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Propagation of Gevrey singularities for a class of microdifferential operators

T. ARISUMI
有隅聡　　東大理

§0 Introduction

We study the microlocal solvability in the space of ultradistributions $\mathcal{D}^{*\prime}$ and the propagation of Gevrey singularities for a microdifferential operator P with multiple involutive characteristics.

Bony and Schapira [3] have shown the microlocal solvability in the space of hyperfunctions $\mathcal{B}$ and the propagation of analytic singularities for a microdifferential operator P with multiple involutive characteristics. Explicitly, they assumed that its real characteristic variety V is regular involutive and P is non-microcharacteristic along $V^{\mathbf{C}}$ (cf.(A) (B) (C) given below). Moreover Bony [2] has shown the microlocal solvability in the space of distributions $\mathcal{D}'$ and the propagation of C^∞−singularities under the Levi condition in addition to the assumptions of Bony-Schapira.

In this article, we interpolate the above two results. That is, we replace the Levi condition by the irregularity condition and show the microlocal solvability in the space of ultradistributions $\mathcal{D}^{*\prime}$ and the propagation of Gevrey singularities corresponding to the irregularity of P.

More explicitly, let $\mathring{T}^*\mathbf{R}^\nu$ denote the cotangent bundle of $\mathbf{R}^\nu$ with the zero section removed. Let $(x;\xi)$ be its coordinate system. Fix a point

$(\overset{\circ}{x};\overset{\circ}{\xi})$ of $\overset{\circ}{T}{}^*\mathbf{R}^\nu$ and a conic neighborhood U of $(\overset{\circ}{x};\overset{\circ}{\xi})$. Let $P(x, D_x)$ be a microdifferential operator on U of order μ (refer to [11],[12] for the sheaf $\mathcal{E}_X$ of microdifferential operators).

We assume the following conditions (A),(B),(C),(D) for P.

(A) $\begin{cases} \text{The real characteristic variety } V = Ch(P) \cap \overset{\circ}{T}{}^*\mathbf{R}^\nu \text{ of } P \\ \text{is a non-singular manifold of } \overset{\circ}{T}{}^*\mathbf{R}^\nu \text{ of codimension } n. \end{cases}$

(B) $\begin{cases} \text{The principal symbol } \sigma(P) \text{ of } P \text{ vanishes on } V \\ \text{exactly of order } m \text{ ; i.e.} \\ \quad \sigma(P)(x+\epsilon\Delta x, \xi+\epsilon\Delta\xi) = a\epsilon^m + o(\epsilon^m) \quad (a \neq 0) \\ \quad for\ \forall(x;\xi) \in V \quad \forall(\Delta x, \Delta\xi) \notin T_{(x;\xi)}V. \end{cases}$

(C) $\begin{cases} V \text{ is regular involutive ; i.e.} \\ \quad \text{there exist n homogeneous functions } q_1(x,\xi), \cdots, q_n(x,\xi) \\ \quad \text{of degree 1 satisfying the conditions } q_i\big|_V = 0, \\ \quad \{q_i, q_j\}\ \big|_V = 0\ (i,j = 1,\cdots,n) \text{ and } dq_1 \wedge \cdots \wedge dq_n \wedge \omega \neq 0, \\ \quad \text{where } \omega \text{ is the canonical 1-form of } \overset{\circ}{T}{}^*\mathbf{R}^\nu. \end{cases}$

(D) Irregularity of P along $V^{\mathbf{C}}$ is not greater than σ on U (refer to §1.1 for its definition).

In this situation, we will show

THEOREM 0.1 (EXISTENCE). *Let v belong to $\mathcal{C}^*_{(\overset{\circ}{x};\overset{\circ}{\xi})}$. We assume that $* \leq (\frac{\sigma}{\sigma-1})$. Then there exists $u \in \mathcal{C}^*_{(\overset{\circ}{x};\overset{\circ}{\xi})}$ satisfying $Pu = v$.*

THEOREM 0.2 (PROPAGATION). *Let U be a neighborhood of $(\overset{\circ}{x};\overset{\circ}{\xi})$ in $S^*\mathbf{R}^\nu$, and $u \in \mathcal{C}^*(U)$ be a solution of $Pu = 0$. We assume that $* \leq (\frac{\sigma}{\sigma-1})$. Then the wave front set $WF_*(u)$ of u in the class $*$ is an union of bicharacteristic leaves of V.*

Refer to §1.1 for $\mathcal{C}^*$, WF_* and the order of $*$.

§1 Notation and reduction

1.1 NOTATION AND DEFINITIONS.

We recall the definitions of irregularity of microdifferential operators, the wave front set in the Gevrey class and so on.

We work in the situation of the Introduction. Let Q_i be microdifferential operators with $\sigma(Q_i)(x,\xi) = q_i(x,\xi)$.

DEFINITION 1.1.1(IRREGULARITY): Assume R has the form

$$R(x,D) = \sum_{|\alpha|\leq m} A_\alpha(x,D)Q^\alpha(x,D)$$

with

$$\sigma(A_\alpha)(\overset{\circ}{x};\overset{\circ}{\xi}) \neq 0.$$

Then we define the irregularity σ of R along $V^{\mathbf{C}}$ at $(\overset{\circ}{x};\overset{\circ}{\xi})$ by

$$\sigma := \max\left\{1, \frac{m-|\alpha|}{\operatorname{ord} R - |\alpha| - \operatorname{ord} A_\alpha}\right\}.$$

Remark that the above definition is independent of the choice of Q_i. Thus the irregularity σ in the above definition is stable under quantized contact transformations. Moreover Laurent[8] has proved the stability of Newton polygons of microdifferential operators under quantized contact transformations. We also remark that the Levi condition coincides with the condition $\sigma = 1$.

REMARK 1.1.2. *Let* $*$ *denote* (s) *or* $\{s\}$. *Here* s *moves in* $]1,\infty[$. *If* $s < s'$, *then* $(s) < \{s\} < (s') < \{s'\}$.

DEFINITION 1.1.3.(WAVE FRONT SET IN THE GEVREY CLASS): Let u be an ultradistribution of class $*$. Then we define the wave front set $WF_*(u)$ of u in the class $*$ as follows. For $(\mathring{x};\mathring{\xi}) \in \mathring{T}^*\mathbf{R}^\nu$,

$(\mathring{x};\mathring{\xi}) \notin WF_*(u) \overset{def}{\Longleftrightarrow}$

there exists an ultradifferentiable function $\chi(x)$ of class $*$ which is equal to 1 in a neighborhood of $\mathring{x}$, and there exists an open cone Γ containing $\mathring{\xi}$ for which $\widehat{\chi u}(\xi)$ (the Fourier transform of u) satisfies the following estimates on Γ in case of $* = (s)$ (resp.$* = \{s\}$); $\forall b$, $\exists C$ (resp.$\exists b$, $\exists C$)

$$|\widehat{\chi u}(\xi)| \leq C \exp(-b|\xi|^{\frac{1}{s}}).$$

DEFINITION 1.1.4: Let $\pi : S^*\mathbf{R}^\nu \longrightarrow \mathbf{R}^\nu$ and $sp : \pi^{-1}\mathcal{B} \longrightarrow \mathcal{C}$. Then we define $\mathcal{C}^*$ by

$$\mathcal{C}^* = \mathrm{Im}(\pi^{-1}\mathcal{D}^{*\prime} \underset{sp}{\longrightarrow} \mathcal{C}).$$

We refer to [6] for the definition of the sheaf of ultradistributions $\mathcal{D}^{*\prime}$, where $\mathcal{D}^{*\prime}$ is characterized by the growth condition from the imaginary axis of defining functions as follows.

$$
\begin{aligned}
&F(x+i\Gamma 0)\in \mathcal{D}^{*\prime}(\Omega) \text{ for } *=(s) \quad (\textit{resp.}\ *=\{s\}) \quad \Longleftrightarrow \\
&\text{for any compact subset } K(\subset \Omega) \quad \exists L,\ C \quad (\textit{resp.}\ \forall L,\ \exists C) \\
&|F(x+iy)|\leq C\exp(L|y|^{-\frac{1}{s-1}}) \quad (x\in K).
\end{aligned}
$$

1.2 Reduction to a partial elliptic operator.

We reduce the theorems in the Introduction to the study of a partial elliptic operator. Let (x,t) be a coordinate system of $\mathbf{R}^\nu = \mathbf{R}^n\times\mathbf{R}^p$ with $x=(x_1,\cdots,x_n)$ and $t=(t_1,\cdots,t_p)$, and (ξ,τ) the dual coordinates of (x,t).

On account of the stability of conditions (A),(B),(C),(D) under quantized contact transformations (Q.C.T. for short), we may assume $V=\{\xi_1=\cdots=\xi_n=0\}$, $(\overset{\circ}{x},\overset{\circ}{\xi})=(0,0;0,\tau_0)$ with $\tau_0=(1,0,\cdots,0)\in\mathbf{R}^p$ by finding a suitable Q.C.T. Moreover, dividing the operator P by an invertible operator of order $\mu-m$, we may assume P is of the form

$$
P(x,t,D_x,D_t)=\sum_{0\leq|\alpha|\leq m} A_\alpha(x,t,D_x,D_t)D_x^\alpha .
$$

Then P satisfies

$$
\text{(B}'\text{)} \qquad \sum_{|\alpha|=m}\sigma_0(A_\alpha)(x,t,0,\tau)\xi^\alpha\neq 0 \quad (\forall\xi\in\mathbf{R}^n\setminus\{0\})
$$

for $(x,t,;0,\tau)\in U$

$$
\text{(D}'\text{)} \qquad 0\leq \operatorname{ord} A_\alpha\leq \frac{\sigma-1}{\sigma}(m-|\alpha|).
$$

In the above situation, Theorem 1 and Therem 2 are reduced to

THEOREM 1.2.1.

a) (existence)

Ω is a neighborhood of the origin in $\mathbf{R}^{n+p}$, and $K(\subset S^{n+p-1})$ a compact set with $\Omega \times K \subset U$. Then for any $K'(\Supset K)$ and for any $v \in \Gamma_{\Omega\times K}(\Omega \times S^{n+p-1}, \mathcal{C}^*)$, there exist a neighborhood Ω' of the origin in $\mathbf{R}^{n+p}$ and $u \in \Gamma_{\Omega'\times K'}(\Omega \times S^{n+p-1}, \mathcal{C}^*)$ satisfying $Pu = v$.

b) (regularity)

Let $u \in \Gamma(U, \mathcal{C}^*)$ satisfy $Pu = v$. Then there exist a neighborhood $\widetilde{U}$ of $(0,0;0,\tau_0)$ in $\mathbf{C}^n \times \mathbf{R}^p \times S^{2n+p-1}$ and $\widetilde{u} \in \Gamma(\widetilde{U}; \mathcal{C}^*)$ which satisfy $\partial_{\bar{z}_i}\widetilde{u} = 0$ $(i = 1, \cdots, n)$ and $\widetilde{u}|_{\mathbf{R}^\nu} = u$.

c) (propagation)

Let $u \in \Gamma(U, \mathcal{C}^*)$ satisfy $Pu = 0$, $(0,0;0,\tau_0) \in WF_*(u)$. Let F denote the connected component of $(0,0;0,\tau_0)$ in $\{(x,t;\xi,\tau) \in; t = \xi = 0, \tau = \tau_0\}$. Then $F \subset WF_*(u)$.

Here $* \leq (\frac{\sigma}{\sigma-1})$.

We can make a further reduction of the operator P, which is used in the next section.

REMARK 1.2.2. *By the division theorem of Weierstrass type, we can assume*

$$P(x,t,D_x,D_t) = D_{x_n}{}^m + \sum_{0\leq|\alpha|\leq m,\ \alpha_n<m} A_\alpha(x,t,D_{x'},D_t)D_{x_n}{}^\alpha$$

with

$$ordA_\alpha \leq \frac{\sigma-1}{\sigma}(m - |\alpha|)$$

Here $x' = (x_1, \cdots, x_{n-1})$.

§2 Cauchy problem for the microdifferential operator in the complex domain

We solve the Cauchy problem in the complex domain with estimates for microdifferential operators as follows.

Let (z, w) be a coordinate system of $\mathbf{C}^\nu = \mathbf{C}^n \times \mathbf{C}^p$ and (ζ, θ) the dual coordinates of (z, w). We set $(z', z_n) = (z_1, \cdots, z_n)$, $(w_1, w') = (w_1, \cdots, w_p)$, $\theta_0 = (1, 0, \cdots, 0) \in \mathbf{R}^p$.

In this situation, we assume that a microdifferential operator P is defined in a neighborhood of $(0, 0; 0, \theta_0) \in T^*\mathbf{C}^\nu$ and has the form

$$P(z, w, D_z, D_w) = D_{z_n}^m + \sum_{\substack{0 \leq |\alpha| \leq m \\ \alpha_n < m}} A_\alpha(z, w, D_{z'}, D_w) D_z^\alpha$$

where $\operatorname{ord} A_\alpha \leq \frac{\sigma - 1}{\sigma}(m - |\alpha|)$, $\quad [z_n, A_\alpha] = 0$.

This can be rewritten as

$$P(z, w, D_z, D_w) = D_{z_n}^m - \sum_{\substack{0 \leq |\alpha| \leq m \\ \alpha_n < m}} D_{z_n}^{\alpha_n} D_{z'}^{\alpha'} D_{w_1}^{\lambda_\alpha} B_\alpha(z, w, D_{z'}, D_w)$$

where $\lambda_\alpha = \operatorname{ord} A_\alpha, \quad \operatorname{ord} B_\alpha \leq 0, \quad [z_n, B_\alpha] = 0$.

REMARK 2.1.
Setting $s = \frac{\sigma}{\sigma - 1}$, then we have $s\lambda_\alpha \leq m - |\alpha|$.

DEFINITION 2.2: We set, in $\mathbf{C}^\nu$, $\Sigma = \{w_1 = \sigma\}$ and $H = \{z_n = h\}$. Let

$\Omega(\subset \mathbf{C}^\nu)$ be an open convex subset. Then

Ω is $z_n - k - \Sigma - flat$ if the conditions

$(z,w) \in \Omega,\ (\tilde{z},\tilde{w}) \in \Sigma,\ z_n = \tilde{z}_n,\ |w_1 - \tilde{w}_1| \geq k|w_i - \tilde{w}_i|\ (i = 2,\cdots,p)$

$|w_1 - \tilde{w}_1| \geq k|z_j - \tilde{z}_j|\ (j = 1,\cdots,n-1)$ imply $(\tilde{z},\tilde{w}) \in \Omega \cap \Sigma$.

Ω is $w - \delta - H - flat$ if the conditions

$(z,w) \in \Omega,\ (\tilde{z},\tilde{w}) \in H,\ w = \tilde{w}$

$|z_n - \tilde{z}_n| \geq \delta|z_i - \tilde{z}_i|\ (i = 1,\cdots,n-1)$ imply $(\tilde{z},\tilde{w}) \in \Omega \cap H$.

DEFINITION 2.3: For $M = (z,w) \in \Omega$, we set

$$d_{z'}(M) = \inf\{\max_{1\leq j\leq n-1}|z_j - \tilde{z}_j|\ ;\ (\tilde{z}', z_n, w) \in \complement\Omega\},$$

$$d_{w'}(M) = \inf\{\max_{2\leq j\leq p}|w_j - \tilde{w}_j|\ ;\ (z, w_1, \tilde{w}') \in \complement\Omega\},$$

$$d_{w_1}(M) = \inf\{|w_1 - \tilde{w}_1|\ ;\ (z, \tilde{w}_1, w') \in \complement\Omega\},$$

$$\lambda_L(w_1) = \exp(L|\Im w_1|^{-\frac{1}{s-1}}).$$

For $v \in \mathcal{O}(\Omega)$, we define the norm of v by

$$\|v\|_L = \sup \frac{|v(M)|}{d_{z'}(M)^{-1} d_{w'}(M)^{-1} \lambda_L(w_1)}.$$

In this situation, we have

THEOREM 2.4. *There exist an open neighborhood Ω_0 of the origin in $\mathbf{C}^\nu$ and constants $k > 0$, $1 > \delta > 0$ enjoying the following property. For any $\Omega(\subset \Omega_0)$ $z_n - k - \Sigma - flat$ and $w - \delta - H - flat$, $g \in \mathcal{O}(\Omega)$ with*

$\|g\|_L < \infty$ *and* $h_j \in \mathcal{O}(\Omega \cap H)$ *with* $\|h_j\|_L < \infty$ $(j = 0, \cdots, m-1)$, *there exist an unique* $f \in \mathcal{O}(\Omega)$ *and* L' *satisfying*

$$\begin{cases} P_\Sigma f = g, \\ D^j_{z_n} f|_H = h_j \quad (j = 0, \cdots, m-1), \\ \|f\|_{L'} < \infty. \end{cases}$$

Here the norm $\| * \|_{L'}$ *is taken on a domain shrinked in the real direction compared with the norm* $\| * \|_L$.

We prepare several lemmas to prove the above theorem.

LEMMA.

A. In the above situation, there exists constant K *and*

$\|f\|_L < \infty \Longrightarrow \|B_{\alpha\Sigma} f\|_L \le K\|f\|_L$.

B. Let Ω *be an open convex set in* $\mathbf{C}^2_{(z,w)}$ *which contains the origin. Assume that* Ω *is flat enough for* $\{z = 0\}$ *and that for some* δ,

$d_w(tz, w) \ge d_w(z, w) + \dfrac{(1-t)|z|}{\delta}$ $(0 \le t \le 1)$ *is satisfied for any* $(z, w) \in \Omega$. *Then if* $f(z, w) \in \mathcal{O}(U)$ *satisfies* $|f(z, w)| \le C d_w(z, w)^{-l}$, *we have* $|D_z^{-k} D_w^k f(z, w)| \le C(e\delta)^k (k + l) d_w(z, w)^{-l}$.

C. Let Ω *be an open convex set in* $\mathbf{C}$ *containing the origin. Then*

$$|f(z)| \le \frac{|z|^l}{l!} \Longrightarrow |D^{-k} f(z)| \le \frac{|z|^{l+k}}{(l+k)!}.$$

The parts A,B are proved in [2],[3]. It is easy to show C.

PROOF OF THEOREM 2.2.4: We decompose f formally as $f = \sum_{l=0}^{\infty} v_l$ in such a way that

$$\begin{cases} D_{z_n}^m v_0 = g \\ D_{z_n}^j v_0|_H = h_j \end{cases} \qquad \begin{cases} D_{z_n}^m v_{l+1} = \sum D_z^\alpha D_{w_1}^{\lambda_\alpha} B_{\alpha\Sigma} v_l \\ D_{z_n}^j v_{l+1}|_H = 0 \qquad (j = 0, \cdots, m-1). \end{cases}$$

Moreover we decompose $v_l = \sum_k v_l^{(k)}$ formally as

$$\begin{cases} v_0^{(k)} = \begin{cases} v_0 & (k = 0) \\ 0 & (k \neq 0) \end{cases} \\ v_{l+1}^{(k)} = \Lambda^{(0)} v_l^{(k)} + \cdots + \Lambda^{(\lambda)} v_l^{(k-\lambda)} \end{cases}$$

where

$$\Lambda^{(k)} = D_{z_n}^{-m} \sum_{\lambda_\alpha = k} D_z^\alpha D_{w_1}^{\lambda_\alpha} B_{\alpha\Sigma},$$

$$\lambda = \max \lambda_\alpha.$$

We put $\Omega_\epsilon = \Omega \cap \{\Im w_1 > \epsilon\}$ and we have $|v_0| d_{z'} d_{w'} \leq M_\epsilon := Const. \exp(L\epsilon^{-\frac{1}{s-1}})$ for $\epsilon \ll 1$. Then we can show the following estimates on $\Omega_{2\epsilon}$ by the above lemmas;

$$|v_l^{(k)}| \leq (\lambda K')^l \left(\frac{e}{\epsilon}\right)^k \frac{|z_n|^{(s-1)k}}{\Gamma((s-1)k)} \delta^{l-k} M_\epsilon d_{w_1}^{-1} d_{z'}^{-1} d_{w'}^{-1}.$$

Then this implies

$$|\sum_l v_l^{(k)}| \leq \sum_l (\lambda K' \delta)^l \left(\frac{e}{\epsilon\delta}\right)^k \frac{|z_n|^{(s-1)k}}{\Gamma((s-1)k)} M_\epsilon d_{w_1}^{-1} d_{z'}^{-1} d_{w'}^{-1}.$$

We put $\delta < \frac{1}{2\lambda K'}$, and remark $|z_n| < 1$. Then we have

$$|f| \leq |\sum_k \sum_l v_l^{(k)}| \leq C \exp\left([L_1 \left(\frac{e}{\delta}\right)^{\frac{1}{s-1}} + L]\epsilon^{-\frac{1}{s-1}}\right) d_{w_1}^{-1} d_{z'}^{-1} d_{w'}^{-1}.$$

Finally the theorem is proved because $d_{w_1}^{-1}(M) \leq \exp(\epsilon^{-\frac{1}{s-1}})$ for $M \in \Omega_{3\epsilon}$. ∎

§3 Proof of the theorems

We work in the situation of Theorem 1.2.1. The proof will be completed in the same way as [2]. First we prepare some notation.
(z, w) is a coordinate system in $\mathbf{C}^{n+p}$ with $z = x + iy$, $w = t + is$, and (ξ, τ) is the associated fiber coordinate system in $\mathring{T}^*\mathbf{R}^\nu$. Let G be an open convex cone in $\mathbf{R}^{n+p}$ with $G \subset \{(y, s) \in \mathbf{R}^{n+p} \; ; \; s_1 \geq 0\}$, and Γ be an open convex cone in $\mathbf{R}^n$. We set $TG := \mathbf{R}^{n+p} + iG$, $T\Gamma := \mathbf{R}^n + i\Gamma$, $B(k) = \{(\xi, \tau) \; ; \; |\xi|^2 + |\tau'|^2 \geq k^2\tau_1^2\}$.

Definition 3.1: We define a subset $\widetilde{O^*}(TG)$ of the stalk of ultradistributions at the origin as follows. For an open neighborhood W of the origin in $\mathbf{C}^{n+p}$, we define the space $O^*(TG \cap W)$ by the equivalence

$$f \in O^*(TG \cap W) \overset{def}{\iff} \begin{cases} f \in \mathcal{O}(TG \cap W), \text{ and satisfies the growth condition} \\ \text{of class } * \text{ for } s_1; \text{ i.e. for } * = (s) \; (resp.* = \{s\}) \\ \exists L, C \; (resp. \forall L, \exists C) \; |f(z, w)| \leq C \exp(L s_1^{-\frac{1}{s-1}}). \end{cases}$$

Then we put $\widetilde{O^*}(TG) := \varinjlim_{0 \in W \subset \mathbf{C}^{n+p}} (TG \cap W)$.

We quote the following lemma from [2].

LEMMA 3.2. *Assume* $G \supset \{s_1 > k_0(\sum_1^n |y_i|^2 + \sum_2^p |s_j|^2)^{\frac{1}{2}}\}$ *and* $\Gamma \supset \{y_n > \delta_0(\sum_2^{n-1} |y_i|^2)^{\frac{1}{2}}\}$ *where* $k_0 < k/\sqrt{n+p}$ $\delta_0 < \delta/\sqrt{n}$. *Then there exists a fundamental system* $\{\Omega_{\sigma h}\}_{\sigma>0,h>0}$ *of the open neighborhoods of the origin for which the following statements hold.*

a) $\Omega_{\sigma h}$ *is* $z_n - k - \Sigma - flat$ *and* $w - \delta - H - flat$,

b) $\Omega_{\sigma h} \cap \Sigma \subset TG + T\Gamma$,

c) $\Omega_{\sigma h} \cap H \cap (TG + \mathbf{C}^n) \subset TG + T\Gamma$,

d) $\Omega_{\sigma h} \cap (TG + T\Gamma)$, $\Omega_{\sigma h} \cap (TG + \mathbf{C}^n)$ *is* $z_n - k - \Sigma - flat$ *and* $w - \delta - H - flat$.

Recall that P is non-microcharacteristic in any direction of z (§1.2.(B')). Thus the preceding argument is valid for any direction of z as well as the direction z_n. Then we can prove the following theorem by Theorem 2.2.4.

THEOREM 3.3. *There exist constants* k_0, δ_0 *for which we have the following statements a) and b) for* $\forall G \subset \mathbf{R}^{n+p} \cap \{s_1 > 0\}$ *with* $G^\circ \subset B(k_0)$ *and for* $\forall \Gamma \subset \mathbf{R}^n$ *with the diameter of* $\Gamma^\circ \leq \delta_0$.

a) *If* $g \in \widetilde{\mathcal{O}^*}(TG + T\Gamma)$ $G' \Subset G$, *then there exist* $f \in \widetilde{\mathcal{O}^*}(TG' + T\Gamma)$ *and* $Pb(f) = b(g)$.

b) *If* $f \in \widetilde{\mathcal{O}^*}(TG + T\Gamma)$ $g \in \widetilde{\mathcal{O}^*}(TG + \mathbf{C}^n)$ *and* $Pb(f) = b(g)$, *then* $f \in \widetilde{\mathcal{O}^*}(TG' + \mathbf{C}^n)$ *for* $\forall G' \Subset G$.

By the aid of the suppleness of $\mathcal{C}^*$ (cf.[5],[4]), we can decompose a given ultradistribution into a sum of ultradistributions whose singular spectra are small enough and we have the edge of the wedge theorem for $\mathcal{D}^{*\prime}$. Moreover, for an ultradistribution whose singular spectrum intersects the characteristic variety of P, we can describe it by the trace of the elements of $\mathcal{D}_t^{*\prime}\mathcal{O}_z$ [9]. Here $\mathcal{D}_t^{*\prime}\mathcal{O}_z$ is the sheaf on $\mathbf{C}_z^n \times \mathbf{R}_t^p$ consisting of ultradistributions with holomorphic parameters in z. Thus from this theorem we can prove Theorem 1.2.1 a) and b).

For the proof of Theorem 1.2.1 c), it suffices to prove the following theorem which shows the propagation of WF_* for an ultradistribution with holomorphic parameters.

THEOREM 3.4. *Let $U \subset \mathbf{C}_z^n \times \mathbf{R}_t^p$ be an open set whose restriction to $\{t = const\}$ is connected and intersects $\mathbf{R}^n \times \mathbf{R}^p$. Let $\tilde{u}(z,t) \in \mathcal{D}^{*\prime}(U)$ satisfy $\frac{\partial}{\partial \bar{z}_i} u = 0 \quad (i = 1, \cdots, n)$, $(x_0, t_0; 0, \tau_0) \notin WF_*(u(x,t))$ where $u(x,t)$ is the restriction of $\tilde{u}(z,t)$ to real axis. Then $(x, t_0; 0, \tau_0) \notin WF_*(u(x,t))$.*

This theorem is proved by a simple result of complex analysis and the partial Foaurier transformation (cf.[2]). Then we can easily conclude theorem 1.2.1 c) from this theorem.

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