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Intrinsic Gap and Final–Double–Offer Arbitration

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Abstract

This paper discusses arbitration procedures assuming that the two disputants are ignorant of their opponents' estimates about the arbitrator's fair point. A measure IG, called intrinsic gap is introduced to describe the difference of the two estimates. Disputants give their arbitration offers only basing on their own estimates. It is assumed that they give prudent offers. We then derive the prudent offers for three typical arbitration procedures Final–Offer Arbitration (FOA), Binding Arbitration (BA) and Combined Arbitration (CA). The result indicate that these arbitration procedures are not powerful enough, in the sense that their contract zones are all empty when IG is positive. Then we propose a new procedure Final–Double–Offer Arbitration (FDOA) and show that it sometimes has nonempty contract zone even if IG is positive.

Key words: binding arbitration (BA), combined arbitration (CA), final–double–offer arbitration (FDOA), final–offer arbitration (FOA), intrinsic gap (IG), prudent offer.

1 Introduction

Arbitration is effective in resolving disputes that appear in politics, economics, psychology, and sociology, particularly in the labor disputes, where arbitration has been adopted as an acceptable alternative to strike or lockout. Three principal arbitration procedures have been extensively studied in the literature (Brams and Merrill III (1986)): Conventional or Binding Arbitration (BA), Final-Offer Arbitration (FOA) and Combined Arbitration (CA).

In BA, the arbitrator is free to choose any settlement that then becomes binding on the two sides. In FOA, which was proposed in 1966 (Stevens (1966)), the arbitrator is restricted to choosing one of the two "final offers" proposed by the two sides. Procedure CA is a combination of FOA and BA. It can be described as follows:

1. If the offers converge or crisscross, the mean of the two offers will be the final result.
2. FOA applies if the arbitrator's fair point falls between the two final offers.
3. BA applies if the arbitrator's fair point falls outside the two final offers, unless they converge.

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The fact that a dispute will be arbitrated if two sides can not reach an agreement will affect the prior negotiation or bargaining of the parties. Further, the effect depends on the arbitration procedure to be used and the disputants' estimates about the arbitrator's fair point. A good procedure should have a deterrent effect of providing an incentive for the parties to reach a negotiated settlement before the arbitration is really used. Usually, the effect is measured by the so-called "contract zone", which is the range of potential settlement that both parties consider preferable to arbitration.

Research to date on arbitration procedures has by and large adopted the following classical model: two disputants share a common estimate of the arbitrator's fair point that lies in interval $[L, H]$, where L (H) is the lower (upper) bound. In this model, it turns out that the common estimate makes the use of arbitration unnecessary, because impasses or disagreement should never occur (Samuelson (1988), Zeng, Chen and Ohnishi (1991), Zeng and Ohnishi (1991)).

On the other hand, the assumption that both disputants have the common estimate seems implausible, because disputant's estimate is determined by many factors, such as risk attitude, temperament which his opponent may be ignorant of. Further, as shown in Zeng, Chen and Ohnishi (1991), they may have different estimates because of the incomplete information about opponents' reservation prices. Thus, it is theoretically and practically important to consider the case in which each disputant is ignorant of his opponent's estimate.

In this paper, we first consider how to measure the difference between two estimates. When two disputants take pessimistic views about the arbitrator's fair point, they should be able to reach an agreement in stead of resorting to arbitration. Therefore, in this case, we hold that the two estimates are not intrinsically different . In terms of expected values and medians of the two estimates, we define a nonnegative quantity IG, called intrinsic gap. A positive IG means that at least one of the disputants is over optimistic about the arbitrator's fair point, and this is the main reason why disputants resort to arbitration, as analyzed in Zeng, Chen and Ohnishi (1991).

When disputants are ignorant of there opponents' estimates, we assume that they give arbitration offers which are "prudent" in the sense as used in game theory (Moulin (1986)). Since the prudent strategy in 0-sum game coincides with Nash equilibrium strategy, prudent offer coincides with the Nash equilibrium offer in the case of classical model, as mentioned above. However, when the two estimates are different, these two kinds of offer are also different.

There are some papers that discussed the case in which two disputants have different estimates. For example, Wittman (1986), Brams and Merrill III (1986) assumed that two estimates are common knowledge. An early paper by Farber and Katz (1979) discussed BA, they researched the case that $IG = 0$ and risk aversion dominates. Their conclusions is that the contract zone is empty if IG is positive and risk aversion does not dominate, and they avoided the case. however, this is an interesting case in our opion.

In Section 2, we describe the basic model, define intrinsic gap and prudent offer. Then we mention two criteria for evaluating arbitration procedures. In Section 3, we analyze three typical arbitration procedures FOA, FOA and CA and derive prudent offers for them. Our results show that FOA, BA and CA are not powerful enough to overcome positive intrinsic gap, in the sense that the contract zones are empty when IG is positive. In Section 4, we present a new arbitration procedure — Final–Double–Offer Arbitration (FDOA). In this procedure, every disputant is required to give two offers. The first is his real offer which may become the final result as in FOA. The second is his estimate about the arbitrator's fair point. Although his estimate is only probabilistic, this second offer is required to be deterministic. By calculating disputants' prudent offers in FDOA, we show that the contract zone of FDOA is always larger than that of BA, FOA or CA under very weak condition. FDOA can sometimes induce a nonempty contract zone even when IG is positive.

2 Basic Model

Two parties, refered to as seller and buyer, must jointly decide on a determinate value of some continuous variable x in $[L, H]$, where L (H) is a finite lower (upper) bound. Disputant s (seller) wants the value to be high — the higher the better — whereas b (buyer) wants the value to be low — the lower the better. We suppose that s and b are risk–neutral.

When s and b cannot reach an agreement by themselves, they resort to arbitration. For arbitration, each disputant is required to give arbitrator his offer that expresses his demand. We denote the offers of s and b by x_s and x_b when they resort to arbitration. The arbitrator's fair point is denoted by x_a.

We describe s's (b's) estimate on the arbitrator's fair point by probabilistic density function $f_s(x)$ ($f_b(x)$) and distribution function $F_s(x)$ ($F_b(x)$) over $[L, H]$. We suppose that $f_s(x)$ and $f_b(x)$ are continuous functions with means E_s, E_b, and medians m_s, m_b respectively, i.e.,

$$E_s = \int_L^H x f_s(x)dx, \qquad E_b = \int_L^H x f_b(x)dx,$$

$$F_s(m_s) = \int_L^{m_s} f_s(x)dx = \frac{1}{2}, \qquad F_b(m_b) = \int_L^{m_b} f_b(x)dx = \frac{1}{2}.$$

As analyzed in introduction, it seems natural for two parties having different and inaccurate estimates about the arbitrator's behavior. The intrinsic gap defined below measures the difference between $f_s(x)$ and $f_b(x)$.

Definition 2.1 *The intrinsic gap between the estimates of two disputants s and b is*

$$IG = \max\{0, \min\{E_s - E_b, m_s - m_b\}\}.$$

We can see that $IG \geq 0$. If s and b are pessimistic about the arbitrator's fair point, i.e., $E_s \leq E_b$, $m_s \leq m_b$, then $IG = 0$. In this case, as s and b are risk–neutral, they should be able to reach agreement in $[m_s, m_b]$, and therefore we hold that the estimates are not intrinsically different. What we are interested in is the case that $E_s > E_b$ and $m_s > m_b$, i.e., IG is positive. Zeng, Chen and Ohnishi (1991) argue that $IG > 0$ is the main reason for two disputants resort to arbitration.

Since disputant is ignorant of the estimate of his opponent, he has to give arbitration offer basing only on his own estimate. It is natural to assume that disputant take prudent strategy (Moulin (1986)). The obtained offer is the following prudent offer:

Definition 2.2 *Value x_s^* (x_b^*) is a prudent offer of s (b) under an arbitration procedure P if and only if*

$$\inf_{x_b \in [L,H]} g_s(x_s^*, x_b|P) = \sup_{x_s \in [L,H]} \inf_{x_b \in [L,H]} g_s(x_s, x_b|P)$$

$$\left(\inf_{x_s \in [L,H]} g_b(x_s, x_b^*|P) = \sup_{x_b \in [L,H]} \inf_{x_s \in [L,H]} g_b(x_s, x_b^*|P) \right)$$

where $g_s(x_s, x_b|P)$ ($g_b(x_s, x_b|P)$) denotes the expected profit of s (b) under the procedure P if s's offer is x_s and b's offer is x_b.

Two criteria have been frequently used to evaluate the effectiveness of arbitration procedures (Farber and Katz (1979)).

The first criterion is frequency. It is commonly thought that a good procedure is one that provides an incentive for the parties to reach a negotiated settlement and hence seldom used. The incentive effect is measured by the so-called *contract zone* defined below. Denote

$$EA_s(P) = \sup_{x_s} \inf_{x_b} g_s(x_s, x_b|P) + L, \tag{2.1}$$

$$EA_b(P) = H - \sup_{x_b} \inf_{x_s} g_b(x_s, x_b|P) \tag{2.2}$$

where $g_s(x_s, x_b|P)$ and $g_b(x_s, x_b|P)$ are the same as in Definition 2.1. They characterize $s's$ and $b's$ secure expected values (H. Moulin (1986)) of arbitration if arbitration procedure P is really used.

Definition 2.3 *The contract zone of procedure P is defined to be the interval $[EA_s(P), EA_b(P)]$ if $EA_s(P) \leq EA_b(P)$, and empty set if $EA_s(P) > EA_b(P)$.*

We can see that contract zone defines the range of potential settlements that both disputants consider preferable to arbitration. Disputants cannot reach agreement by themselves if the contract zone is empty. The larger the contract zone is, the larger the possibility that two disputants reach agreememt by themselves is. By the definition, we know that the length of contract zone is determined by two factors: what arbitration procedure will be employed and what the disputants' estimates are? A good procedure should produce nonempty contract zone even if $IG > 0$.

The second criterion is the extent to which the arbitrated settlement differs from the bargaining settlement that could have been reach in an environment that did not include the procedure. An arbitration procedure is said to be *of low quality* if the arbitrated result is more extreme than the bounds of the contract zone when this zone is nonempty.

3 Prudent Offers in Procedures FOA, BA and CA

In this section, we derive prudent offers in three typical arbitration procedures FOA, BA and CA. Then we calculate their contract zones and evaluate these procedures by the two criteria described in Section 2. For simplicity, we suppose that $f_s(x)$ ($f_b(x)$) is symmetric around $x = m_s$ ($x = m_b$) on its support, which implies that $E_s = m_s$ ($E_b = m_b$).

As stated in Section 2, $g_s(x_s, x_b|P)$ ($g_b(x_s, x_b|P)$) denotes the expected profit of s (b) under the procedure P if s's offer is x_s and b's offer is x_b. For simplicity, we omit the indication of procedure P, and the notation becomes $g_s(x_s, x_b)$ ($g_b(x_s, x_b)$) if it does not make confusion.

For FOA, if the offer of s is x_s, and the offer of b is x_b, then the expected profit of s is

$$g_s(x_s, x_b) = \begin{cases} g_s^1(x_s, x_b) = x_s \int_L^{\frac{x_s+x_b}{2}} f_s(x)dx + x_b \int_{\frac{x_s+x_b}{2}}^H f_s(x)dx - L & \text{if } x_s > x_b \\ g_s^2(x_s, x_b) = \frac{x_s+x_b}{2} - L & \text{if } x_s \leq x_b, \end{cases}$$

and the expected profit of b is

$$g_b(x_s, x_b) = \begin{cases} g_b^1(x_s, x_b) = H - x_s \int_L^{\frac{x_s+x_b}{2}} f_b(x)dx - x_b \int_{\frac{x_s+x_b}{2}}^H f_b(x)dx & \text{if } x_s > x_b \\ g_s^2(x_s, x_b) = H - \frac{x_s+x_b}{2} & \text{if } x_s \leq x_b. \end{cases}$$

The following theorem is an immediate consequence of the main theorem of Brams and Merrill III (1983).

Theorem 3.1 *Assume that $f_s'(m_s)$ and $f_b'(m_s)$ exist and $f_s(m_s) > 0$, $f_b(m_b) > 0$ hold. Then, in FOA,*

$$x_s^* = m_s + \frac{1}{2f_s(m_s)}, \qquad x_b^* = m_b - \frac{1}{2f_b(m_b)}$$

are prudent offers of s and b respectively, if

$$f_s(x) \leq f_s(m_s) + 4f_s^2(m_s)|x - m_s| \quad \text{for } x \text{ such that } |x - m_s| \leq \frac{1}{4f_s(m_s)},$$

$$f_b(x) \leq f_b(m_b) + 4f_b^2(m_b)|x - m_b| \quad \text{for } x \text{ such that } |x - m_b| \leq \frac{1}{4f_b(m_b)}$$

and there exist $c_{s1} \in [L, m_s]$, $c_{s2} \in [m_s, H]$, $c_{b1} \in [L, m_b]$ and $c_{b2} \in [m_b, H]$ such that

$$\begin{aligned} f_s(x) &\geq f_s(m_s)exp(-2f_s(m_s)|x - m_s|) && \text{for } c_{s1} \leq x \leq c_{s2}, \\ f_s(x) &\leq f_s(m_s)exp(-2f_s(m_s)|x - m_s|) && \text{for } x \leq c_{s1} \text{ and } x \geq c_{s2}, \\ f_b(x) &\geq f_b(m_b)exp(-2f_b(m_b)|x - m_b|) && \text{for } c_{b1} \leq x \leq c_{b2}, \\ f_b(x) &\leq f_b(m_b)exp(-2f_b(m_b)|x - m_b|) && \text{for } x \leq c_{b1} \text{ and } x \geq c_{b2}. \end{aligned}$$

We can also calculate $EA_s(FOA)$ and $EA_b(FOA)$ defined by (2.1) and (2.2) when f_s and f_b satisfy the conditions of Theorem 3.1:

$$\begin{aligned} EA_s(FOA) &= \sup_{x_s} \inf_{x_b} \{x_b F_s(\frac{x_s + x_b}{2}) + x_s[1 - F_s(\frac{x_s + x_b}{2})]\} = m_s \\ EA_b(FOA) &= \sup_{x_b} \inf_{x_s} \{x_b F_b(\frac{x_s + x_b}{2}) + x_s[1 - F_b(\frac{x_s + x_b}{2})]\} = m_b. \end{aligned}$$

As we have assumed that f_s and f_b are symmetric around m_s and m_b on their support respectively, $EA_b(FOA) - EA_s(FOA) = m_b - m_s = E_b - E_s$. Hence, contract zone is nonempty if and only if $IG = 0$, where IG is the intrinsic gap of Definition 2.1. In this sense, we say that FOA is not powerful enough to overcome positive intrinsic gap.

Farber (1980) relaxed the assumption that the two disputants are risk-neutral and discussed FOA with the condition that $f_s = f_b$. In this case, there may exist a contract zone only if risk aversion dominates (S. H. Farber and H. C. Katz (1979)). But even though the contract zone exits, their offers are out of the contract zone. Thus FOA is of low quality according to the second criterion of Section 2.

Now let us analyze BA and CA. To ensure the uniqueness of their offers, we introduce arbitration fees (Żeng, Chen and Ohnishi (1991)). We denote the total arbitration fee by (a positive number) α. The fee is paid in the following manner:

1. Every party pays $\alpha/2$, if $x_s \leq x_b$ or $|x_a - x_s| = |x_a - x_b|$,
2. s pays α if $|x_a - x_s| > |x_a - x_b|$ and b pays α if $|x_a - x_s| < |x_a - x_b|$.

We denote by $x_s^\dagger$ and $x_b^\dagger$ the solutions of the following equations:

$$x_s - E_s = \alpha\left(\frac{1}{2} - \int_L^{x_s} f_s(x)dx\right), \tag{3.1}$$

$$x_b - E_b = \alpha\left(\frac{1}{2} - \int_{x_b}^{H} f_b(x)dx\right). \tag{3.2}$$

It is easy to prove the following lemma:

Lemma 3.1 *Equation (3.1) ((3.2)) has a unique solution $x_s^\dagger$ ($x_b^\dagger$) in $[L, H]$ and*

$$|x_s^\dagger - E_s| \leq \frac{\alpha}{2} \quad (|x_b^\dagger - E_b| \leq \frac{\alpha}{2}).$$

Remark 3.1 a) When $f_s(x) = f_b(x)$, $x_s^\dagger = x_b^\dagger$.
b) If $f_s(x)$ ($f_b(x)$) is symmetric around m_s (m_b) on its support, then $x_s^\dagger = E_s$ ($x_b^\dagger = E_b$).
c) $x_s^\dagger$ and $x_b^\dagger$ converge to E_s and E_b respectively when α converges to 0.

The following two theorems tells us the prudent offers in procedures BA and CA.

Theorem 3.2 *The above $x_s^\dagger$ and $x_b^\dagger$ are the prudent offers of s and b respectively under BA with arbitration fee α.*

Theorem 3.3 *If $f_s(x)$ is nondecreasing in $[L, x_s^\dagger]$ and $f_b(x)$ is nonincreasing in $[x_b^\dagger, H]$, then $x_s^\dagger$ and $x_b^\dagger$ are the prudent offers of s and b respectively, under CA procedure with arbitration fee α.*

By Theorems 3.2, 3.3 and Remark 3.1, $EA_s(BA) = EA_s(CA) = m_s = E_s$, $EA_b(BA) = EA_b(CA) = m_b = E_b$, when f_s and f_b are unimodal, symmetric around m_s and m_b on their supports respectively. Hence, contract zone is nonempty only if $IG = 0$, i.e., neither BA nor CA is powerful enough to overcome positive IG.

The reason that we introduce arbitration fee is to ensure the uniqueness of offers. For the convenience of comparing with FOA and the following FDOA, we are interested in the case of

$\alpha \to 0$. In this limit case, by Remark 3.1, $x_s^\dagger = E_s$ and $x_b^\dagger = E_b$, and they form the bounds of contract zone. Thus procedure CA always make the final result equal to or more extreme than the bounds if the contract zone is nonempty, therefore CA is of low quality. As to procedure BA, the final result may fall into the contract zone. In this sense, BA is better than FOA and CA.

4 Final–Double–Offer Arbitration

By the theorems in Section 3, we can see that, if the arbitration fee is small, BA, FOA and CA can have a nonempty contract zone only if $IG = 0$. It is desired to introduce a procedure that can induce the existence of contract zone even at the presence of a positive intrinsic gap.

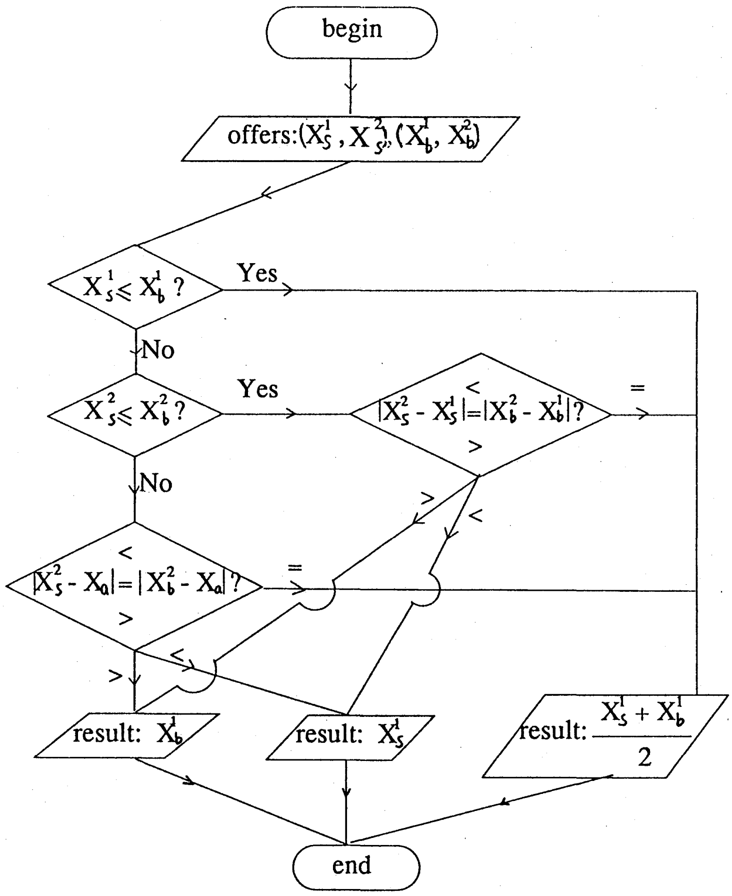

Fig. 1: flow chart of FDOA procedure

By Theorem 3.1 and Chatterjee (1981), we see that a reason why FOA fails to induce convergence is that estimates of s and b are probabilistic. However the idea of FOA to enforce the closeness between disputant's epected value of estimate and his offer is useful. Based on this observation, we introduce here a new arbitration procedure Final–Double–Offer Arbitration (FDOA). In FDOA, disputants s and b are requested to give pairs of two offers (x_s^1, x_s^2), and

(x_b^1, x_b^2) respectively. The first components x_s^1 and x_b^1 are their real offers, which may become the final result as in FOA. The second components x_s^2 and x_b^2 are their estimates of the arbitrator's fair point. Although their estimates may be probabilistic, x_s^2 and x_b^2 are required to be accurate.

As illustrated in the flow chart of Figure 1, the arbitrator will first compare x_s^2, x_b^2 with his fair point x_a. If $x_s^2 > x_b^2$, their estimates are not compatible. This time, as in FOA, the arbitrator will choose x_s^1 (x_b^1 or $(x_s^1 + x_b^1)/2$) as the final result if x_s^2 is closer (farther or equal) to x_a than x_b^2 to x_a. By this rule, we discourage the disputants to give an estimate that is too optimistic. If $x_s^2 \le x_b^2$, their estimates are compatible. This time, the arbitrator will compare $|x_s^1 - x_s^2|$ with $|x_b^1 - x_b^2|$. If $|x_s^1 - x_s^2| < |x_b^1 - x_b^2|$, s has made more concession and thus s's offer x_s^1 will be the final result. By this rule, we encourage the disputants to make their offers x_s^1 and x_b^1 close to their estimates x_s^2 and x_b^2 respectively.

Theorem 4.1 *Let $(x_s^{1\sharp}, x_s^{2\sharp})$, $(x_b^{1\sharp}, x_b^{2\sharp})$ be the prudent offers of s and b. Then $x_s^{2\sharp}$ maximizes*

$$(x_s - L)\frac{1 - F_s(x_s)}{2 - F_s(x_s)} \tag{4.1}$$

and $x_b^{2\sharp}$ maximizes

$$(H - x_b)\frac{F_b(x_b)}{1 + F_b(x_b)}, \tag{4.2}$$

furthermore $x_s^{1\sharp}$ and $x_b^{1\sharp}$ are respectively given by

$$x_s^{1\sharp} = x_s^{2\sharp} + (x_s^{2\sharp} - L)\frac{F_s(x_s^{2\sharp})}{2 - F_s(x_s^{2\sharp})}, \qquad x_b^{1\sharp} = x_b^{2\sharp} - (H - x_b^{2\sharp})\frac{1 - F_b(x_b^{2\sharp})}{1 + F_b(x_b^{2\sharp})}. \tag{4.3}$$

The resulting secure expected profit of s is

$$2(x_s^{2\sharp} - L)\frac{1 - F_s(x_s^{2\sharp})}{2 - F_s(x_s^{2\sharp})} \tag{4.4}$$

and the expected profit of b is

$$2(H - x_b^{2\sharp})\frac{F_b(x_b^{2\sharp})}{1 + F_b(x_b^{2\sharp})}. \tag{4.5}$$

Corollary 4.1 *By (2.1), (2.2), (4.4) and (4.5), we have*

$$\begin{aligned} EA_s(FDOA) &= L + 2(x_s^{2\sharp} - L)\frac{1 - F_s(x_s^{2\sharp})}{2 - F_s(x_s^{2\sharp})}, \\ EA_b(FDOA) &= H - 2(H - x_b^{2\sharp})\frac{F_b(x_b^{2\sharp})}{1 + F_b(x_b^{2\sharp})}. \end{aligned}$$

Corollary 4.2 *If $x_s^{2\sharp} \in (L, H)$ ($x_b^{2\sharp} \in (L, H)$) and $f_s(x)$ ($f_b(x)$) is differentiable, then $x_s^{2\sharp}$ ($x_b^{2\sharp}$) is a solution of*

$$\begin{aligned} (x - L)f_s(x) &= [1 - F_s(x)][2 - F_s(x)] \\ ((H - x)f_b(x) &= F_b(x)[1 + F_b(x)]) \end{aligned} \tag{4.6}$$

Now, we show the effectiveness of procedure FDOA.

Theorem 4.2 *If $f_s(x)$ and $f_b(x)$ are unimodal and symmetric around m_s and m_b on their supports respectively, then $EA_s(FDOA) \leq m_s$ and $EA_b(FDOA) \geq m_b$.*

Proof[3]: By Corollary 4.1, it is sufficient to prove

$$2(x - L)(1 - \frac{1}{2 - F_s(x)}) \leq m_s - L, \text{ for all } x \in [L, H]. \tag{4.7}$$

(i) Let $m_s = (H + L)/2$. When $x \leq (H + L)/2$, the inequality (4.7) is obvious. When $(H + L)/2 < x \leq (3H + L)/4$, we have $F_s(x) \geq 1/2$, therefore $1 - 1/(2 - F_s(x)) \leq 1/3$. Hence

$$2(x - L)(1 - \frac{1}{2 - F_s(x)}) \leq 2[\frac{3}{4}(H - L)]\frac{1}{3} = \frac{H - L}{2} = m_s - L.$$

When $x > (3/4)H + (1/4)L$, we have $F_s(x) \geq 3/4 > 2/3$, therefore

$$2(x - L)(1 - \frac{1}{2 - F_s(x)}) \leq 2(H - L)(1 - \frac{3}{4}) = \frac{H - L}{2} = m_s - L.$$

(ii) Let $m_s > (H + L)/2$. By an argument similar to (i), when $x \in [2m_s - H, H]$,

$$2(x - L)(1 - \frac{1}{2 - F_s(x)}) \leq m_s - (2m_s - H) = H - m_s \leq m_s - L.$$

When $x \in [L, 2m_s - H)$, $F_s(x) = 0$, therefore

$$2(x - L)(1 - \frac{1}{2 - F_s(x)}) = 2(x - L)\frac{1}{2} = x - L \leq m_s - L.$$

(iii) Let $m_s < (H + L)/2$. By an argument similar to (i), when $x \in [L, 2m_s - L)$,

$$2(x - L)(1 - \frac{1}{2 - F_s(x)}) \leq m_s - L.$$

When $x \in [2m_s - L, H]$, as $F_s(x) = 1$, we have

$$2(x - L)(1 - \frac{1}{2 - F_s(x)}) = 0 \leq m_s - L.$$

As (4.7) holds in all cases, we have completed our proof for s. The conclusion for b similarly holds. *Q.E.D.*

When $f_s(x)$ and $f_b(x)$ satisfy the conditions of Theorem 4.2, the contract zone may exist even if IG is positive. We show this by the following example.

Example 1: Let $L = -0.3$, $H = 1$, and we assume symmetric triangular distributions $f_s(x)$ and $f_b(x)$:

$$f_s(x) = \begin{cases} 4x & \text{if } x \in [0, \frac{1}{2}] \\ 4 - 4x & \text{if } x \in [\frac{1}{2}, 1] \\ 0 & \text{otherwise,} \end{cases} \qquad f_b(x) = \begin{cases} 4(x + 0.3) & \text{if } x \in [-0.3, 0.2] \\ 2.8 - 4x & \text{if } x \in [0.2, 0.7] \\ 0 & \text{otherwise.} \end{cases}$$

[3]To conserve space, details of the proofs of other threorems, lemmas and consequences are available from the authors.

These are unimodal and symmetric on their support around their medians $m_s = 0.5$ and $m_b = 0.2$ respectively, as shown in Figure 2. By Definition 2.1, $IG = 0.3 > 0$. Thus, according to the conclusions of Section 3, if either of FOA, BA and CA is used, the contract zone will be empty.

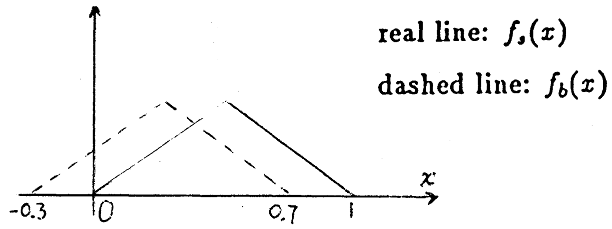

Fig.2

For $x \geq 1/2$, expression (4.1) becomes

$$\frac{x\int_x^1 (4-4x)dx}{1+\int_x^1 (4-4x)dx} = \frac{2x(2-4x+2x^2)}{3-4x+2x^2}.$$

Let

$$y(x) = \frac{2x(2-4x+2x^2)}{3-4x+2x^2}.$$

As $y'(x) < 0$, $\sup_{x \geq 1/2} y(x) = y(1/2) = 1/3$.

For $x \leq 1/2$, the maximum of expression (4.1) becomes

$$\sup_x \frac{x(1-2x^2)}{1-x^2} = \left.\frac{x(1-2x^2)}{1-x^2}\right|_{x=\frac{\sqrt{5-\sqrt{17}}}{2}} \approx 0.33674 > y(\frac{1}{2}).$$

Thus the estimate of s $x_s^{2\sharp}$ is $\frac{\sqrt{5-\sqrt{17}}}{2} \approx 0.46821$, and his offer is $x_s^{1\sharp} \approx 0.59967$ by (4.3). By (4.4), $EA_s(FDOA) \approx 0.33674$.

Similarly, $x_b^{2\sharp} \approx 0.23719$, $x_b^{1\sharp} \approx 0.10033$, $EA_b(FDOA) \approx 0.36326$. As $EA_b(FDOA) > EA_s(FDOA)$, their contract zone exists, and the disputants prefer reaching agreement in $[0.33674, 0.36326]$ to arbitration.

Example 2: $f_s(x)$ is the same as in Example 1, but $f_b(x) = f_s(x)$, so that IG=0. Of course $x_s^{1\sharp}$, $x_s^{2\sharp}$ and $EA_s(FDOA)$ is the same as Example 1. Similar to Example 1, we can show that $x_b^{1\sharp} \approx 0.40033$, $x_b^{2\sharp} \approx 0.53179$ and $EA_b(FDOA) \approx 0.66325$. Therefore, the contract zone is $[0.33674, 0.66325]$. It is interesting to note that both $x_s^{1\sharp}$ and $x_b^{1\sharp}$ belong to this interval. As the final result of FDOA will be either $x_s^{1\sharp}$, or $x_b^{1\sharp}$ or their mean, the result will fall into this interval. Therefore according to the second criterion described in Section 2, FDOA is not of low quality in this case.

Example 3: Consider the case of uniform distribution, $L = 0$, $H = 1$, $f_s(x) = 1$. As $f_s(x)$ is not strictly unimodal function, unless we consider (positive) arbitration fee, arbitration procedure CA fails to induce the convergence of offers if $f_b(x) = f_s(x)$ (Brams and Merrill

(1986), Zeng, Chen and Ohnishi (1991)). Now, we consider FDOA. As $x = 2-\sqrt{2} \approx 0.58578$ is the unique solution of (4.9), by Corollary 4.2, $x_s^{2\sharp} = 2-\sqrt{2} \approx 0.58578$, and $x_s^{1\sharp} = 2(\sqrt{2}-1) \approx 0.82842$ by (4.3). By (4.4), $EA_s(FOA) = 6-4\sqrt{2} \approx 0.34314 < 0.5$. Therefore, FDOA is powerful enough to induce a nonempty contract zone if $f_b(x) = f_s(x)$.

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