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Anticipatory system and Systems failures

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1. Introduction

We may consider that many systems which we encouter consist of a controller(CR) and a controlled system(CS). It is also effective to understand organizational decision making systems through a framework based on the control system paradigm(Kickert,1979). In large-scale complex systems, though controller CR can't always control correctly controlled system CS and can't predict perfectly changes of CS in the changeable environment, each system 'survives' holding some kinds of desired relation between CR and CS, or CR and the environment. For examples, engineering systems hold high reliability or safety and the immune systems prevent self from non-self.

One of most important relation is high reliability in sense that fatal system failures don't occur. Therefore, all engineering systems are required to obtain certain desired relation and to retain(maintain) it in order to survive in changeable environment. Each constitutes a process, respectively.

(1) A process of retaining the relation is realized by anticipatory systems based on an internal model.

An 'anticipatory system' introduced and researched by Rosen(1974) is a new type of system in sense that present behavior depends in some fashion upon "predicted future states" or "future inputs". This means that present behavior doesn't depend upon past states so that an anticipatory system involves feedforwards control system rather than feedback one. The basic methodological presuppositions are that anticipatory systems clearly violate the principles of causality and embody a form of teleology.

$$
\begin{array}{ccccc}
\text{Past State} & \longrightarrow & \text{Present State} & \longleftarrow & \text{Future State} \\
 & \uparrow & & \uparrow & \\
 & \text{Newtonian Physics} & & \text{Anticipatory System} & \\
 & \text{Input/Output Model} & & \text{Feedforwards} &
\end{array}
$$

In order to control system anticipatively, systems must have internal models to determine adequate control actions for system states or environment (Kijima,1986). In this paper, we'll call a system which has an internal model an 'anticipative system'.

Moreover, systems are required to have a mechanism for improving the internal model. Because if the internal model is fixed, the systems may fail in a global way without any localized failure in a specific subsystem, that is called global failures(Rosen,1978b) which result from change of environment. This is the second process (2).

(2) A process of obtaining or improving the desired relation is equal to a process of building or improving the internal model.

Repeats of both process constitutes a process of self-improving internal model, which makes systems without global failures possible. This paper tries to mathematical approach to the first process of self-improvement, which we consider as a decision making process in system management. We adopt a set-theoretical approach based on Goal Seeking System Model known as a hierarchical decision making model(Mesarovic et al.,1970). This method is different from an analytical approach utilizing differential equations.

We introduce a framework for analysis in section 2. In section 3, we define systems failures as undesired relations between CR and CS, which includes the concept of global failures. We propose a new concept of "admissible pair" as a desired relation. After we discuss process of obtaining admissible pairs briefly in section 4, we try to formal approach to realization of retaining process by an anticipative system in section 5. Consequently, we'll be able to learn a mechanism of anticipative system.

2. A Framework for Analysis

This paper pays an attention to the feature that a lot of systems failures in large-scale complex systems are concerned with (1)cognitive limits of controller and (2)the doubled structure of evaluation.

(1) Firstly, as systems are concentrated or large, any controller can't observe directly any real state in object system CS. Controller can observe states only on a 'cognitive model' which describes certain cognitive level for observing states. For example, controller observes states at some subsystem level in structure, and every interval time. Usually, we build a mathematical model on assumption that such a cognitive model is implicitly given. However, in large scale systems, cognitive levels aren't trivial from the cognitive limits.

(2) Secondly, we should admit that there is a doubled structure in evaluation system. One is a prior evaluation under which control rules are decided and the other is a posterior evaluation for the controlled results. A lot of systems failures result from inconsistent between the two different evaluations.

We adopt a framework (S, W, G) to analyze these two factors which are much concerned with system failures(Fig.1).

(1) S is a management system as a controller and W is a set of the real states of controlled object system. S is modeled by a goal seeking system which is one of decision making system model.

$$S = (W/\theta, A, P, g, \phi) \rightarrow D$$

where

$W/\theta = \{[x] | x \in W\}$: the cognitive model, partition of W

A : the set of actions

$P : W/\theta \times A \rightarrow W/\theta$: the state transition (probability) function

$g : W/\theta \times A \rightarrow R$: the real-valued prior performance function

$\phi : R \rightarrow \{True,\ False\}$: the prior decision principle

D: a set of control rules which is a subset of $W/\theta \times A$

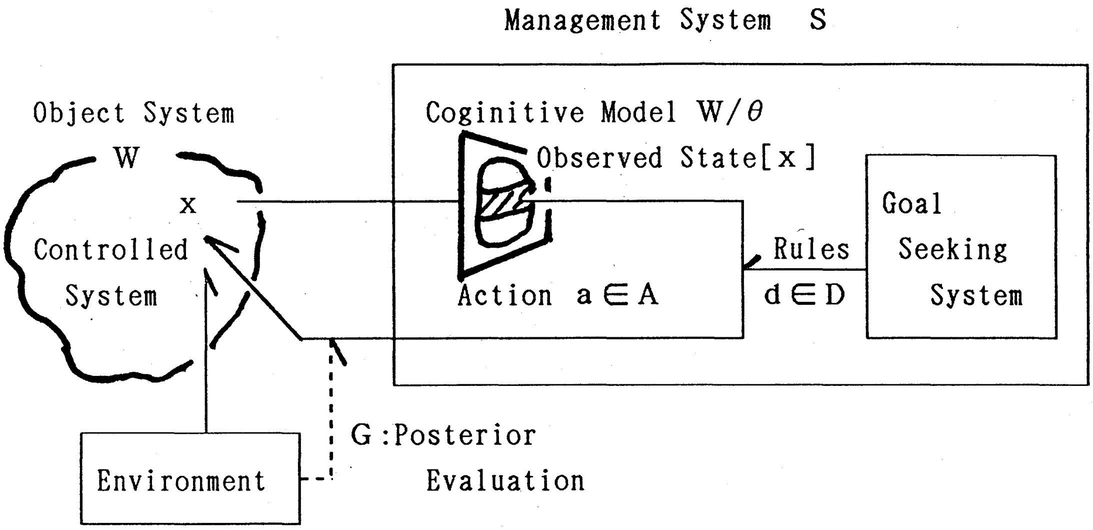

Figure 1. Framework for Analyzing complex systems

Main decision problem in S is to decide a control rule set $D \subset W/\theta \times A$ based on prior evaluation(g, ϕ). The problem has analyzed by Operations Research or decision theory. We notice that cognitive model is expressed by a state partition(Rosen,1978a).

(2)$G := \{g_E, \phi_E\}$ is a posterior evaluation.

$g_E : W \times A \rightarrow R$: the posterior performance function

$\phi_E : R \rightarrow \{True, False\}$: the posterior decision principle

We should notice that domain of the prior performance function g is different from one of the posterior performance function g_E. Also we can often find that while ϕ is an optimal principle, ϕ_E is a satisfactory principle. In some system, the posterior evaluation is in the management system S, in other system, out of S, that is in the environment. For example of the latter, in safety problem for nuclear power station, we should distinguish a decision principle on which controlled states are evaluated by inhabitants in neighborhood from another decision principle on which operating rules are predetermined.

Table 1 Doubled structure in evaluation

	Prior Evaluation	Posterior Evaluation
Function	$g : W/\theta \times A \rightarrow R$	$g_E : W\times A \rightarrow$R
Principle	ϕ : Optimal	ϕ_E : Satisfaction

Thus, in the triple (S, W, G), we may expect to analyze complex or large-scale systems, distinguishing (1)real states of object system and observed states, (2)the prior decision principle and the posterior one, respectively. Therefore we call the triple a framework of complex system.

3. Systems Failures and Desired Relation

In (S, W, G), though physical failures are expressed as states in W, many systems failures in system management are considered as undesired relations between the management system S and the object system as follows. The definition will includes ideas which Bignell & Fortune(1984) and Gigch(1986) proposed.

Definition 3.1 *A management system S is called to fail at real state $x(\in W)$ when*

(a) $(\forall d \in D)(\phi \cdot g([x], d([x])) = False)$*or*

(b) $(\exists d \in D)(\phi_E \cdot g_E(x, d([x])) = False)$

We also call this situation systems failure.

(a) means that the management system doesn't have solution in its rules D based on (ϕ, g). (b) means that the action selected on the basis of prior evaluation(ϕ, g) can't satisfy the posterior evaluation(ϕ_E, g_E). We should distinguish (b) from (a) because of two reasons. One is that they results from different causes. Though (a) results from change of environment, (b) results from inadequate cognitive model the management system has. That is, the former is concerned with limits of prediction, but the latter with limits of cognition. The other is that there is a difference about controllability for causes. Environment is an uncontrollable factor, but the cognitive model is a controllable one so that (b) is avoidable. To distinguish them more clearly, we introduce the next concept.

Definition 3.2 *A rule $d \in D$ is said to a mis-chosen rule at real state x, when*

$$\phi \cdot g([x], d([x])) = True \ \ \& \ \ \phi_E \cdot g_E(x, d([x])) = False$$

A mis-chosen rule causes systems failure (b). The reason why such rules are selected is concerned with a difference between the domain W/θ of the prior performance function g and the domain W of posterior one g_E. Hence management systems must have a desired cognitive model θ as well as adequate rules D (See Tanaka(1989) about applying this concept to problems for safety monitoring systems) .

Definition 3.3 *Let (θ, D) be a pair which consists of a cognitive model θ and a set of control rules D. A pair (θ, D) is called admissible pair when* $(\forall x \in W)(\forall d \in D)$

$$(\phi \cdot g([x], d([x])) = True \Rightarrow \phi_E \cdot g_E(x, d([x])) = True).$$

The admissible pair guarantees the relation that for any real state x the action $d([x])$ decided on the basis of the observation $[x]$ always satisfies the posterior evaluation.

Let $\Phi(x)$ be a set of actions which satisfy the posterior evaluation principle, as follows.

$$\Phi(x) = \{a \in A \mid \phi_E \cdot g_E(x, a) = True\} \quad for \ \ \forall x \in W$$

By this notation, the definition of the admissible pair of (θ, D) for

(W, G) is also expressed by $(\forall x \in W)(\forall d \in D)(\Phi\ (x) \ni d([x]))$. In this sense, we may call $\Phi\ (\cdot)$ a solution.

4. Process of Obtaining

Management systems are required to hold admissible pairs(θ, D) for any various environment. Main problems we must solve are

(1) how management systems obtain or improve admissible pairs, and once an admissible pair is obtained,

(2) how management systems retain the admissible pair.

(1) is very difficult problem. Because any real management system can't admit whether its own pair is admissible since the property depends upon the real state which management system can't observe directly. Sequential improvement is most effective method. The improvement process is a structure control which controls cognitive model by refining state partitions or integrating them to reach an alternative admissible pair. We have shown that management system can improve itself only utilizing the information from occurred systems failures by mis-chosen rules and that the process constitutes positive feedback rather than negative feedback(Tanaka,1988a;1988b;1990b). This paper doesn't refer to mathematical approach to their dynamics, but analyze the problem (2) in details in the next section.

Systems Failures by Change of Environment ↓   Systems Failure by Mis-choice ↓ ↓

——————×— — — — — — ×— — — —×——————→

Feedforwards Retention Process   Positive Feedback Process of Obtaining (Improvement Process)   Feedforwards Retention Process

Figure 2 Two Processes of Obtaining and Retaining
(×:Occurrence of Systems Failure)

5. Retention Process and Anticipative System

In this section, we try to show that retaining admissible pair is equivalent to hold anticipative property and analyze conditions for systems to be anticipative.

Let (S, W, G) be a framework of complex system. We consider a pair (W, α) where $\alpha : W \to W$ is a state transition function over real states of the object system. This one step transition map is a generator of the real state, that is, if an initial state of the object system is $x \in W$, then α generates a sequence of x, $\alpha(x)$, $\alpha^2(x) \ldots \alpha^t(x) \ldots$ and so on. We write W for (W, α) briefly.

We assume two conditions as follows (A1)-(A2) where,

(A1) dynamics α of W is a deterministic function, but the management system S has no knowledge about the dynamics,

(A2) control rule is unique, i.e. $D = \{d\}$.

(A2) holds when the prior principle is optimal one, however it is not an essential assumption.

Now we'll define anticipative system as a system which has an internal model of W by extending the definition which Kijima(1986) introduced.

Definition 5.1 *If there exists a function $\beta : A \to A$ such that*

$$(\forall x \in W)(\forall \mathrm{t} \in \{0, 1, 2, \ldots\})(\Phi(\alpha^t(x)) \ni \beta^t d([x])), \tag{5.1}$$

β is called an internal model of W with respect to Φ. In this case, the management system S is called to be anticipative for (W, G).

When $\mathrm{t} = 0$, (5.1) says that (θ, D) is an admissible pair for(W, G). Accordingly, if a management system has an internal model of W, then initial desired pair is kept by action sequence selected on the basis of the internal model. Therefore, our main problem is to find conditions for a management system S to have an internal model β of W.

We'll discuss the conditions according to two decision principles in the posterior evaluation. One is optimal decision principle and the other satisfactory decision principle.

(1) Optimal Decision Principle

In this principle, $\Phi\ (x)$ is a singleton set for each x, i.e. $\Phi\ (x) = \{a\}$. Then (5.1) especially holds in equality. By applying results in Kijima(1986) to this framework, a condition for existence of internal model is obtained by Proposition 5.1 below. Before proposing Proposition 5.1, we introduce a consistent property between an object system W and a posterior evaluate G.

Definition 5.2 *(W, G) is called consistent when*

$$(\forall x, x' \in W)(\Phi(x) = \Phi(x') \Rightarrow \Phi(\alpha(x)) = \Phi(\alpha(x'))) \tag{5.2}$$

The consistent property implies that state transition over object system does not change so rapidly with respect to the posterior decision principle. Mathematically, when (W, G) is consistent, Φ constitutes a congruence relation with respect to α. We notice that this concept is independent on a management system S.

Proposition 5.1 *For a management system S to be anticipative, it is necessary and sufficient that the following two conditions be satisfied:*
(a) (W, G) is consistent,
(b) (θ, D) is an admissible pair for (W, G).

Proposition 5.1 suggests that if (W, G) isn't consistent, system S can't have an internal model to retain the admissible pair. The proof of Proposition 5.1(see Theorem 4 (Kijima 1986)) is strongly dependent upon the characteristics of a singleton set of $\Phi\ (x)$. However, we can also verify a similar proposition in case of a general set of Φ.

(2) Satisfactory Principle

In this principle, since $\Phi\ (x) = \{a_1, a_2, \ldots\}$ is a subset of A, the concept of consistent is required to be extended.

Definition 5.3 *(W, G) is called consistent if*

$$(\forall K \subset W)(\cap_{x \in K} \Phi(x) \neq 0 \Rightarrow \cap_{x \in K} \Phi(\alpha(x)) \neq 0)(0 : \textit{empty set}) \tag{5.3}$$

This means that for any subset K of real states if there is a common element in all solution sets for each x in K, then a common element can be always found after one step transition. We should remark this relation is not congruence relation. However, we will not confuse even if we use the same label(consistent) since Definition 5.3 coincide with Definition 5.1 when $\Phi(x)$ is a singleton set.

In a general set of Φ, though the consistent property guarantees the existence of an internal model for management system which has an admissible pair, the property is not a necessary condition but a sufficient one.

Proposition 5.2 *When (W, G) is consistent, the following conditions are equivalent:*

(a) (θ, D) is an admissible pair for (W, G).

(b) S has a internal model of W with respect to Φ.

When we restrict the consistent property to S, we can obtain a necessary and sufficient condition.

Definition 5.4 *For any $a \in A$, we set $K(a) = \{x \mid \Phi(x) \ni a$ and $a = d([x])\} (\subset W)$. When*

$$K(a) \neq 0 \Rightarrow \cap_{x \in K(a)} \Phi(\alpha(x)) \neq 0, \tag{5.4}$$

(W, G) is called consistent relevant to S.

As $K(a)$ is also expressed by $K(a) = \Phi^{-1}(a) \cap (d \cdot \theta)^{-1}(a)$ where $\theta(x) = [x]$, the next relation holds trivially.

Lemma 5.3 *If (W, G) is consistent, then (W, G) is consistent relevant to S for any S.*

Now we can propose our main theorem.

Theorem 5.4 *For a management system S to be anticipative, it is necessary and sufficient that the following two conditions be satisfied:*

(a)(W, G) is consistent relevant to S,

(b)(θ, D) is an admissible pair for (W, G).

By Theorem 5.4, if the consistency restricted to the cognitive model is satisfied, management system S can be always anticipative once S has desired pair. And it is also a necessary condition.

(3) Action depended State Transition

We have considered state transition α independent of action selected by management system so far. If we extend the transition to $\alpha : W \times A \to W$, that is $\alpha(x_t, a_t) = x_{t+1}(t \in \{0, 1, 2, \ldots\})$, then we can obtain similar results by simple modification. Internal model is naturally defined by

$$(\forall x_0 \in W)(\forall t \in \{0, 1, 2, \ldots\})(\Phi(x_{t+1})) \ni \beta^t d([x_0])) \tag{5.1$'$}$$

In optimal principle, Proposition 5.1 holds by modifying Definition 5.2 of the consistent property to

$$(\forall x, x' \in W)(\Phi(x) = \Phi(x') \Rightarrow \Phi(\alpha(x, \Phi(x))) = \Phi(\alpha(x', \Phi(x)))). \tag{5.2$'$}$$

In satisfactory principle, Proposition 5.4 also holds by modifying Definition 5.3 of the consistent property to

$$K(a) \neq 0 \Rightarrow \cap_{x \in K(a)} \Phi(\alpha(x, a)) \neq 0, \tag{5.4$'$}$$

6. Summary

We first showed that there is a new type of systems failures, mis-choice, in large-scale complex systems which results from double structured evaluation and the discrepancies between the real states and the observed states in the cognitive model. As a relation which guarantees that a system doesn't reach failures by mis-choice, we proposed an "admissible pair" which consists of the cognitive model and the control rules based on the cognitive model.

Next, we revealed that a management system must be anticipative by an internal model to retain the admissible pair. By mathematical analysis, a consistent property between W and G was obtained as a necessary and sufficient condition for systems to have an internal model. The condition is also one for realizing system management without global failures.

When the environment around the control system changes variously for long time interval, real state distribution or the dynamics over the state transition may also change. Hence the management system is required to improve its internal model as well as to retain the model.

Repeats of both process of retaining and improving the internal model constitutes an adaptive self-improving process of management system by learning activities. This paper has researched only the process of retaining by formal approach.

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