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HOLE DYNAMICS OF ONE-DIMENSIONAL PLASMA

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ABSTRACT

A system of N negatively-charged sheets with uniform positive background is studied numerically by transforming the equations of motion into finite-time mapping of $2N$ degrees of freedom. Evolution of the system from two-stream instability towards large-vortex states is shown.

1. INTRODUCTION

A collisionless plasma is described by Vlasov equation

$$\frac{\partial f_{\pm}(x,v)}{\partial t}+v\frac{\partial f_{\pm}(x,v)}{\partial x}\mp\frac{e}{m_{\pm}}\frac{\partial\phi(x)}{\partial x}\frac{\partial f_{\pm}(x,v)}{\partial v}=0 \tag{1}$$

where $f_{+}(x,v)$ and $f_{-}(x,v)$ are the distribution function of ions and the distribution function of electrons, respectively, in the phase space of the position x and the velocity v. $+e$ is the charge of an ion and $-e$ is the charge of an electron. m_{+} and m_{-} are the masses of an ion and an electron, respectively. Eq.(1) is supplemented with the Poisson's equation for the potential $\phi(x)$,

$$\frac{\partial^2\phi(x)}{\partial x^2}=-4\pi e(n_{+}-n_{-}) \tag{2}$$

where the number density $n_{\pm}$ is given by

$$n_{\pm}(x)=\int_{-\infty}^{\infty}dv\ f_{\pm}(x,v) \tag{3}$$

Bernstein, Greene and Kruskal[1] found steady state solutions of the one-dimensional Vlasov equations

$$v\frac{\partial f_{\pm}(x,v)}{\partial x}\mp\frac{e}{m_{\pm}}\frac{\partial\phi(x)}{\partial x}\frac{\partial f_{\pm}(x,v)}{\partial v}=0 \tag{4}$$

with the Poisson's equation (2)-(3).

The general solution of Eq.(4) is

$$f_{\pm}(x,v)=f_{\pm}(E_{\pm}) \tag{5}$$

with

$$E_{\pm}\equiv\frac{1}{2}m_{\pm}v^2\pm e\phi \tag{6}$$

Therefore, for an arbitrarily given functions of $f_{\pm}(E_{\pm})$, $\phi(x)$ is the solution of the following integro-differential equation

$$\frac{d^2\phi(x)}{dx^2} = 4\pi e\{\int_{-e\phi}^{\infty} dE \frac{f_{-}(E)}{[2m_{-}(E+e\phi(x))]^{1/2}} - \int_{e\phi}^{\infty} dE \frac{f_{+}(E)}{[2m_{+}(E-e\phi(x))]^{1/2}}\} \tag{7}$$

The solutions for the integro-differential equations are listed in $[2-3]$. However, the stability of the steady state solutions have been studied only for restricted conditions analytically.$[2-4]$.

On the other hand, Dupree[5] and Kadometsev-Pogutse [6] suggested the existence of highly correlated structures in a turbulent plasma, called "clumps". Therefore it is important to study not only the steady state solution of the Vlasov equation but also its time evolution.

In this report, we show results of our numerical study on a one-dimensional plasma model. Our model is a mechanical model. Namely, we solve the equation of motions for the system of charged particles. We should note that the Vlasov equation is based on the assumption that the electric field due to the presence of charged particles is a self-consistent field; namely the electric field is determined by the ditribution function of charged particles. This is valid when there are sufficiently many particles within the Debye length. So it is important to perform particle simulations for various densities of charged particles in order to see the effects of individual collisions, which is neglected in the Vlasov equation, on the evolution of the distribution functions of charged particles.

In Section 2, we describe the model and the results are summarized in Section 3. Section 4 is devoted to concluding remarks.

2. Dawson's Model

The model system, which was proposed by Dawson $[7-8]$, consists of N electron sheets and of positive uniform ion background, confined within a distance L. Since the system is one-dimensional, the electric field is a linear function of the space x and it jumps with a finite amount $-4\pi\sigma$ at the position of a sheet, where $-\sigma$ is the charge per unit area on the electron sheet. We denote by x_i the position of the i-th electron sheet from the left boundary of the system. Then, the equation of motion is written as

$$\frac{d^2x_i}{dt^2} = -\omega_p^2[x_i - (i-\frac{1}{2})\Delta] \tag{8}$$

where ω_p is the plasma frequency

$$\omega_p \equiv \frac{4\pi\sigma^2}{m\Delta} \tag{9}$$

m is the mass per unit area of the electron sheet and Δ is the average distance of neighboring pairs of sheets, $\Delta = L/N$.

Eq(1) is completely solved for a given initial condition of positions $\mathbf{x}^0 = (x_1^0, ..., x_N^0)$ and velocities $\mathbf{v}^0 = (v_1^0, ..., v_N^0)$. Thus,

$$x_i(t) = X_i(x_i^0, v_i^0, t), \qquad v_i(t) = V_i(x_i^0, v_i^0, t) \tag{10}$$

where

$$X_i(x, v, t) \equiv \Delta(i - \frac{1}{2}) + [x - \Delta(i - \frac{1}{2})]\cos(\omega_p t) + \frac{v}{\omega_p}\sin(\omega_p t) \tag{11}$$

$$V_i(x, v, t) \equiv v\cos(\omega_p t) - \omega_p[x - \Delta(i - \frac{1}{2})]\sin(\omega_p t) \tag{12}$$

Then, it is possible that a pair of neighboring sheets may cross each other. Suppose at time $t_c(i)$, the i-th sheet and the $(i+1)$-th sheet cross each other for the first time from a given initial condition. Then $t_c(i)$ is given as the solution of $X_i(x_i^0, v_i^0, t) = X_{i+1}(x_{i+1}^0, v_{i+1}^0, t)$, namely

$$[x_{i+1}^0 - x_i^0 - \Delta]\cos(\omega_p t) + \frac{1}{\omega_p}(v_{i+1}^0 - v_i^0)\sin(\omega_p t) + \Delta = 0 \tag{13}$$

which can be expressed analytically. After finding crossing times for all pairs of neighboring sheets $t_c(i)$, we look for the minimum of them, $t_c = \min_i t_c(i)$ and for the pair i_c which corresponds to t_c. That is, the pair of neighboring sheets, i_c and $i_c + 1$, are the ones which cross each other at the earliest time of all neighboring pairs in the system. After the two sheets exchange the velocities, their subsequent evolution is again given by Eq.(11). However, in updating the initial conditions, we should find the first crossing pair i_c and the corresponding crossing time, which is given by $t_c = \min_i t_c(i)$. Thus after the crossing, the evolution of the system is described by

$$\begin{cases} x_i(t) = X_i(x_i(t_c), v_i(t_c), t - t_c) \\ v_i(t) = V_i(x_i(t_c), v_i(t_c), t - t_c) \end{cases} (i \neq i_c, i_c + 1) \tag{14}$$

$$\begin{cases} x_{i_c}(t) = X_{i_c}(x_{i_c}(t_c), v_{i_c+1}(t_c), t - t_c) \\ v_{i_c}(t) = V_{i_c}(x_{i_c}(t_c), v_{i_c+1}(t_c), t - t_c) \end{cases} \tag{15}$$

$$\begin{cases} x_{i_c+1}(t) = X_{i_c+1}(x_{i_c+1}(t_c), v_{i_c}(t_c), t - t_c) \\ v_{i_c+1}(t) = V_{i_c+1}(x_{i_c+1}(t_c), v_{i_c}(t_c), t - t_c) \end{cases} \tag{16}$$

Thus, we can again determine the subsequent crossing time $t_c^{(k+1)}$ and the configuration of N particles in the phase space at this crossing time from the configuration $(\mathbf{x}^{(k)}, \mathbf{v}^{(k)})$ at the previous crossing time $t_c^{(k)}$. Namely, the time interval $\tau_c^{(k+1)} \equiv t_c^{(k+1)} - t_c^{(k)}$ between the k-th crossing and the $(k+1)$-th crossing is a function of the configuration at the k-th crossing. Therefore, the configuration $(\mathbf{x}^{(k+1)}, \mathbf{v}^{(k+1)})$ at the $(k+1)$-th crossing is specified completely by the time interval $\tau_c^{(k+1)}$ and by the configuration $(\mathbf{x}^{(k)}, \mathbf{v}^{(k)})$ at the k-th crossing. Thus we have the $2N$-dimensional mapping.

$$\tau_c^{(k+1)} = T(\mathbf{x}^{(k)}, \mathbf{v}^{(k)}), \quad \mathbf{x}^{(k+1)} = \mathbf{X}(\mathbf{x}^{(k)}, \mathbf{v}^{(k)}, \tau_c^{(k+1)}), \quad \mathbf{v}^{(k+1)} = \mathbf{V}(\mathbf{x}^{(k)}, \mathbf{v}^{(k)}, \tau_c^{(k+1)}) \tag{17}$$

3. Results

We have analyzed the two-stream instability of $N = 1260$ particles in the one-dimensional space of length $L = 4\pi$ as shown in the figures below. We initially put 630 electron sheets with positive velocities and 630 electron sheets with negative velocities. The signs of the velocities of neighboring sheets are opposite. The initial positions of the sheets are randomly distributed and the distribution of velocities is chosen to be narrow ("cold beams"). In fact, we put initially

$$v(x) = \pm v_0[1 + a\cos(4\pi x/L) + bR] \tag{18}$$

where $a \approx 10^{-4}$, $b \approx 10^{-8}$ and R is a random number with $|R| < 1$. We take periodic boundary condition; namely, when we have $x_N(t) - x_1(t) = L$, we consider that the 1-st sheet and the N-th sheet cross each other. The time is scaled by $1/\omega_p$.

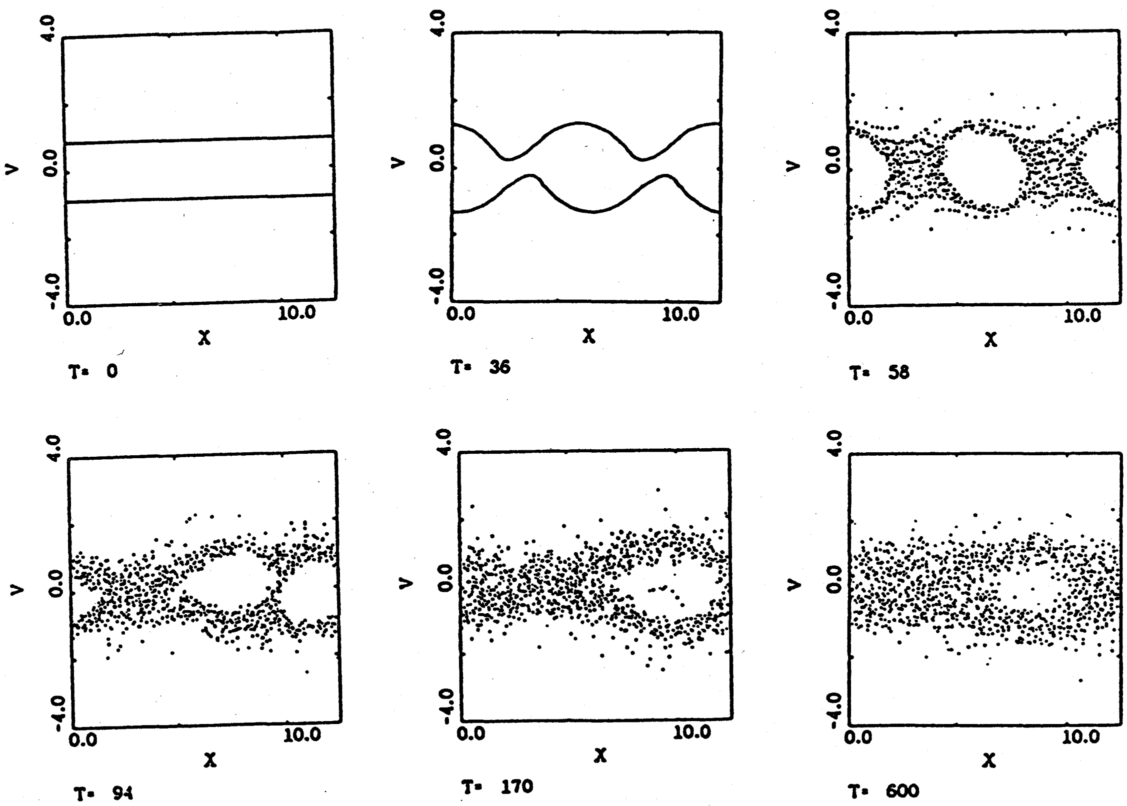

Fig. A typical evolution of 1260 sheets in the phase space.
The V axis is the velocity axis and the X axis is the position axis.
Time T is scaled by the plasma frequency.

We can see the quick evolution of a large vortices in the phase space and the subsequent collapse into a single vortex. The single vortex structure seems very stable and it decays after a long time. This type of structure formation was observed in the computer simulation by Ghizzo et.al.[10].

We analyzed also the mean square of relative displacement of two sheets, $< [x_i(t) - x_j(t)]^2 >$ at the later stage ($\omega_p t \approx 2000$). It turns out to be proportional to t^3 for large t, implying that the system is in a turbulent state.

We examined the statistics of the electric field in order to see whether the Gaussian assumption, which has been used in theretical works [11], really holds. The ratio of the fourth cumulant versus the square of the second cumulant seems to fluctuate between 1 and -1. Thus we cannot verify that the Gaussian approximation may hold.

4. CONCLUDING REMARKS

We have shown that the system of charged sheets can be handled exactly in numerical calculation by transforming the equations of motion into a mapping. This is due to the peculiarity of the one-dimensional system. We can use this simple model to check several important aspects of the system of charged particles, for example, as we mentioned above, the transition from collisional regime to collisionless regime, charecteristics of plasma turbulence and so on.

We may generalize the idea of mapping to the case of ion plasma [11], in which electrons are in thermal equilibrium for a given configuration of ions, although the formulation is far more complicated.

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