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On the zeroes of Artin L-series of irreducible characters of the symmetric group S_n

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1 Introduction

Let E/F be a finite normal extension of algebraic number fields with Galois group $Gal(E/F) = G$. E. Artin [1] constructed to each virtual complex character η of G an L-series $L(s, \eta, E/F)$ which is meromorphic in the whole complex plane $\mathbf{C}$ as was proved by R. Brauer [3] by means of his famous induction theorem and fundamental classical results of E. Artin and E. Hecke. So far, no counterexample has been found to E. Artin's conjecture [1] which asserts that for each complex character χ of G its L-series $L(s, \chi, E/F)$ is holomorphic in $\mathbf{C} - \{1\}$. He showed that the Dedekind zeta function $\zeta_F(s) = L(s, 1_G, E/F)$ of the field F has a pole of order 1 at $s = 1$, where 1_G denotes the trivial character of G.

According to E. Hasse [6], p.163, it is also conjectured that in the vertical strip $0 < Re(s) < 1$ of the complex plane $\mathbf{C}$ the L-series $L(s, \chi, E/F)$ of all characters χ of G have all their zeroes on the line $Re(s) = \frac{1}{2}$. Riemann's conjecture for the classical zeta function $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ is a special case of this conjecture, because $\zeta(s)$ is the Dedekind zeta function for $E = F = \mathbf{Q}$, the field of rational numbers.

In this note we consider finite normal extensions E/F of algebraic number fields with Galois group $Gal(E/F) = S_n$, the symmetric group of degree n. Let $k = [\frac{n}{2}]$. In Theorem 5.3 it is shown that the truth of Artin's conjecture would imply that all the zeroes of the L-series $L(s, \chi, E/F)$ of all irreducible characters χ of S_n are contained in the union of the set of zeroes of the Dedekind zeta function $\zeta_\Omega(s)$ of the proper intermediate field $\Omega = E^{V_k}$ corre-

sponding to the wreath product $V_k = C_2 \wr S_k$ of the cyclic group C_2 of order 2 with the symmetric group S_k, and the union of sets of zeroes of the L-series of the sign characters σ_{n-2t} of the symmetric groups S_{n-2t} for $0 \leq t \leq k-1$. Furthermore, if also Riemann's conjecture holds for these $k+1$ L-series $\zeta_\Omega(s)$ and $L(s, \sigma_{n-2t})$, then the zeroes of the L-series $L(s, \chi, E/F)$ of all irreducible characters χ of the symmetric group S_n with $0 < Re(s) < 1$ lie on the vertical line $Re(s) = \frac{1}{2}$, see Corollary 5.4. This shows that the Dedekind zeta function $\zeta_\Omega(s)$ of finite normal extensions E/Ω of algebraic number fields with Galois group $Gal(E/\Omega) = C_2 \wr S_k$, $k = 1, 2, \ldots$, and the L-series $L(s, \sigma_n, E/F)$ of the sign characters σ_n of $Gal(E/F) = S_n$, $n = 1, 2, \ldots$, are the critical cases for the so called Riemann's conjecture on the zeroes of the Artin L-series of the irreducible characters χ of S_n.
In Theorem 5.2 the truth of Artin's conjecture is <u>not</u> assumed. It asserts that for each point s_0 of $\mathbf{C} - \{1\}$ there are at least $k+1$ irreducible characters χ_ν of S_n whose L-series $L(s, \chi, E/F)$ are holomorphic at s_0. The partitions $\nu \vdash n$ parametrizing these $k+1$ irreducible characters have different numbers of odd parts.
As in Foote-Murty [4] and Foote-Wales [5] Heilbronn's virtual character Θ_G of $G = Gal(E/F)$ [7] is used essentially in the proofs of these results. Its main properties are described in section 3. Another important tool is the explicit model for the complex characters of the symmetric group S_n given by Inglis, Richardson and Saxl [8]. This is a set $\{\pi_{t,n-2t} \mid 0 \leq t \leq k\}$ of monomial representations $\pi_{t,n-2t}$ of S_n which together contain each irreducible representation $\chi \in Irr_{\mathbf{C}}(S_n)$ of S_n exactly once. The main result of [8] is explained in section 4. The basic definitions and properties of the Artin L-series $L(s, \eta, E/F)$ are stated in section 2.
Concerning notation and terminology of the representation theory of finite groups we refer to the books by Nagao and Tsushima [11], and James and Kerber [9]. The standard reference for the results in algebraic number theory is S. Lang's book [10].
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2 Artin L-functions

In this section the basic definitions and notations from representation theory and number theory are given.
Let G be a finite group, then $k(G)$ denotes the number of conjugacy classes of G. The set of all inequivalent irreducible characters of G is denoted by $IrrG$. In particular, we write $IrrG = \{\chi_i \mid 1 \leq i \leq k(G)\}$. The set $char(G) = \{\sum_{i=1}^{k(G)} n_i\chi_i \mid n_i \geq 0, n_i \in \mathbf{Z}\}$ is the set of all ordinary characters. $vchar(G) = \{\sum_{i=1}^{k(G)} n_i\chi_i \mid n_i \in \mathbf{Z}\}$ is the ring of all virtual characters.

Definition. Let p be a prime number. A subgroup H is called p-elementary, if $H = P \times C$, where P is a p-subgroup and C is a cyclic p'-subgroup of G. $\mathcal{E}_p = \{H \mid H\ p\text{–elementary subgroup of } G\}$. $\mathcal{E} = \bigcup_p \mathcal{E}_p$ is the set of all elementary subgroups of G.

Brauer's Induction Theorem. For each $\eta \in vchar(G)$ there are elementary subgroups $H_i \in \mathcal{C}$ and linear characters $\lambda_{ij} \in IrrH_i$, $1 \leq j \leq h_i$ such that

$$\eta = \sum_{i=1}^{s}\sum_{j=1}^{h_i} a_{ij}\lambda_{ij}^G$$

for some integers $a_{ij} \in \mathbf{Z}$.

A short proof of this fundamental result in the representation theory of finite groups is given in [11], p.207.

Let E/F be a finite normal extension of number fields E, F with Galois group $Gal(E/F) = G$. Let $\mathcal{O}_F$ and $\mathcal{O}_E$ be the ring of algebraic integers in F and E, respectively.

Definition. Let $\eta \in char(G)$. Let $\mathcal{P}$ be the set of prime ideals p of $\mathcal{O}_F$. Then each $p \in \mathcal{P}$ splits into a product

$$p = (P_1 \dots P_r)^e$$

of prime ideals $P \in \{P_i \mid 1 \leq i \leq\}$ of $\mathcal{O}_E$. If f is the degree of the residue class field extension then $|E : F| = efr$ by [10], p.26.
For any P the norm $NP = (Np)^f$, where $N_p = |\mathcal{O}_F/p|$. Let G_P be the inertial group of P in G. Let $T_P = \{\tau \in G_P \mid \tau$ induces the identity automorphism

of the residue class field extension}. Then the Frobenius automorphism $\sigma = \sigma(P, E/F) = \sigma(P, E/F) \in G_P$ is defined by

$$\sigma\alpha = \alpha^{Np} mod P, \ \ \alpha \in \mathcal{O}_E.$$

σ is determined only up to multiplication with some $\tau \in T_P$. For each $m \geq 1$ and $\sigma = (P, E/F)$ let $\eta(\sigma^m T_P) = \sum_{t \in T_P} \eta(\sigma^m \tau)$, and

$$\eta(p^m) = \frac{1}{e}\,\eta(\sigma^m T_P), \ p \in \mathcal{P}.$$

Then the <u>L-series</u> $L(s, \eta, E/F)$ is defined by

$$log L(s, \eta, E/F) = \sum_{p \in \mathcal{P}} \sum_{m \geq 1} \frac{\eta(p^m)}{m(Np)^{sm}}$$

$L(s, \eta, E/F)$ is holomorphic in the half plane $Re(s) > 1$. It has a continuation to the entire plane $\mathbf{C}$.
In [1] and [2] E. Artin proved or stated the following fundamental results. For precise references for its complete proof see also Foote-Wales [5], p.227.

<u>Lemma 2.1</u>. The L-series have the following properties:

1. $L(s, \eta_1 \oplus \eta_2, E/F) = \Pi_{i=1}^2 L s, \eta_i, E/F)$, for all $\eta_i \in char G$.
2. If $H \leq G$, $\sigma \in char(H)$, then $L(s, \sigma^G, E/F) = L(s, \sigma, E/E^H)$, where σ^G denotes the induced character of G.
3. For $\psi \in char G$ let $H = ker\psi$, and ψ' the character of G/H induced by ψ, then $L(s, \psi, E/F) = L(s, \psi', E^H/F)$.
4. (Hecke) If χ is a non-principal linear character of G, then $L(s, \chi, E/F)$ is holomorphic in the entire complex plane $\mathbf{C}$.
5. The <u>Dedekind zeta function</u> $\zeta_F(s) = L(s, 1_G, E/F)$ has a simple pole at $s = 1$, $\zeta_F(1) \neq 0$, and $\zeta_F(s)$ is holomorphic everywhere except for $s = 1$.

6. Let $\chi \in IrrG$ and $\bar{\chi}$ be its complex conjugate. Artin multiplies $L(s,\chi,E/F)$ and $L(s,\bar{\chi},E/F)$ with appropriate powers of the Γ-function $\Gamma(s)$ and obtains meromorphic functions $\xi(s,\chi,E/F)$ and $\xi(s,\bar{\chi},E/F)$ satisfying a functional equation

$$\xi(1-s,\chi,E/F) = W(\chi)\xi(s,\bar{\chi},E/F),$$

where $\xi(s,\chi,E/F)$ and $L(s,\chi,E/F)$ have the same zeroes in

$$0 < Re(s) < 1.$$

__Remark 2.2__. If the Galois group $Gal(E/F) = S_n$, the symmetric group of degree n, then assertion 6 of Lemma 2.1 implies that in the vertical strip $0 < Re(s) < 1$ the zeroes of all L-series $L(s,\chi,E/F)$ of all irreducible characters χ of S_n lie symmetric with respect to the vertical line $Re(s) = \frac{1}{2}$, because by Theorem 2.1.12 of James and Kerber [9], p.37 the rational field $\mathbf{Q}$ is a splitting field for S_n, which implies $\chi(g) = \bar{\chi}(g)$ for all $g \in S_n$.

In [3] R. Brauer proved the following fundamental results on the Artin L-functions $L(s,\chi,E/F)$ by means of Lemma 2.1 and his induction theorem.

__Theorem 2.3__. The Artin L-series $L(s,\chi,E/F)$, $\chi \in char(G)$, are all meromorphic in the complex plane $\mathbf{C}$.

__Artin's conjecture__: Let $\eta \in char(G)$. If the inner product $< 1_G, \eta >= 0$, then $L(s,\eta,E/F)$ has an analytic continuation for all $s \in \mathbf{C}$.

3 Heibronn's virtual character

In [4] Foote and Murty showed that the set of zeroes and poles of the Artin L-functions $L(s,\chi,E/F)$, $\chi \in char(G)$, are contained in the set of zeroes of the Dedekind zeta function $\zeta_E(s)$ of the extension field E. In the proof of this result they apply some subsidiary results on a virtual chracter, originally introduced by H. Heilbronn [7]. Its definition and properties are restated in this section.
Let $s_0 \in \mathbf{C} - \{1\}$ be fixed. For each $\psi \in IrrG$ let $n_\psi(s_0) = n_\psi = ord_{s=s_0} L(s,\eta,E/F)$ be the order of zero or pole of the meromorphic function $L(s,\psi,E/F)$ at the point s_0. Heilbronn's virtual character is defined in [7], p.871, by

$$\ominus_G = \sum_{\psi \in Irr(G)} n_\psi \psi,$$

The following subsidiary results are due to Heilbronn [7], Foote-Murty [4] and Foote-Wales [5].

Lemma 3.1. a) $\ominus_G$ is a virtual character of G.
b) For each $\psi \in char(G)$

$$ord_{s=s_0} L(s,\psi,E/F) = <\ominus_G,\psi> .$$

Assertion a) is proved in [4], p.116, and b) is shown in [5], p.228.

Lemma 1 of Foote-Wales [5], p.230, is restated as

Lemma 3.2. For each subgroup H of G the restriction $\ominus_{G|H} = \ominus_H$.

Lemma 3.3. If $\zeta_E(s)$ is the Dedekind zeta function of E, then

$$\ominus_G(1) = ord_{s=s_0} \zeta_E(s) \geq 0.$$

This result is proved in [5], p.228.

4 The model of the symmetric group

In [8] Inglis, Richardson and Saxl constructed an explicit model for the irreducible representations of the symmetric group S_n. It consists of a finite set of monomial representations defined over the integers $\mathbf{Z}$.
It is well known that the field $\mathbf{Q}$ of rational numbers is a splitting field for any symmetric group S_n.
Throughout this section the integer n is fixed. Let A_n be the alternating subgroup of S_n. The irreducible representations of S_n are parametrized by the partitions $\lambda \vdash n$ of n. The set of all partitions λ of n is denoted by $\mathcal{P}(n)$, and its cardinality $|\mathcal{P}(n)|$ by $p(n)$. If χ_λ is the character corresponding to $\lambda \vdash n$, then $d_\lambda = \chi_\lambda(1)$ is the degree of χ_λ.
The construction of the monomial representations of S_n given in [8] requires the following subgroups of S_n and linear (one dimensional) representations. Let t be any integer with $0 \leq t \leq [\frac{n}{2}]$. Let $V_t = C_2 \wr S_t$ be the wreath product of the cyclic group C_2 of order 2 with the symmetric group S_t of degree t. Let $U_t = V_t \times S_{n-2t}$, and $W_t = V_t \times A_{n-2t}$. Let σ_{n-2t} be the sign character of S_{n-2t}, and 1_t the trivial representation of V_t. Then $\mu_t = 1_t \otimes \sigma_{n-2t}$ is a linear representation of U_t. Therefore, the induced representation $\pi_{t,n-2t} = (\mu_t)^{S_n}$ is a monomial representation of S_n. Furthermore, $\pi_{t,n-2t}$ has degree $m_t = dim_F \pi_{t,n-2t} = \frac{n!}{2^t t!(n-2t)!}$. This notation is kept throughout this section.
The Corollary of Inglis, Richardson and Saxl [8] is restated as

<u>**Proposition 4.1**</u>. a) The representation $\sum_{0 \leq t \leq [\frac{n}{2}]} \pi_{t,n-2t}$ of S_n is the direct sum of all irreducible representations of S_n, each appearing with multiplicity one.
b) The irreducible character χ_λ of S_n corresponding to the partition λ of n is a constituent of $\pi_{t,n-2t}$ if and only if $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_s)$ has precisely $n - 2t$ odd parts.

<u>**Remark**</u>. In the special cases $n = 2t$ and $n = 2t + 1$, the monomial representations $\pi_{t,0}$ and $\pi_{t,1}$ are in fact transitive permutation representations of S_n.

5 Artin's conjecture and the zeroes of the L-series of the irreducible characters of S_n

Throughout this section E/F denotes a finite normal extension of algebraic number fields E and F with Galois group $Gal(E/F) = S_n$, the symmetric group of degree n.
The L-series $L(s, \pi_{t,n-2t}, E/F)$ of the monomial model characters $\pi_{t,n-2t}$ of S_n are described by

Lemma 5.1. Let t be any integer with $0 \le t \le [\frac{n}{2}]$. Let $V_t = C_2 \wr S_t$, $U_t = V_t \times S_{n-2t}$, and $W_t = V_t \times A_{n-2t}$. Let σ_{n-2t} be the sign character of S_{n-2t}, 1_t be the trivial character of V_t, and $\mu_t = 1_t \otimes \sigma_{n-2t}$. Then the following assertions hold:
a) If $0 \le n-2t \le 1$, then $L(s, \pi_{t,n-2t}, E/F) = L(s, (1_t)^{S_n}, E/F) = L(s, 1_t, E/E^{U_t}) = \zeta_\Omega(s)$, where $\zeta_\Omega(s)$ denotes the Dedekind zeta function of the intermediate field $\Omega = E^{U_t}$ corresponding to the subgroup $U_t = V_t = C_2 \wr S_t$ of S_n.
b) If $n-2t > 1$, then $L(s, \pi_{t,n-2t}, E/F) = L(s, (\mu_t)^{S_n}, E/F) = L(s, \mu_t, E/E^{U_t}) = L(s, \sigma_{n-2t}, E^{V_t}/E^{U_t}) = L(s, \sigma'_{n-2t}, E^{W_t}/E^{U_t})$, where $Gal(E^{V_t}/E^{U_t}) \cong S_{n-2t}$ and $|Gal(E^{W_t}/E^{U_t})| = 2$.

Proof. a) follows immediately from assertions (2) and (5) of Lemma 2.1.
b) Certainly $L(s, (\mu_t)^{S_n}, E/F) = L(s, \mu_t, E/E^{U_t})$ by (2) of Lemma 2.1. The linear character $\mu_t = 1_t \times \sigma_{n-2t}$ of $U_t = V_t \times S_{n-2t}$ has V_t in its kernel, and it induces the sign character σ_{n-2t} in the factor group $U_t/V_t \cong S_{n-2t}$. Therefore, assertion (3) of Lemma 2.1 implies that

$$L(s, \mu_t, E/E^{U_t}) = L(s, \sigma_{n-2t}, E^{V_t}/E^{U_t}).$$

Furthermore, $Gal(E^{V_t}/E^{U_t}) \cong S_{n-2t}$ by the main theorem of Galois theory. As $A_{n-2t} = ker(\sigma_{n-2t}) \triangleleft S_{n-2t}$, another application of Lemma 2.1 (3) yields that $L(s, \sigma_{n-2t}, E^{V_t}/E^{U_t}) = L(s, \sigma'_{n-2t}, E^{W_t}/E^{U_t})$, where σ_{n-2t} denotes the non-trivial character of the cyclic group $U_t/W_t \cong S_{n-2t}/A_{n-2}$ of order 2. This completes the proof.

Theorem 5.2. Let t be any integer with $0 \le t \le [\frac{n}{2}]$, and s_0 any point of the Gaussian plane $\mathbf{C} - \{1\}$. Then for at least one irreducible character χ_ν

such that $\nu \vdash n$ has $n-2t$ odd parts, the L-series $L(s,\chi,E/F)$ is holomorphic at s_0.

Proof. Let $k(t)$ be the number of irreducible characters χ_{ν_i} of S_n corresponding to the partitions $\nu_i \vdash n$ of n with precisely $n-2t$ odd parts. Then

$$\pi_{t,n-2t} = (\mu_t)^{S_n} = \sum_{i=1}^{k(t)} \chi_{\nu_i}$$

by Proposition 4.1, where $\mu_t = 1_t \otimes \sigma_{n-2t}$ denotes the linear character of $U_t = V_t \times S_{n-2t}$ described in the previous section. By Brauer's theorem 2.3, all L-series $L(s,\chi_{\nu_i},E/F)$ are meromorphic at s_0. Let n_i be the order of a pole or a zero of $L(s,\chi_{\nu_i},E/F)$ at s_0, and let $\ominus = \sum_{\psi\in Irr(S_n)} n_\psi \psi$ be Heilbronn's virtual character of S_n with respect to s_0. As $L(s,\pi_{t,n-2t},E/F)$ is holomorphic at s_0 by assertions (2) and (4) or (5) of Lemma 2.1, it follows from Lemma 3.1 b) that

$$0 \leq ord_{s=s_0} L(s,\pi_{t,n-2t},E/F) =< \ominus, \pi_{t,n-2t} >= \sum_{i=1}^{k(t)} n_i.$$

Therefore, at least one $n_i \geq 0$ for some $1 \leq i \leq k(t)$. Thus $L(s,\chi_{\nu_i},E/F)$ is holomorphic at s_0.

Theorem 5.3. Let E/F be a finite normal extension of algebraic number fields with Galois group $Gal(E/F) = S_n$. For each $0 \leq t \leq [\frac{n}{2}] = k$ let $V_t = C_2 \wr S_t$, $U_t = V_t \times S_{n-2t}$ and σ_{n-2t} be the sign character of the symmetric group S_{n-2t}. Let $\zeta_\Omega(s)$ be the Dedekind zeta function of the intermediate field $\Omega = E^{V_k}$.
If the L-series $L(s,\chi,E/F)$ of all the irreducible characters χ of S_n are holomorphic in $\mathbf{C}-\{1\}$ then the zeroes of all L-series $L(s,\chi,E/F)$ are contained in the set of zeroes of the Dedekind zeta function $\zeta_\Omega(s)$ and of the k Artin L-series $L(s,\sigma_{n-2t},E^{V_t}/E^{U_t})$ of the sign characters σ_{n-2t} of the Galois groups $Gal(E^{V_t}/E^{U_t}) \cong S_{n-2t}$, $0 \leq t \leq k-1$.

Proof. Let s_0 be a point in $\mathbf{C}-\{1\}$ such that $\zeta_\Omega(s_0) \neq 0$ and $L(s_0,\sigma_{n-2t},E^{V_t}/E^{U_t}) \neq 0$ for all $0 \leq t \leq k-1$. Let χ be any irreducible character of S_n. Then there is a uniquely determined partition $\nu \vdash n$ corresponding to $\chi = \chi_\nu$. Suppose that ν has $n-2t$ odd parts. Then by

Proposition 4.1 χ_ν occurs in the monomial model character $\pi_{t,n-2t}$ of S_n with multiplicity 1, and $< \chi_\nu, \pi_{s,n-2s} >= 0$ for all $0 \le s \le k$ and $s \ne t$.
Let $k(t)$ be the number of irreducible characters χ_{ν_i} of S_n corresponding to the partitions $\nu_i \vdash n$ of n with precisely $n-2t$ parts. We may assume that $\nu = \nu_1$.
Let n_i be the order of a zero of the holomorphic function $L(s, \chi_{\nu_i}, E/F)$ at s_0, and let $\ominus = \sum_{\psi \in Irr(S_n)} n_\psi \psi$ be Heilbronn's virtual character of S_n with respect to s_0. Then $n_i \ge 0$ for $i = 1, 2, \dots, k(t)$, and by Lemma 3.1 b)

$$ord_{s=s_0} L(s, \pi_{t,n-2t}, E/F) =< \ominus, \pi_{t,n-2t} >= \sum_{i=1}^{k(t)} n_i.$$

Now Lemma 5.1 asserts that

$$ord_{s=s_0} L(s, \pi_{t,n-2t}, E/F) = ord_{s=s_0} \zeta_\Omega(s) = 0 \text{ for } t = k, \text{ and}$$

$$ord_{s=s_0} L(s, \pi_{t,n-2t}, E/F) = ord_{s=s_0} L(s, \sigma_{n-2t}, E^{V_t}/E^{U_t}) = 0,$$

because $\zeta_\Omega(s_0) \ne 0$, $L(s_0, \sigma_{n-2t}, E^{V_t}/E^{U_t}) \ne 0$ and both functions are holomorphic at s_0. Hence, all $n_i = 0$ for $1 \le i \le k(t)$. In particular, $L(s_0, \chi, E/F) \ne 0$, completing the proof.

<u>Corollary 5.4</u>. Let E/F be a finite normal extension with Galois group $Gal(E/F) = S_n$ such that the L-series $L(s\chi, E/F)$ of all irreducible characters χ of the symmetric group S_n are holomorphic in $\mathbf{C} - \{1\}$. Let $k = [\frac{n}{2}]$.
If Riemann's conjecture holds for
a) all L-series $L(s, \sigma_{n-2t}, E_t/F_t)$ of the sign characters σ_{n-2t} of the symmetric groups S_{n-2t} and all finite extensions E_t/F_t with Galois groups $Gal(E_t/F_t) = S_{n-2t}$ for $0 \le t \le k-1$, and
b) the Dedekind zeta function $\zeta_\Omega(s)$ of all finite normal extensions E_k/F_k with Galois group $Gal(E_k/F_k) = C_2 \wr S_k$,
then the zeroes of the L-series $L(s, \chi, E/F)$ of all irreducible characters χ of S_n with $0 < Re(s) < 1$ lie on the vertical line $Re(s) = \frac{1}{2}$.

<u>Proof</u> follows immediately from Theorem 5.3

References

[1] E. Artin, Über eine neue Art von L-Reihen. Hamburger Abhandlungen 3 (1923), 147-156.
[2] E. Artin, Zur Theorie der L-Reihen mit allgemeinen Gruppencharakteren. Hamburger Abhandlungen 7 (1929), 46-51.
[3] R. Brauer, On Artin L-series with general group characters. Annals of Math. 48 (1947), 502-514.
[4] R. Foote, V. K. Murty, Zeroes and poles of Artin L-series. Math. Proc. Cambridge Phil. Soc. 105 (1989), 115-133.
[5] R. Foote, D. Wales, Zeroes of order 2 of Dedekind zeta functions and Artin's conjecture. J. Algebra 131 (1990), 226-257.
[6] H. Hasse, Bericht über neuere Untersuchungen und Probleme aus der Theorie der algebraischen Zahlkörper. Physica Verlag, Würzburg (1965).
[7] H. Heilbronn, On real zeroes of Dedekind ζ-functions. Canad. J. Math. 25 (1973), 870-873.
[8] N. F. J. Inglis, R. W. Richardson, J. Saxl, An explicit model for the complex representations of S_n. Archiv d. Math. 54 (1990), 258-259.
[9] G. James, A. Kerber, The representation theory of the symmetric group. Addison-Wesley, Reading, Mass. (1981).
[10] S. Lang, Algebraic number theory. Springer, Heidelberg (1986).
[11] H. Nagao, Y. Tsushima, Representations of finite groups. Acad. Press, New York (1988).

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