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COMBINATORIAL INVARIANTS OF $\mathcal{G}$-MANIFOLDS.

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1. INTRODUCTION.

In this work we define and investigate some combinatorial invariants of a manifold $\mathcal{M}$ on which compact Lie group $\mathcal{G}$ acts (shortly $\mathcal{G}$-manifold). For this aim we first define an addition structure on $\mathcal{M}$ which is called "admissible section σ" on $\mathcal{M}$. σ is really piecewise continuous section of a topological bundle $\kappa : \mathcal{M} \to \Omega$ of $\mathcal{M}$ over its orbit space $\Omega = \mathcal{G} \backslash \mathcal{M}$. The "pieces" on which σ is continuous are strata of some stratification of orbit space Ω. The stratification is compatible with orbit types.

So the main object of the work is a pair $(\mathcal{M}, \sigma)$ which consists of a $\mathcal{G}$-manifold $\mathcal{M}$ and its admissible section σ.

Two invariants of a pair $(\mathcal{M}, \sigma)$ are constructed. One of them is (co-) representation θ of the poset $\mathcal{R}$ of all strata on Ω into the poset of closed subgroups of a group $\mathcal{G}$. Another is 1-cocycle ξ of the poset with values in combinatorial sheaf of Lie groups which is defined in the work.

We prove that this invariants are the full system of invariants of a pair $(\mathcal{M}, \sigma)$. There is a construction of a topological space $\mathcal{M}_\Lambda$ on which group $\mathcal{G}$ acts ($\mathcal{G}$-space) with given list of invariants Λ and fixed admissible section $\sigma_\Lambda \subset \mathcal{M}_\Lambda$. $(\mathcal{M}, \sigma)$ is equivalent to $(\mathcal{M}_\Lambda, \sigma_\Lambda)$ if $\Lambda = \Lambda(\mathcal{M}, \sigma)$.

This construction is in some sense the natural generalization of the construction of the homogeneous space $\mathcal{M}$ as factor $\mathcal{G}/\mathcal{H}$ of a group by its subgroup $\mathcal{H}$ but the general construction needs more combinatorial approach described here. It seems that this construction may be useful in different questions concerned with Lie group actions on manifolds. It would be very interesting to define different additional structures on $\mathcal{M}_\Lambda$ in regular way. Smooth structure can be defined so and some other. Particularly the questions about hypergeometrical systems and its generalizations was stimulated for this work.

We start this investigations with D. Alexeevski in [5] where classification of Rimannian manifolds with one dimensional orbit space is done. Different partial cases of the suggested construction arises in publications. Note at least the works of Palais, Terng [3] and Szente [4] on Rimannian $\mathcal{G}$-manifolds and its submanifolds which is orthogonal to any orbit.

In this paper we try to formulate some basic notions and statements of the suggested approach without proves which is not very complicated.

2. INVARIANTS OF A $\mathcal{G}$-MANIFOLD.

Denote by $\mathcal{M}$ a $\mathcal{G}$-manifold. That is a topological manifold $\mathcal{M}$ on which Lie group $\mathcal{G}$ acts continuously. Some restrictions on $\mathcal{G}$ and $\mathcal{M}$ are suggested in the paper.

1) $\mathcal{G}$ is a compact Lie group not necessary connected.

2) There is a smooth structure on $\mathcal{M}$ in which $\mathcal{G}$ acts smoothly. However a smooth structure on $\mathcal{M}$ isn't fixed. (This suggestion may be changed by a more combinatorial one.)

3) There are only finite number of types of $\mathcal{G}$-orbits on $\mathcal{M}$. Equivalently $|\mathcal{R}| < \infty$ in notations introduced below.

Denote by Ω the orbit space $\mathcal{G} \setminus \mathcal{M}$ and by $\kappa : \mathcal{M} \to \Omega$ the natural projection. It is known that Ω is Haussdorff topological space.

To define stratification $\mathcal{R}$ of Ω on orbit types we say that two points $a, a' \in \Omega$ are equivalent iff they have representatives $x \in \kappa^{-1}(a), x' \in \kappa^{-1}(a')$ in $\mathcal{M}$ with the same stabilizers $\mathcal{G}_x = \mathcal{G}_{x'}$. A connected component of a class of equivalent elements we call stratum. So we have division $\Omega = \bigsqcup_{r\in\mathcal{R}} \Omega_r$ of Ω into strata $\Omega_r \subset \Omega$. We denote by $\mathcal{R}$ the set of all strata and $r \in \mathcal{R}$ corresponds to a stratum Ω_r which is the subset of points of Ω.

Next statements are well known [1].

PROPOSITION 1.1. *Let $\mathcal{G}$ be compact Lie group and let $\mathcal{M}$ be smooth $\mathcal{G}$-manifold. Then*

(1) *each stratum Ω_r is a manifold;*
(2) *boundary $\partial\Omega_r$ consists of some strata $\Omega_{r'}$ of a less dimension;*
(3) *there is only one maximal stratum Ω_m which does not contains in a boundary of any stratum.*

Using the statement (2) of the preposition 1.1 we define a partial ordering on the set $\mathcal{R}$ of all strata: $r' < r$ iff $\Omega_{r'} \subset \partial\Omega_r$.

Generally we use the term "stratification Q on Ω" for a division $\Omega = \bigsqcup_{q\in Q} \Omega_q$ of Ω into parts Ω_q marked by elements $q \in Q$ of partial ordered set (poset) Q which satisfies the statements (1) and (2) of proposition 1.1 and for which $q' < q$ iff $\partial\Omega_q \supset \Omega_{q'}$.

Stratification $Q : \Omega = \bigsqcup_{q\in Q} \Omega_q$ is said to be subdivision of a stratification $\mathcal{R} : \Omega = \bigsqcup_{r\in\mathcal{R}} \Omega_r$ iff for $q \in Q$ there is unique $r \in \mathcal{R}$ such that $\Omega_q \subset \Omega_r$. So we have the morphism π of poset Q onto poset $\mathcal{R}$.

The central role in the work plays additional structure on $\mathcal{M}$ which is named admissible section σ. To define it we fix a subdivision $Q : \Omega = \bigsqcup_{q\in Q} \Omega_q$ of a stratification $\mathcal{R}$ of Ω on orbit types. For $q \in Q$ let σ_q be a continuous section of a bundle $\kappa_q : \mathcal{M}_q \to \Omega_q$ where $\mathcal{M}_q = \kappa^{-1}(\Omega_q)$ and $\kappa_q = \kappa|_{\mathcal{M}_q}$. We denote the image of σ_q in $\mathcal{M}$ by the same sign σ_q.

DEFINITION 1.2. A joint $\sigma = \bigsqcup_{q\in Q} \sigma_q$ of continuous sections of bundle $\kappa_q : \mathcal{M}_q \to \Omega_q$ is called admissible section of $\mathcal{G}$-manifold $\mathcal{M}$ if

(1) for any $q \in Q$ all points of $\sigma_q \subset \mathcal{M}$ have the same stabilizer $\mathcal{G}_q$;
(2) for $q, q' \in Q, q > q'$ the section σ_q may be continued to the boundary $\Omega_{q'}$ of stratum Ω_q (this continuation we denote by $\sigma_q|_{q'}$ and call a restriction of σ_q on q');
(3) the restriction $\sigma_q|_{q'}$ is a section of $\kappa_{q'} : \mathcal{M}_{q'} \to \Omega_{q'}$ which satisfy the (1) and (2) conditions above;
(4) for $q, q', q'' \in Q, q > q' > q''$ holds an equality $\sigma_q|_{q''} = (\sigma_q|_{q'})|_{q''}$;
(5) for $q, q' \in Q, q > q'$ common stabilizer of points of the $\sigma_q|_{q'}$ is equal to the $\mathcal{G}_{q'}$.

Note that admissible section may be defined in the similar way for a topological space $\mathcal{M}$ on which compact Lie group acts, not nessesary manifold.

If $\sigma = \bigsqcup_{q\in Q} \sigma_q$ is an admissible section of $\mathcal{M}$, then obviously $\mathcal{G}_{q'} \supset \mathcal{G}_q$ for $q' < q$ and $\mathcal{G}_q = \mathcal{G}_{q'}$ if $\pi(q) = \pi(q') = r \in \mathcal{R}$. so there is a map $\theta : \mathcal{R} \to \mathrm{Sub}(\mathcal{G})$ of poset $\mathcal{R}$ into the poset of all closed subgroups of the Lie group $\mathcal{G} : \theta(r) = \mathcal{G}_r$ is a common stabilizer of all points of $\sigma_q \subset \mathcal{M}$ with $\pi(q) = r$.

The map θ is an exact corepresentation of $\mathcal{R}$, i.e. if $r' < r$ than $\theta(r') \supset \theta(r)$ and $\theta(r') \neq \theta(r)$.

Obviously $\theta(\mathcal{M},\sigma) = \theta(\mathcal{M}',\sigma')$ if there exists a $\mathcal{G}$-homeomorphism $\varphi : \mathcal{M} \simeq \mathcal{M}'$ which is identical on Ω and $\varphi(\sigma) = \sigma'$.

The pair $(\mathcal{M},\sigma)$ which consists of a $\mathcal{G}$-manifold $\mathcal{M}$ and its admissible section σ has also a cohomological invariant which is 1-cocycle of a poset Q with coefficients in a constructive sheaf $\mathcal{N}$ of noncommutative Lie groups.

We define first the sheaf $\mathcal{N}$. For $q \in Q$ denote by $\mathcal{N}_q$ the factor group $\mathcal{N}_{\mathcal{G}}(\mathcal{G}_q)/\mathcal{G}_q$ of a normalizer $\mathcal{N}_{\mathcal{G}}(\mathcal{G}_q)$. Let $\eta : \Omega_q \to \mathcal{N}_q$ be a continuous map of Ω_q into the group $\mathcal{N}_q$ and $\tilde{\mathcal{N}}_q$ be the group of all those maps.

Let $q > q'$. Then the group $\mathcal{N}_{q,q'} = \mathcal{N}_{\mathcal{G}}(\mathcal{G}_q) \cap \mathcal{N}_{\mathcal{G}}(\mathcal{G}_{q'})$ has images $\mathcal{N}^{(q)}_{q,q'} = \mathcal{N}_{q,q'}/\mathcal{G}_q$ and $\mathcal{N}^{(q')}_{q,q'} = \mathcal{N}_{q,q'}/\mathcal{G}_{q'}$ both in the groups $\mathcal{N}_q$ and $\mathcal{N}_{q'}$. The group $\mathcal{G}_{q'}$ contains $\mathcal{G}_q$ so there is the homomorphism $\mathcal{N}^{(q)}_{q,q'} \to \mathcal{N}^{(q')}_{q,q'}$ of a subgroup $\mathcal{N}^{(q)}_{q,q'}$ of $\mathcal{N}_q$ into the group $\mathcal{N}^{(q')}_{q,q'} \subset \mathcal{N}_{q'}$.

A map $\eta_{q'} \in \tilde{\mathcal{N}}_{q'}$ is called a restriction of a map $\eta_q \in \tilde{\mathcal{N}}_q$ if for any point $a' \in \Omega_{q'}$ there is a limit $\lim_{a\to a'}\eta_q(a), a \in \Omega_q$, in factor-space $\mathcal{N}_q/(\mathcal{G}_{q'} \cap \mathcal{N}_q)$, it lies in a group $\mathcal{N}^{(q)}_{q,q'}/(\mathcal{G}_q \cap \mathcal{N}_q) = \mathcal{N}^{(q')}_{q,q'}$ and $\eta_{q'}(a') = \lim_{a\to a'}\eta_q(a)(\mathrm{mod}(\mathcal{G}_{q'} \cap \mathcal{N}_q))$. We denote the restriction of η_q on q' as $\eta_q|_{q'}$.

DEFINITION 1.3. A continuous map $\eta_q : \Omega_q \to \mathcal{N}_q$ is called an admissible map if

(1) for a pair $q, q' \in Q, q > q'$ there exists the restriction $\eta_q|_{q'}$;

(2) for triple $q, q', q'' \in Q, q > q' > q''$ holds an equality $\eta_q|_{q''} = (\eta_q|_{q'})|_{q''}$.

Denote by $\hat{\mathcal{N}}_q$ the subset of all admissible maps $\eta_q \in \tilde{\mathcal{N}}_q$. Obviously $\hat{\mathcal{N}}_q$ is a subgroup of the group $\tilde{\mathcal{N}}_q$ and by definition for a pair $q, q' \in Q, q > q'$ there is a restriction homomorphism $\hat{\mathcal{N}}_q \to \hat{\mathcal{N}}_{q'}, \eta_q \to \eta_q|_{q'}$.

It can be shown using the technique of Bjorner [2] that the set $\mathcal{N} = \{\hat{\mathcal{N}}_q | q \in Q\}$ of groups $\hat{\mathcal{N}}_q$ and restriction homomorphisms $\hat{\mathcal{N}}_q \to \hat{\mathcal{N}}_{q'}$ is an object which is equivalent to a combinatorial sheaf (that is a sheaf which is constant on every stratum) of a Lie groups on the stratified topological space $\Omega = \bigsqcup_{q\in Q} \Omega_q$.

In standard way one can define a noncommutative cohomologies of degree 0 and 1 of Q (equivalently of Ω) with values in $\mathcal{N}$.

DEFINITION 1.4. Let $\xi = \{\xi_{q,q'} | q, q' \in Q, q > q'\}$ be a set of admissible maps $\xi_{q,q'} \in \hat{\mathcal{N}}_{q'}$. ξ is called 1-cocycle if for a triple $q, q', q'' \in Q, q > q' > q''$ holds an equality $\xi_{q,q''} = \xi_{q,q'}|_{q''} \bullet \xi_{q',q''}$ in the group $\hat{\mathcal{N}}_{q''}$.

Turn now to a $\mathcal{G}$-manifold $\mathcal{M}$ with fixed admissible section $\sigma = \bigsqcup_{q\in Q} \sigma_q$.

PROPOSITION 1.5. *Let $\sigma = \bigsqcup_{q\in Q} \sigma_q$ be an admissible section of a $\mathcal{G}$-manifold $\mathcal{M}$. Then*

(1) *for $q, q' \in Q, q > q'$ there is a uniquely defined map $\xi_{q,q'} : \Omega_{q'} \to \mathcal{N}_{q'}$ such that $\sigma_q|_{q'}(a) \equiv \xi_{q,q'}(a)\sigma_{q'}(a)$ for any point $a \in \Omega_{q'}$;*

(2) *the set $\xi = \{\xi_{q,q'} | q > q'\}$ is a 1-cocycle of Q with values in a combinatorial sheaf $\mathcal{N} = \{\hat{\mathcal{N}}_q | q \in Q\}$.*

The proposition may be easily proved basing on the definitions of an admissible section, the sheaf $\mathcal{N}$ and 1-cocycle.

Finally for a $\mathcal{G}$-manifold $\mathcal{M}$ with fixed admissible section $\sigma = \bigsqcup \sigma_q$ we have two invariants: corepresentation $\theta : \mathcal{R} \to \mathrm{Sub}(\mathcal{G})$ and 1-cocycle $\xi \in \mathcal{Z}^1(Q, \mathcal{N})$.

3. CONSTRUCTION OF A $\mathcal{G}$-MANIFOLD WITH GIVEN INVARIANTS.

To construct a $\mathcal{G}$-manifold we fix next list of date: $\Lambda = \{\mathcal{G}, \Omega, \mathcal{R}, \theta, Q, \xi\}$

Here $\mathcal{G}$ is a compact Lie group.

Ω is Haussdorff topological space.

$\mathcal{R}$ is a stratification $\Omega = \bigsqcup_{r\in\mathcal{R}} \Omega_r$. It is suggested that $\mathcal{R}$ is finite poset with the unique maximal element $m \in \mathcal{R}$.

θ is an exact corepresentation of the poset $\mathcal{R}$ into the poset $\mathrm{Sub}(\mathcal{G})$ of all closed subgroups of the group $\mathcal{G}$.

$Q : \Omega = \bigsqcup_{q\in Q} \Omega_q$ is a stratification of Ω which is subdivision of $\mathcal{R}$ and $\pi : Q \to \mathcal{R}$ is the morphism defined for a subdivision Q of a stratification $\mathcal{R}$.

ξ is 1-cocycle of Q with values in combinatorial sheaf $\mathcal{N} = \{\hat{\mathcal{N}}_q | q \in Q\}$ which is defined in the section 2 above for any corepresentation θ.

Denote by $\mathcal{M}_\Lambda$ the joint of sets $\Omega_q \times \mathcal{G}/\mathcal{G}_{\pi(q)}$ where $\mathcal{G}_r = \theta(r) \subset \mathcal{G}$ for $r = \pi(q) \in \mathcal{R}$. So $\mathcal{M}_\Lambda = \bigsqcup_{q\in Q}(\Omega_q \times \mathcal{G}/\mathcal{G}_{\pi(q)})$.

The group $\mathcal{G}$ acts naturally on $\mathcal{M}_\Lambda : g_1(a, g_2\mathcal{G}_{\pi(q)}) = (a, g, g_2\mathcal{G}_{\pi(q)})$. Obviously the set of orbits $\mathcal{G} \setminus \mathcal{M}_\Lambda$ may be identified with Ω.

Denote by $\sigma_{\Lambda,q}$ the set of points: $\sigma_{\Lambda,q} = \{(a, e\mathcal{G}_{\pi(q)}) | a \in \Omega_q\} \subset \mathcal{M}_\Lambda$ and $\sigma_\Lambda = \bigsqcup_{q\in Q} \sigma_{\Lambda,q}$.

PROPOSITION 3.1. *There is unique up to an equivalency topology on $\mathcal{M}_\Lambda$ which satisfy properties:*

(1) *$\mathcal{G}$ acts continuously on $\mathcal{M}_\Lambda$;*

(2) *the topology on $\Omega = \mathcal{G} \setminus \mathcal{M}_\Lambda$ induced from $\mathcal{M}_\Lambda$ is equivalent to a given topology on Ω;*

(3) *$\sigma_\Lambda = \bigcup \sigma_{\Lambda,q}$ is an admissible section on $\mathcal{G}$-space $\mathcal{M}_\Lambda$;*

(4) *1-cocycle $\xi(\mathcal{M}_\Lambda, \sigma_\Lambda)$ is equal to the given 1-cocycle $\xi \in \mathcal{Z}^1(Q, \mathcal{N})$.*

The proof of the proposition can be done by standard methods of topological group theory.

So there is the canonical topology on $\mathcal{M}_\Lambda$. Note that for a sequence of points $x_i = (a_i, g_i\mathcal{G}_{\pi(q)}), a_i \in \Omega_q$ with $\lim a_i = a_0 \in \Omega_{q'}$ a point x_0 is the limit of $\{x_i\}$ iff there exists the limit $\lim\{g_i\mathcal{G}_{\pi(q')}\} = g_0\mathcal{G}_{\pi(q')}$ in factor-space $\mathcal{G}/\mathcal{G}_{\pi(q')}$ (not in $\mathcal{G}/\mathcal{G}_{\pi(q)}$!). In this case $\lim x_i = (a_0, \xi(q, q')(a_0)g_0\mathcal{G}_{\pi(q')})$.

THEOREM 3.2. *Let $\mathcal{M}$ be a $\mathcal{G}$-manifold with compact Lie group $\mathcal{G}$ and let $\sigma = \bigcup_{q\in Q} \sigma_q$ be an admissible section of $\mathcal{M}$ defined for a subdivision $Q : \Omega = \bigsqcup_{q\in Q} \Omega_q$ of a stratificaton $\mathcal{R}$ of Ω on orbit types. Denote by Λ the list of date: $\Lambda = \{\mathcal{G}, \Omega, \mathcal{R}, \theta(\mathcal{M}, \sigma), Q, \xi(\mathcal{M}, \sigma)\}$ (invariants $\theta(\mathcal{M}, \sigma)$ and $\xi(\mathcal{M}, \sigma)$ of a pair $(\mathcal{M}, \sigma)$ was defined in section 2) and construct $\mathcal{G}$-space $\mathcal{M}_\Lambda$ (defined in the section 3). Then the map $\varphi : \mathcal{M}_\Lambda \to \mathcal{M}$ defined with formula: $\varphi(a, g\mathcal{G}_{\pi(q)}) = g\sigma_q(a), a \in \Omega_q, g \in \mathcal{G}$ is $\mathcal{G}$-homeomorphism of $\mathcal{M}_\Lambda$ and $\mathcal{M}$ and φ is identical on the common orbit space Ω of $\mathcal{M}$ and $\mathcal{M}_\Lambda$.*

The next question is about $\mathcal{G}$-homeomorphity of $\mathcal{M}_\Lambda$ and $\mathcal{M}_{\Lambda'}$. Obviously we may suppose that orbit spaces $\mathcal{G}\setminus\mathcal{M}_\Lambda$ and $\mathcal{G}\setminus\mathcal{M}_{\Lambda'}$ are identified with the same topological space Ω with fixed stratification $\mathcal{R}$ on orbit types. Suppose additionally that subdivisions Q and Q' of a stratification $\mathcal{R}$ are the sama: $Q = Q'$.

THEOREM 3.3. *Let $\Lambda = \{\mathcal{G},\Omega,\mathcal{R},\theta,Q,\xi\}$ and $\Lambda' = \{\mathcal{G},\Omega,\mathcal{R},\theta,Q,\xi'\}$. Then $\mathcal{G}$-spaces $\mathcal{M}_\Lambda$ and $\mathcal{M}_{\Lambda'}$ are $\mathcal{G}$-homeomorphic with $\mathcal{G}$-homeomorphism which induce identity homeomorphism on Ω iff 1-cocycles ξ and ξ' are cohomological: $\xi'_{q,q'} = \eta_q|_{q'} \bullet \xi_{q,q'} \bullet \eta_{q'}^{-1}$ for a chain $\eta = \{\eta_q | q \in Q, \eta_q \in \hat{\mathcal{N}}_q\}$.*

We say that corepresentations θ and θ' are equivalent if for any $r \in \mathcal{R}$ there is $g_r \in \mathcal{G}$ such that $\theta'(r) = g_r\theta(r)g_r^{-1}$.

For $g \in \mathcal{G}$ we can define a conjugated corepresentation θ^g by formula $\theta^g(r) = g\mathcal{G}_r g^{-1}$. Obviously θ^g is equivalent to θ.

Generally denote by ν a set $\{g_r | r \in \mathcal{R}, g_r \in \mathcal{G}\}$ of elements of the group $\mathcal{G}$. It can be easily shown that θ^ν defined as $\theta^\nu(r) g_r\theta(r)g_r^{-1}$ is a corepresentation iff for any pair $r, r' \in \mathcal{R}, r > r'$ holds an equality $g_r\mathcal{G}_{r'}g_r^{-1} = g_{r'}\mathcal{G}_{r'}g_{r'}^{-1}$. By definition if θ' is equivalent to θ then $\theta' = \theta^\nu$ for some $\nu = \{g_r | r \in \mathcal{R}\}$.

It seems that there exist equivalent θ and θ' which is not conjugated but we don't know examples.

PROPOSITION 3.4.

(1) *If $\mathcal{M}_\Lambda$ is $\mathcal{G}$-homeomorphic to $\mathcal{M}_{\Lambda'}$ with $\mathcal{G}$-homeomorphism which is identical on Ω for $\Lambda = \{\mathcal{G},\Omega,\mathcal{R},\theta,Q,\xi\}$, $\Lambda' = \{\mathcal{G},\Omega,\mathcal{R},\theta',Q,\xi'\}$ then θ' is equivalent to θ.*

(2) *For any corepresentation θ^ν which is equivalent to θ a $\mathcal{G}$-space $\mathcal{M}_{\Lambda'}$ for $\Lambda' = \{\mathcal{G},\Omega,\mathcal{R}, \theta^\nu,Q,\xi\}$ is $\mathcal{G}$-homeomorphic to $\mathcal{M}_{\Lambda''}$ with $\Lambda'' = \{\mathcal{G},\Omega,\mathcal{R},\theta,Q,\xi''\}$.*

Generally $\mathcal{M}_\Lambda$ is not a manifold. So there is a question for which lists of data Λ the $\mathcal{G}$-space $\mathcal{M}_\Lambda$ is a manifold. We formulate here nessesary and sufficient conditions for this in partial case $\xi \equiv 1$ and Q is a triangulation of Ω.

Let $\Lambda = \{\mathcal{G},\Omega,\mathcal{R},\theta,Q,\xi\}$ is a list of data with above suggestions. For $q \in Q$ denote by $\mathcal{Z}_q$ the star of stratum Ω_q in $\Omega : \mathcal{Z}_q = \bigsqcup_{q' \geq q}\Omega_{q'}$. It can be easily proved that topological space $\mathcal{Z}_q$ is homeomorphic to a $\Omega_q \times \mathcal{C}^\star(\triangle_q)$ where $\triangle_q \approx \partial\Omega_q$ and $\mathcal{C}^\star(\mathcal{X})$ is a neighborhood of a vertex of a cone over $\mathcal{X}$.

Then we can restrict Λ on $\triangle_q$ and denote by Λ_q the next list of data $\Lambda_q = \{\mathcal{G}_q, \triangle_q, \mathcal{R}_q, \theta_q, Q_q, \xi \equiv 1\}$. Here stratifications $\mathcal{R}_q : \triangle_q = \bigsqcup_{r\in\mathcal{R}_q}\triangle_r$ and $Q_q : \triangle_q = \bigsqcup_{p\in Q_q}\triangle_p$ are induced on $\triangle_q$ from $\mathcal{R}$ and Q using homeomorphism $\mathcal{Z}_q \approx \Omega_q \times \mathcal{C}^\star(\triangle_q)$, $Q_q = \{p \in Q | p \geq q\} \subset Q$, $\mathcal{R}_q = \{r \in \mathcal{R} | r \geq \pi(q)\} \subset \mathcal{R}$ and θ_q is obviously defined restriction of θ on $\mathcal{R}_q$.

THEOREM 3.5. *Let $\Lambda = \{\mathcal{G},\Omega,\mathcal{R},\theta,Q,\xi \equiv 1\}$ and $\mathcal{M}_\Lambda$ is $\mathcal{G}$-space defined above. Then $\mathcal{M}_\Lambda$ is a manifold iff for any vertex $q \in Q$ the space $\mathcal{M}_{\Lambda_q}$ for $\Lambda_q = \{\mathcal{G}_q, \triangle_q, \mathcal{R}_q, \theta_q, Q_q, \xi \equiv 1\}$ is homeomorphic to a sphere $\mathcal{S}^m$ for some $m \geq 0$.*

Note that analoguos statement is true and in the case $\xi \not\equiv 1$ but it has more complicated formulation.

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