),

(4) $\text{c-dim}_G X \geq \text{c-dim}_G Y$ *for every abelian group* G ([6]).

Here a compactum X has *cohomological dimension* $\leq n$ *with respect to* G, written $\text{c-dim}_G X \leq n$, provided that every map $\alpha: A \longrightarrow K(G,n)$ of a closed subset A of X to an Eilenberg-MacLane space K(G,n) of type (G,n) has a continuous extension $\bar{\alpha}: X \longrightarrow K(G,n)$ of α.

In [6], we posed the problem *whether the converse inequality of (4) hold or not*. In this note we will announce a partial answer.

Namely, in the case of $G = \mathbb{Z}$ or $\mathbb{Z}_p$, the converse one is valid. For the purpose, we introduce a new dimension-like function, a-dim_G, called *approximable dimension with respect to an abelian group* G (see [7]).

Definition 1. Let K be an ANR and let n be a natural number. Let $\varepsilon > 0$ be a positive number. A map $\psi: Q \longrightarrow P$ between compact polyhedra is (K,n,ε)-*approximable* provided that there exists a triangulation L of P such that for any triangulation M of Q, there is a map $\psi': |M^{(n)}| \longrightarrow |L^{(n)}|$ satisfying the following conditions:

1) $d(\psi', \psi | |M^{(n)}|) \leq \varepsilon$,
2) for any map $\alpha: |L^{(n)}| \longrightarrow K$, the map $\alpha \circ \psi': |M^{(n)}| \longrightarrow K$ admits a continuous extension over Q.

Definition 2. Let K be an ANR. A compactum X has *approximable dimension with respect to K less than n*, written $\text{a-dim}_K X \leq n$, provided that for every compact polyhedron P, every map $f: X \longrightarrow P$ and every positive number $\varepsilon > 0$, there exists a compact polyhedron Q and maps $\varphi: X \longrightarrow Q$, $\psi: Q \longrightarrow P$ such that

3) $d(f, \psi \circ \varphi) \leq \varepsilon$,
4) ψ is (K,n,ε)-approximable.

Specially, let G be an abelian group and let consider $K = K(G,n)$ in the above definitions. Then we use the terminology (G,n,ε)-*approximability* and *approximable dimension with respect to* G instead of $(K(G,n),n,\varepsilon)$-approximability and approximable dimension with respect to $K(G,n)$, respectively. And we denote by a-$\dim_G$ X ≤ n instead of a-$\dim_{K(G,n)}$ X ≤ n.

Concerning the relation between approximable dimension and ordinal and cohomological dimension, we have the following:

Lemma ([7]). *Let* X *be a compactum. Then we have*

(i) dim X ≤ n *if and only if* a-$\dim_{S^n}$ X ≤ n,

(ii) *in the case of* G = $\mathbb{Z}$ or $\mathbb{Z}_p$, c-$\dim_G$ X ≤ n *if and only if* a-$\dim_G$ X ≤ n.

Note that approximable dimension does not coincide with cohomological dimension with respect to the group $\mathbb{Q}$ of all rational numbers (see [8]).

In order to have the converse inequality of (4), we have shown the following:

Theorem. *Let* K *be an* ANR *and let* r: X ⟶ Y *be a refinable map between compacta. Then* a-$\dim_K$ X ≤ n *if and only if* a-$\dim_K$ Y ≤n.

As its consequence, we have the following:

Corollary. *Let* r: X ⟶ Y *be a refinable map between compacta. Then we have*:

(i) dim X = dim Y,

(ii) c-$\dim_{\mathbb{Z}}$ X = c-$\dim_{\mathbb{Z}}$ Y,

(iii) c-$\dim_{\mathbb{Z}_p}$ X = c-$\dim_{\mathbb{Z}_p}$ Y.

Note that some of our results can be generalized to compact

Hausdorff spaces or to non-compact normal spaces. The detail will be appeared elsewhere.

References

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