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A Method for Evaluating Randomness of Random Sequence Based on Perron-Frobenius Operator

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SUMMARY

A new statistical test has been recently presented in which one transform a real-valued random sequence into a binary sequence using any threshold function and determine whether such a transformed binary sequence precisely mimics Bernoulli trials $B(p, 1-p)$ with probabilities of 0 and of 1, p and $1-p$, each being equal to ones of the binary sequence, or not. This paper gives a theoretical test based on such a stringent test and shows its usefulness. This method uses the ensemble average technique under the assumption that the pseudorandom-number generator is mixing with respect to an absolutely continuous invariant measure. The existence of such a measure permits us to theoretically calculate the ensemble average of several statistics in the newly introduced statistical tests by using the Perron-Frobenius integral operator. Furthermore, this operator releases us from cumbersome and tedious procedures to calculate several joint probability distributions, in connection with several statistical tests. Three kinds of tests, the runs test, poker test, and serial correlation test are presented.

To overcome difficulties concerning the infinite-dimensionality property of this integral operator, a functional space is defined on which the absolutely continuous invariant measure is precisely approximated. The Galerkin approximation to the operator on such a suitably selected functional space is also introduced which provides a finite dimensional matrix(referred to as a *Galerkin-approximated matrix of the Perron-Frobenius operator*). The ratio of the largest eigenvalue of such a matrix to 1 is a kind of measures determing whether the Galerkin approximation to the invariant measure is good or not. The eigenvector with the largest eigenvalue of the matrix gives the approximated invariant measure. Each theoretical value of three tests for $B(p, q)$ shows that the magnintude of the second largest eigenvalue plays an important role in determing randomness of the sequence generated by the generation.