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Microlocal *a priori* estimates and the Cauchy problem

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1. Microlocal *a priori* estimates.

Microlocal analysis has made a great contribution to the recent development of the theory of partial differential equations, and many authors have proved that microlocal analysis is a poweful and useful tool in studying partial differential equations. In particular, it leads us to studies on standard forms of pseudodifferential operators and clarifies essential parts to be analyzed. However, for general problems one only obtains results modulo C^∞. To remove "modulo C^∞" one must make an effort according to the problems. In the studies of the Cauchy problem one must distinguish the time variable from other variables, and only limited application of microlocal analysis was permitted. Our purpose is to show that microlocal analysis is fairly applicable to the studies of uniqueness and well-posedness of the Cauchy problem. In [7] we showed that microlocal *a priori* estimates (Carleman type estimates) give theorems on propagation of singularities. Here we shall show that similar microlocal estimates also give theorems on uniqueness and well-posedness of the Cauchy problem. Let $P(x,\xi) = \xi_1^m + \sum'_{|\alpha|\le m, \alpha_1<m} a_\alpha(x)\xi^\alpha$, where $a_\alpha(x) \in C^\infty(\mathbf{R}^n)$.

<u>Microlocal *a priori* estimate at $z^0 \in S^*\mathbf{R}^n (\simeq \mathbf{R}^n \times S^{n-1})$</u> $(0 < A \le \infty)$

$P(x,\xi;\gamma) \in S^m_{1,0}$ satisfies $(E; z^0, \{t_\tau(x,\xi)\}_{\tau\in\mathcal{T}}, A) \overset{def}{\Longleftrightarrow}$
$\exists \psi_j(x,\xi) \in C^\infty(T^*\mathbf{R}^n\backslash 0)$ ($j=1,2$) (pos. homo. of deg. 0), $\exists l_j \in \mathbf{R}$ ($1 \le j \le 3$) s.t. $\psi_j(x,\xi) = 1$ in a conic nbd of z^0 ($j=1,2$) and $\forall \tau \in \mathcal{T}$, $\forall b \in \mathbf{R}$, $1 \le \exists K < A$, $\exists \gamma_0 \ge 1$, $\exists C > 0$ *satisfying*

$$\| \langle D\rangle_\gamma^{l_1} v \| \le C\{\| \langle D\rangle_\gamma^{l_2} P_\Lambda(x,D;\gamma)v \| + \| \langle D\rangle_\gamma^{l_3}(1-\psi_{1,h}(x,D))v \|\}$$

if $v \in H^\infty(\mathbf{R}^n)$, $\gamma \ge \gamma_0$, $h = K\gamma$ and $\Lambda(x,\xi) = (t_\tau(x,\xi) - b)\log\langle\xi\rangle(1-\Theta(4|\xi|/h))\psi_2(x,\xi)$, where $\Theta(t) \in C_0^\infty(\mathbf{R})$, $\Theta(t) = 1$ ($|t| \le 1$), supp $\Theta \subset (-2,2)$, $\psi_{1,h}(x,\xi) = (1-\Theta(|\xi|/h))\psi_1(x,\xi)$, $P_\Lambda(x,D;\gamma)v = (e^{-\Lambda})(x,D)P(x,D;\gamma)(e^\Lambda)(x,D)v$, $\langle\xi\rangle_\gamma = (\gamma^2 + |\xi|^2)^{1/2}$ and $\|\cdot\|$ denotes the L^2-norm.

(P-1) $p(x,\xi)$ (=the principal part of $P(x,\xi)$) is hyperbolic w.r.t. $\vartheta = (1,0,\dots,0) \in \mathbf{R}^n$

Put $P(x,D;\gamma)u = e^{-\gamma\zeta(x)}P(x,D)(e^{\gamma\zeta(x)}u)$ for $\zeta(x) \in \mathcal{B}^\infty(\mathbf{R}^n)$.

(P-2) $\forall z^0 = (x^0,\xi^0) \in S^*\mathbf{R}^n$ with $x_1^0 > 0$ and $dp(z^0) = 0$, $\forall \vartheta^0 \in \Gamma(p(x^0,\cdot),\vartheta)$, $\forall k > 0$, $\exists a_0 > 0$ s.t. '$P(x,\xi;\gamma)$ for some $\zeta(x)$ which is equal to $(x-x^0)\cdot\vartheta^0 + k\times |x-x^0|^2$ near x^0 (*resp.* ${}^tP(x,\xi+i\gamma\vartheta)$) satisfies $(E; z^0, \{at_+(x,\xi;z^0)\}_{a\ge a_0}, \infty)$ (*resp.* $(E; z^0, \{at_-(x,\xi;z^0)\}_{a\ge a_0}, \infty)$),' where $t_\pm(x,\xi;z^0) = \pm(x_1 - x_1^0) + |x-x^0|^2$

$+ \mid \xi / \mid \xi \mid - \xi^0 \mid^2$ in a conic nbd of z^0, $\Gamma(p(x^0,\cdot),\vartheta)$ denotes the component of $\{\xi \in \mathbf{R}^n;\ p(x^0,\xi) \neq 0\}$ containing ϑ, and ${}^tP(x,D)$ denotes the transposed of $P(x,D)$.

Assume that for each $z^{0\prime} = (x^{0\prime},\xi^{0\prime}) \in S^*\mathbf{R}^{n-1}$ $P(x,D)$ can be written as $P(x,D) = P^1(x,D;z^{0\prime}) \cdots P^\nu(x,D;z^{0\prime}) + R(x,D;z^{0\prime})$ in $\mathcal{C}'(z^{0\prime})$, where $\mathcal{C}'(z^{0\prime})$ is a conic nbd of $(0,z^{0\prime})$ in $\mathbf{R}\times(T^*\mathbf{R}^{n-1}\setminus 0)$, $x' = (x_2,\dots,x_n)$, $\nu \equiv \nu(z^{0\prime})$, $P^k(x,\xi,z^{0\prime}) \in S^{m_k,0}_{1,0}$ (the coef. of $\xi_1^{m_k}=1$), $R(x,\xi;z^{0\prime}) \in \bigcap_l S^{m,l}_{1,0}$, $a(x,\xi) \in S^{m,l}_{1,0} \Longleftrightarrow^{def} a(x,\xi) = \sum_{j=0}^{[m]} a_j(x,\xi')\xi_1^j$, $a_j(x,\xi') \in S^{m+l-j}_{1,0}$ (classical). Put $t'_\pm(x,\xi';z^{0\prime}) = \pm x_1 + x_1^2 + \mid x' - x^{0\prime} \mid^2 + \mid \xi' / \mid \xi' \mid \pm \xi^{0\prime} \mid^2$ in a conic nbd of $(0,z^{0\prime})$.

(P-3) $\forall z^{0\prime} = (x^{0\prime},\xi^{0\prime}) \in S^*\mathbf{R}^{n-1}$, $1 \le \forall k \le \nu(z^{0\prime})$, $\exists \mathcal{P}^k(x,\xi;z^{0\prime}) \in S^{m_k,0}_{1,0}$ (hyp. w.r.t. ϑ, the coef. of $\xi_1^{m_k} = 1$) s.t. (1) $\mathcal{P}^k(x,\xi;z^{0\prime}) = P^k(x,\xi;z^{0\prime})$ ($(x,\xi') \in \mathcal{C}'(z^{0\prime})$, $\mid \xi' \mid \ge 1$). (2) $\mathcal{P}^k(x,\xi;z^{0\prime}) = \mathcal{P}^k(\xi;z^{0\prime})$ ($x \notin \Omega(z^{0\prime})$ ($\subset\subset\mathbf{R}^n$): a nbd of $(0,x^{0\prime})$), where $\mathcal{P}^k(\xi;z^{0\prime})$ is pos. homo. for $\mid \xi' \mid \ge 1$. (3) $\forall z^1 = (x^1,\xi^1) \in S^*\mathbf{R}^n$ with $x^1 \in \overline{\Omega(z^{0\prime})}$ and $d\mathcal{P}^k_0(z^1;z^{0\prime}) = 0$, $\exists a_0, a_0' > 0$ s.t. '${}^t\mathcal{P}^k(x,\xi - i\gamma\vartheta;z^{0\prime})$ ($(+)$*sign*) (*resp.* ${}^t\mathcal{P}^k(x,\xi + i\gamma\vartheta;z^{0\prime})$ ($(-)$*sign*)) satisfies $(E;z^1,\{at_\pm(x,\xi;z^1) + a't'_\pm(x,\pm\xi';z^{0\prime})\log\langle\xi'\rangle \times 1/\log\langle\xi\rangle\}_{a\ge a_0, a'\ge a_0'},\infty)$,' where $\mathcal{P}^k_0(x,\xi;z^{0\prime})$ denotes the principal symbol of $\mathcal{P}^k(x,\xi;z^{0\prime})$.

For $x^0 \in \mathbf{R}^n$ define $K^\pm_{x^0} = \{x(t);\ \pm t \ge 0$, and $\{x(t)\}$ is a Lipschitz cont. curve in $\mathbf{R}^n$ satisfying $(d/dt)x(t) \in \Gamma(p(x(t),\cdot),\vartheta)^*$ (*a.e.* t) and $x(0) = x^0\}$, where $\Gamma^* = \{y \in \mathbf{R}^n;\ y\cdot\eta \ge 0$ for $\forall\eta \in \Gamma\}$.

(P-4) $K^-_{x^0} \cap \{x_1 \ge 0\}$ is bdd for every $x^0 \in \mathbf{R}^n$.

Theorem 1. *Assume that* (P-1)–(P-4) *are satisfied. Then* $\forall f \in \mathcal{D}'$ *(resp.* C^∞*) with* supp $f \subset \{x_1 \ge 0\}$, $\exists! u \in \mathcal{D}'$ *(resp.* C^∞*) s.t.* $P(x,D)u = f$, supp $u \subset \{x_1 \ge 0\}$. *Moreover,* supp $u \subset \{x \in \mathbf{R}^n;\ x \in K^+_y$ *for some* $y \in$ supp $f\}$.

To prove Theorem 1 we first derive local *a priori* estimates (Carleman type estimates) from microlocal *a priori* estimates. This proves uniqueness of the Cauchy problem. This argument can be also applicable to operators which are not necessarily hyperbolic. Next we derive hyperbolic estimates for ${}^tP(x,D)$ from (P-3), which proves existence of solutions to the Cauchy problem. In deriving hyperbolic estimates the condition (2) of (P-3) plays an important role. For the detail we refer to [?].

2. The Cauchy problem.

We shall assume that $P(x,D)$ satisfies at least one of the conditions $(\mathrm{A})_{z^0}$, $(\mathrm{B})_{z^0}$ and $(\mathrm{C})_{z^0}$ for $z^0 \in S^*\mathbf{R}^n$ with $dp(z^0) = 0$, which will be defined below. Let $z^0 = (x^0,\xi^0) \in S^*\mathbf{R}^n$ satisfy $dp(z^0) = 0$. We say that $P(x,D)$ satisfies the condition $(\mathrm{A})_{z^0}$ if $(\mathrm{A}\text{-}1)_{z^0}$ and $(\mathrm{A}\text{-}2)_{z^0}$ below are fulfilled.

$(\mathrm{A}\text{-}1)_{z^0}$ $\exists$conic nbd $\mathcal{C}$ of z^0, $\exists$conic nbd $\tilde{\mathcal{C}}$ of (y^0,η^0), $\exists$homo. canonical transf. $\chi : \tilde{\mathcal{C}} \xrightarrow{\sim} \mathcal{C}$, $\exists$symbols $q(\eta)$, $e(y,\eta)$ s.t. $z^0 = \chi(y^0,\eta^0)$, $q(\eta)$: pos. homo. of deg. $m' \in \mathbf{N}$ and $p(\chi(y,\eta)) = e(y,\eta)q(\eta)$, $e(y,\eta) \neq 0$ for $(y,\eta) \in \tilde{\mathcal{C}}$.

Let F_1 and F_2 be classical Fourier integral operators corresponding to χ and χ^{-1} which are elliptic at (y^0,η^0) and z^0, respectively. Under $(\mathrm{A}\text{-}1)_{z^0}$ we can write

$$\sigma(F_2P(x,D)F_1)(y,\eta) = \tilde{e}(y,\eta)(q(\eta) + s(y,\eta))$$

in a conic nbd $\tilde{\mathcal{C}}_0$ of (y^0,η^0) if $|\eta| \geq 1$, where $\tilde{e}(y,\eta)$ is a classical symbol, which is elliptic in $\tilde{\mathcal{C}}_0$, and $s(y,\eta) \in S^{m'-1}_{1,0}$ is a classical symbol.

(A-2)$_{z^0}$ $\exists C_0 > 0$, $\exists C_\alpha > 0$ for $\forall\alpha$ s.t.

$$|s_{(\alpha)}(y,\eta)| \leq C_\alpha |q(\eta;\tilde{\vartheta})| \quad \text{for } (y,\eta) \in \tilde{\mathcal{C}}_0 \text{ with } |\eta| \geq C_0,$$

where $\tilde{\vartheta} = d\eta_{z^0}(0,\vartheta)$, $\chi^{-1}(x,\xi) = (y(x,\xi),\eta(x,\xi))$ and $q(\eta;\zeta) = \sum_{|\alpha|\leq m}(-i\zeta)^\alpha q^{(\alpha)}(\eta) \times\alpha!^{-1}$ for $\zeta \in \mathbf{R}^n$.

We note that $q(\eta)$ is microhyperbolic with respect to $\tilde{\vartheta}$ at (y^0,η^0) (see [16]). By the definition of microhyperbolicity we may assume that $|q(\eta; s\tilde{\vartheta})| \geq cs^m$ for $(y,\eta) \in \tilde{\mathcal{C}}_0$ and $0 < s \leq s_0$, where $c > 0$ and $s_0 > 0$. This implies that $\tilde{\vartheta} \neq 0$ (see, also, Lemma 3.2 in [15]). We say that $P(x,D)$ satisfies the condition (B)$_{z^0}$ if (B-1)$_{z^0}$–(B-3)$_{z^0}$ below are fulfilled.

(B-1)$_{z^0}$ $\exists$conic nbd $\mathcal{C}$ of z^0, $\exists$conic nbd $\tilde{\mathcal{C}}$ of (y^0,η^0), $\exists$homo. canonical transf. $\chi : \tilde{\mathcal{C}} \xrightarrow{\sim} \mathcal{C}$, $\exists$real-valued funs $t_j(y) \in \mathcal{B}^\infty(\mathbf{R}^n)$ ($1 \leq j \leq N$), $\exists$real-valued symbols $\lambda(y,\eta')$, $\alpha(y,\eta')$, $e(y,\eta)$ and $\exists C > 0$ s.t. $z^0 = \chi(y^0,\eta^0)$, $\eta^0 = (0,\cdots,0,1)$, $d\chi_{(y^0,\eta^0)}(0,\vartheta), d\chi_{(y^0,\eta^0)}(0,\nabla t_j(y^0)) \in \Gamma(p_{z^0},(0,\vartheta))$ ($1 \leq j \leq N$), $\lambda(y,\eta')$, $\alpha(y,\eta')$: pos. homo. of deg. 1, 2 (resp.), $p(\chi(y,\eta)) = e(y,\eta)(\eta_1(\eta_1 - \lambda(y,\eta')) - \alpha(y,\eta'))$ and $e(y,\eta) \neq 0$ for $(y,\eta) \in \tilde{\mathcal{C}}$, $\lambda(y^0,\eta^{0\prime}) = \alpha(y^0,\eta^{0\prime}) = 0$, and $\alpha(y,\eta') \geq 0$ and $T(y)\partial\alpha/\partial y_1(y,\eta') \leq C\alpha(y,\eta')$ for $(y,\eta') \in \tilde{\mathcal{C}}'$, where $p_{z^0}(\delta z)$ ($\not\equiv 0$ in $\delta z \in \mathbf{R}^{2n}$) is defined as $p(z^0+s\delta z) = s^\mu(p_{z^0}(\delta z)+o(1))$ when $s \to 0$, $\Gamma(p_{z^0},(0,\vartheta))$ is the connected component of $\{\delta z \in \mathbf{R}^{2n}; p_{z^0}(\delta z) \neq 0\}$ containing $(0,\vartheta)$ ($\in \mathbf{R}^{2n}$), $T(y) = \min_{1\leq j\leq N}|t_j(y)|$ and $\tilde{\mathcal{C}}' = \{(y,\eta'); (y,\eta) \in \tilde{\mathcal{C}} \text{ for some } \eta_1\}$.

Let F_1 and F_2 be classical Fourier integral operators corresponding to χ and χ^{-1} which are elliptic at (y^0,η^0) and z^0, respectively. Under (B-1)$_{z^0}$ we can write

$$\sigma(F_2P(x,D)F_1)(y,\eta) = \tilde{e}(y,\eta)\{\eta_1(\eta_1 - \lambda(y,\eta')) - \alpha(y,\eta') + \beta(y,\eta)\}$$

in a conic nbd $\tilde{\mathcal{C}}_0$ of (y^0,η^0) if $|\eta| \geq 1$, where $\tilde{e}(y,\eta)$ is a classical symbol, which is elliptic in $\tilde{\mathcal{C}}_0$, and $\beta(y,\eta) \in S^1_{1,0}$ is a classical symbol.

(B-2)$_{z^0}$ $\exists$classical symbol $A(y,\eta') \in S^0_{1,0}$, $\exists C > 0$ s.t.

$$T(y)|\Im\beta(y,0,\eta') - \sum_{j=2}^n(\partial^2\alpha/\partial y_j\partial\eta_j)(y,\eta')/2 - A(y,\eta')\lambda(y,\eta')|$$
$$\leq C(\alpha(y,\eta')^{1/2}+1) \quad \text{for } (y,\eta') \in \tilde{\mathcal{C}}_0'.$$

We say that the condition (B-3)$_{z^0}$ is satisfied if at least one of the following conditions (B-3-1)$_{z^0}$ and (B-3-2)$_{z^0}$ is satisfied:

(B-3-1)$_{z^0}$ $\exists$classical symbol $B(y,\eta') \in S^0_{1,0}$, $\exists C > 0$ s.t.

$$T(y)|\Re\beta(y,0,\eta') - B(y,\eta')\lambda(y,\eta')| \leq C(\alpha(y,\eta')^{1/2}+1)$$

for $(y,\eta') \in \tilde{\mathcal{C}}_0'$.

$(\text{B-3-2})_{z^0}$ $\quad \Re P_{m-1}(z^0) < \mathrm{Tr}^+ F_p$,
where F_p denotes the Hamilton map (fundamental matrix) corresponding to the Hessian of $p(z)/2$ at $z = z^0$ and $\mathrm{Tr}^+ F_p = \sum \lambda_j$, the $i\lambda_j$ are the eigenvalues of F_p with pos. imaginary parts.

We say that $P(x, D)$ satisfies the condition $(\mathrm{C})_{z^0}$ if the following conditions $(\text{C-1})_{z^0}$ and $(\text{C-2})_{z^0}$ are fulfilled:

$(\text{C-1})_{z^0}$ $\exists$conic nbd $\mathcal{C}$ of z^0, $\exists$ conic nbd $\tilde{\mathcal{C}}$ of (y^0, η^0), $\exists$homo. canonical transf. $\chi : \tilde{\mathcal{C}} \xrightarrow{\sim} \mathcal{C}$, $\exists$real-valued funs $t_j(y) \in \mathcal{B}^\infty(\mathbf{R}^n)$ ($1 \le j \le N$), $\exists c > 0$ s.t. $z^0 = \chi(y^0, \eta^0)$, $d\chi_{(y^0,\eta^0)}(0, \nabla t_j(y^0)) \in \Gamma(p_{z^0}, (0, \vartheta))$ ($1 \le j \le N$),

$$h_{m-1}(x,\xi) \ge c h_{m-2}(x,\xi) T(x,\xi)^2$$

for $(x,\xi) \in \mathcal{C}$ with $|\xi| = 1$, where $h_k(x,\xi)$ ($0 \le k \le m$) are defined as $|p(x, \xi - is\vartheta)|^2 = \sum_{j=0}^m s^{2j} h_{m-j}(x,\xi)$ for $(x,\xi) \in T^*\mathbf{R}^n$ and $s \in \mathbf{R}$, $T(x,\xi) = \min_{1 \le j \le N} |t_j(y(x,\xi))|$ and $\chi^{-1}(x,\xi) = (y(x,\xi), \eta(x,\xi))$.

$(\text{C-2})_{z^0}$ $\exists C > 0$ s.t.

$$|P_j(x,\xi)| \le C h_{2j-m}(x,\xi)^{1/2}$$

for $(x,\xi) \in \mathcal{C}$ with $|\xi| = 1$ and $[m/2] + 1 \le j \le m-1$, where $P_j(x,\xi) = \sum_{|\alpha|=j} a_\alpha(x)\xi^\alpha$ ($0 \le j \le m-1$).

Theorem 2. *Assume that* (P-1) *and* (P-4) *are satisfied and that at least one of the conditions* $(\mathrm{A})_{z^0}$, $(\mathrm{B})_{z^0}$ *and* $(\mathrm{C})_{z^0}$ *is satisfied if* $z^0 = (x^0, \xi^0) \in S^*\mathbf{R}^n$, $x_1^0 \ge 0$ and $dp(z^0) = 0$.*Then* $\forall f \in \mathcal{D}'$ *(resp.* C^∞*) with* supp $f \subset \{x_1 \ge 0\}$, $\exists! u \in \mathcal{D}'$ *(resp.* C^∞*) s.t.* $P(x, D)u = f$, supp $u \subset \{x_1 \ge 0\}$. *Moreover,* supp $u \subset \{x \in \mathbf{R}^n;\ x \in K_y^+$ *for some* $y \in$ supp $f\}$.

Theorem 2 can be proved, applying Theorem 1 (or an improved version) (see Theorem 5.2 in [8]). If one finds other conditions to give microlocal *a priori* estimates at $z^0 \in S^*\mathbf{R}^n$, one can add these conditions to $(\mathrm{A})_{z^0}$, $(\mathrm{B})_{z^0}$ and $(\mathrm{C})_{z^0}$ and improve Theorem 2. The condition $(\mathrm{A})_{z^0}$ is satisfied if the characteristic roots of $p(x,\xi) = 0$ are involutive and the lower order terms of $P(x, D)$ satisfy the so-called Levi conditions, which was studied by Zeman [17] (see, also, [1], [10]). The condition $(\mathrm{B})_{z^0}$ implies that $p(x,\xi)$ has double characteristics at z^0. If $P(x, D)$ is effectively hyperbolic, then $P(x, D)$ satisfies both the conditions $(\mathrm{B})_{z^0}$ and $(\mathrm{C})_{z^0}$. C^∞ well-posedness of the Cauchy problem for effectively hyperbolic operators was proved by Iwasaki [6] (see, also, [3], [5], [11], [12], [14]). The operators, which was treated by Ivrii [4], satisfy the condition $(\mathrm{B})_{z^0}$ with $T(y) \equiv 1$ (see, also, [2]). We can treat both effectively hyperbolic cases and non effectively hyperbolic cases at the same time by introducing $T(y)$ in $(\mathrm{B})_{z^0}$. The condition $(\mathrm{C})_{z^0}$ is closely related to the conditions given in [13]. Some results on propagation of singularities were essentially obtained in [15] if the operators are microhyperbolic at z^0 and satisfy $(\mathrm{A})_{z^0}$. For microhyperbolic operators satisfying $(\mathrm{B})_{z^0}$ we obtained theorems on propagation of singularities in [13]. Applying the methods developed in [7], we can also obtain theorems on propagation of singularities for microhyperbolic operators which satisfy at least one of the conditions $(A)_{z^0}$, $(B)_{z^0}$ and $(C)_{z^0}$. For the proof of Theorem 2 and the detail we refer to [9].

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