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Exchange Algebras and Economic Field

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ABSTRACT

In this paper we analyze economic systems as complex economic field by using exchange algebras. Exchange algebras is the extension of accounting vector space which was introduced by H.Deguchi and B.Nakano[1]. By using this algebras we describe systemic properties of economic exchange and properties of economic field which gives a formal model for SNA(System of National Account).

1. EXCHANGE ALGEBRAS

1.1 CONCEPT OF EXCHANGE AND ECONOMICS

The notions of exchange in economic systems are usually discussed for analyzing the structure of a market. In this paper, however, we study the notions of exchange as a descriptive basis of economic systems and investigate the notion for analyzing common structure of different economic systems.

1.2 REDUNDANT ALGEBRAS

We previously introduced the notion of accounting vector space as an abstraction of the book-keeping system[1]. Now we give a further abstraction and give basic notions to develop economic analysis over the algebras. In this paper we omit some proofs of the proposition.

At first we introduce the notion of redundant algebras corresponding to vector spaces used in natural sciences. Redundant algebras do not use the notion of negative elements. Instead of using negative elements we add redundant elements to the algebras by using the notion of dual basis. In order to formulate the notion of exchange in economics we need to add several axioms to this algebras. Then we can formulate the mathematical structure of exchange which has been ambiguous in economics.

[Definition 1.1] Redundant algebras Ψ

Let Ψ be a non-empty set and T be a set of non negative integers Z^+ or non-negative real numbers R^+. We call T the set of scalars. We define a binary operation + and unary operations ^, $^{-}$ on Ψ and a scalar multiplication $ax \in \Psi$ for $a \in T$ $x \in \Psi$, which satisfy the following axioms.

(1) $x + y = y + x$	(2) $(x + y) + z = x + (y + z)$
(3) $x + 0 = x$	(4) $a(bx) = (ab)x$
(5) $1x = x$, $0x = 0$	(6) $(a + b)x = ax + bx$
(7) $a(x + y) = ax + ay$	(8) $^{-}{}^{-}x = {}^{-}x$
(9) $\hat{}\hat{}x = x$	(10) $^{-}(x + y) = {}^{-}({}^{-}x + {}^{-}y)$
(11) $\hat{}(x + y) = \hat{}x + \hat{}y$	(12) $^{-}(x + \hat{}y) = 0 \equiv {}^{-}x = {}^{-}y$
(13) $^{-}(\hat{}x) = \hat{}({}^{-}x)$	(14) $^{-}(ax) = a({}^{-}x)$, $\hat{}(ax) = a(\hat{}x)$
(15) $x + y = 0 \rightarrow x = 0 \wedge y = 0$	(16) $ax = 0 \rightarrow a = 0 \vee x = 0$

Then we call Ψ a redundant algebras or a redundant space. We speak of a real redundant algebras or a Z^+-redundant algebras according as $T=R^+$ or $T=Z^+$ respectively.

[Proposition 1.1]

(1) $^{-}(x + \hat{}x) = 0$

(2) $x + z = x \rightarrow z = 0$

Proof:(1) From axiom (12)' we get $^{-}(x + \hat{}x) = 0 \equiv {}^{-}x = {}^{-}x$ and $^{-}x = {}^{-}x$ is clear. (2) Let us show contraposition. Let $z \neq 0$. If we assume $x + z = x$ then $\hat{}x + x + z = \hat{}x + x$. From (1) $^{-}(x + \hat{}x) = 0$. Thus $^{-}(\hat{}x + x + z) = {}^{-}((\hat{}x + x) + z) = {}^{-}({}^{-}(\hat{}x + x) + {}^{-}z) = {}^{-}(0 + {}^{-}z) = {}^{-}{}^{-}z = z = 0$ ■

[Definition 1.2] Subspace : A non-empty subset M of a redundant space Ψ is called a redundant subspace of Ψ、 if M satisfies the following conditions.

(1) if $x, y \in M$ then $x + y \in M$ (2) $\forall x \in M, a \in T$ $ax \in M$

(3) $\forall x \in M$ $\hat{}x \in M$ (4) $\forall x \in M$ $^{-}x \in M$

If M satisfies only the condition (1), (2), then M is called a weak redundant subspace of Ψ. Next we introduce a similar notion of the norm of usual linear spaces.

[Definition 1.3] Redundant norm

A norm on a redundant space Ψ is a function which assigns to each element x in the Ψ a real number $|x|$ in such a manner that

(1) $|x|>0$ and $|x|=0 \equiv x=0$ (2) $|ax|=a|x|$

(3) $|x + y|<|x| + |y|$ (4) $|\hat{}x|=|x|$

(5) $|{}^{-}x|<|x|$ and $|{}^{-}x|=|x| \equiv {}^{-}x=x$

[Proposition 1.2] $|{}^{-}(x + y)|<|x| + |y|$

We can introduce the notions of linear combination, linear dependence, linear independence, dimension, and basis according to linear spaces but we omit it in this paper[1, 2, 3].

[Definition 1.4] Linear combination and weak linear combination

Let Ψ be a redundant space, and if $\{x_1, ..., x_n\}$ be a finite nonempty set of elements in Ψ, then an element $x = a_1x_1 + a_2x_2 + ...anx_n + b_1x_1...b_nx_n$ is called a linear combination of $x_1, ..., x_n$, where $a_1, ..., a_n, b_1, ..., b_n \in T$. And $y = a_1x_1 + a_2x_2 + ... + a_nx_n$ is called a weak linear combination of $x_1, ..., x_n$, where $a_1, ..., a_n \in T$.

[Definition 1.5] Linear independence, linear dependence, dimension, basis Let Ψ be a redundant space, and let $S = \{x_1, ..., x_n\}$ be a finite non-empty set of elements in Ψ. S is said to be linearly dependent if there exist scalars $a_1, a_2, ..., a_n, b_1, b_2, ..., b_n$, for not all of $i(1<i<n)$ $a_i=b_i$, such that

$$\overline{\ }(a_1x_1 + a_2x_2 + ... + a_nx_n + b_1\hat{\ }x_1 + ... + b_n + \hat{\ }x_n)=0. \qquad (1)$$

If S is not linearly dependent, then it is called linearly independent ;and this clearly means that if equation (1) holds for certain scalar coefficients $a_1, ..., a_n, b_1, ..., b_n$, then for all these scalar $a_i=b_i(1<i<n)$ necessarily , i.e., if $\overline{\ }(a_1x_1 + a_2x_2 + ... + a_nx_n + b_1\hat{\ }x_1 + ... + b_n\hat{\ }x_n)=0$ then $a_1=b_1, ..., a_n=b_n$.

Let Ψ be a redundant space. If Ψ contains n independent elements and every n + 1 elements always become dependent, then Ψ is called n-dimensional. If there exist n independent elements for $\forall n \in N$, then Ψ is called infinite-dimensional. Let Ψ be a n-dimensional redundant space, then a set of n independent elements is called a basis for Ψ.

[Proposition1.3]

Let Ψ be a n-dimensional redundant space. If $e_1, e_2, ..., e_n$ are independent elements in Ψ, then $\forall x \in \Psi$ is uniquely expressed such as $x=a_1e_1 + ... + a_ne_n + b_1\hat{\ }e_1 + ... + b_n\hat{\ }e_n$.

Proof: Let $x = \Sigma a_ie_i + \Sigma b_i\hat{\ }e_i$, $y = \Sigma a_i'e_i + \Sigma b_i'\hat{\ }e_i$ and $x = y$. Then we get $\overline{\ }(x+\hat{\ }y) = 0$ from axiom (12) and $\overline{\ }x = \overline{\ }y$. Thus $\overline{\ }(\Sigma(a_i+b_i')e_i + \Sigma(a_i'+b_i)\hat{\ }e_i) = 0$ holds. We assume that e1, e2, ⋯, en are independent, then $a_i+b_i' = a_i'+b_i$, $i = 1, \cdots, n$ holds. Let $c_i = a_i - a_i' = b_i - b_i'$, then we show that c_i become 0 for any i. From proposition 1.1 (2), $x + y = 0 \rightarrow x = 0 \wedge y = 0$ holds. We can assume that $c_i \geqq 0$, $a_i = a_i' + c_i$, $b_i = b_i' + c_i$. Thus we can assume $z = \Sigma c_ie_i + \Sigma c_i\hat{\ }e_i$ and $y+z=x$ and $z=0$ hold. From axiom (15), $c_ie_i=0$, $c_i\hat{\ }e_i=0$, for $i=1, \cdots, n$ hold. Then we get $c_i=0$ from $e_i \neq 0$, $\hat{\ }e_i \neq 0$.

Q. E. D.

[Definition 1.6]

Let A be an arbitrary non-empty subset of a n-dimensional redundant space Ψ. We denote the smallest subspace which contains A by $[A]_r$. We denote the set of all weak linear combinations of elements in A by [A]. Usually [A] is not a redundant space.

[Definition 1.7] Basic basis, dual basis, extended basis.

Let Λ be a basis of Ψ. If $|x|=1$ for $\forall x \in \Lambda$, then Λ is called a basic basis of Ψ and $\hat{\ }\Lambda = \{\hat{\ }e \mid e \in \Lambda\}$ is called the dual basis of Λ and $\Gamma = \Lambda \cup \hat{\ }\Lambda$ is called the extended basis of Λ.

[Proposition 1.4]

(1)If Λ be a basis of a n-dimensional redundant space Ψ, then $\hat{\ }\Lambda$ is a basis of Ψ.

(2)If Λ be a basis of a n-dimensional redundant space Ψ, then $[\Lambda]_r = [\hat{\ }\Lambda]_r =$[

$\Lambda \cup {}^\wedge\Lambda] = \Psi$

1.3 EXCHANGE ALGEBRAS

We add the axioms to redundant algebras and define exchange algebras.

[Definition 1.8] Exchange Algebras

Let Λ be a basic basis of a redundant space Ψ and $\Gamma = \Lambda \cup {}^\wedge\Lambda$ be the extended basis of Λ. If there exist a relation $\Leftrightarrow$ on Γ satisfying next six axioms, then Ψ is called an exchange algebras. We call the relation $\Leftrightarrow$ an exchange relation and the basic basis Λ on which the relation $\Leftrightarrow$ is defined is called an exchange basis of an exchange algebras and Γ is called an extended exchange basis.

(1) $\forall x, y \in \Gamma \;\; x \Leftrightarrow y \equiv {}^\wedge x \Leftrightarrow {}^\wedge y$ (2) $\forall x, y, z \in \Gamma \;\; x \Leftrightarrow y$ and $y \Leftrightarrow z \rightarrow \neg(x \Leftrightarrow z)$

(3) $\forall x, y \in \Gamma \;\; x \Leftrightarrow y \equiv y \Leftrightarrow x$ (4) $\forall x, y \in \Gamma \;\; x \Leftrightarrow y \rightarrow \neg(x \Leftrightarrow {}^\wedge y)$

(5) $\forall x, y, z \in \Gamma \;\; \neg(x \Leftrightarrow y)$ and $\neg(y \Leftrightarrow z) \rightarrow \neg(x \Leftrightarrow z)$

(6) $\forall x \in \Gamma \;\; \exists y \in \Gamma \;\; x \Leftrightarrow y$

Remark: In exchange algebras the selection of an exchange basis on which an exchange relation is defined is important. We do not assume that an exchange relation is preserved in a transformation between basis.

[Definition 1.9] Trivial basis

Let Λ be an exchange basis of an exchange algebras Ψ. If $\forall x, y \in \Lambda \;\; \neg(x \Leftrightarrow y)$, then Ψ is called trivial and Λ is called a trivial exchange basis.

[Proposition 1.5]

Let Λ be an non-empty exchange basis of an exchange algebras Ψ.

$\forall x \in \Gamma \;\; x \Leftrightarrow {}^\wedge x$, where $\Gamma = \Lambda \cup {}^\wedge\Lambda$

[Proposition 1.6]

(1) $\forall x \in \Gamma \;\; \neg(x \Leftrightarrow x)$ (2) $\neg(x \Leftrightarrow {}^\wedge y) \rightarrow x \Leftrightarrow y$

[Example] Accounting Algebras

Let Λ={assets, liabilities, capital, profit, cost } and let ${}^\wedge\Lambda$={^assets, ^liabilities, ^capital, ^profit, ^cost } and $\Gamma = \Lambda \cup {}^\wedge\Lambda$. Let $[\Gamma]_f$ denote the commutative free semi-group generated by Γ with unit element 0. Then the operations $\bar{\ }$ and ^ can be defined on $[\Gamma]_f$ (cf.[2]) and it is easy to show that $[\Gamma]_f$ is a redundant algebras. If the exchange relation $\Leftrightarrow$ on Γ is defined below, then $[\Gamma]_f$ becomes an exchange algebras: assets $\Leftrightarrow$ liabilities, assets $\Leftrightarrow$ profit, assets $\Leftrightarrow$ capital, cost $\Leftrightarrow$ liabilities, cost $\Leftrightarrow$ profit, cost $\Leftrightarrow$ capital and other relations on Γ are determined uniquely by the axioms.

[Proposition 1.7]

Let Λ be a basic basis of a redundant space Ψ and $\Leftrightarrow$ be a relation on Λ which satisfy the axioms (2), (3), (5), (6). Then the relation $\Leftrightarrow$ can be extended to the relation on the extended basis Γ uniquely.

2. ECONOMIC FIELD

2.1 DEFINITION OF ECONOMIC FIELD

In this section we extend the notion of exchange algebras for describing multi units economic systems. Let Ω be a set of economic units, then we define the state of economic field as a function from Ω to accounting vector space. The notion of economic field corresponds to the notion of vector field or tensor field in the physical systems description. Nevertheless its similarity is small. Vector space is defined on the three dimensional space. In the economic field a set of economic units(Ω) corresponds to the space and the structure of Ω can change in its self-organizing process and each unit $\omega \in \Omega$ might have different decision making rules and different roles in the system. Next we introduce the basic definitions for discussing these problems.

[Definition 2.1] R-L decomposition, P-M decomposition

Let $\Gamma = \Lambda \cup \hat{}\Lambda$=credit side $\cup$ debit side, is a basis of exchange algebras. Then the following operators are introduced depending on proposition 1.4 of the previous part.

(1) $x \in [\Gamma]$ is decomposed uniquely as follows,

$x = x_r + x_l$, $x_r \in$[debit side], $x_l \in$[credit side].

We define operators R and L as $R(x)=x_r$, $L(x)=x_l$. Then the R-L decomposition of x is defined as $x=R(x) + L(x)$.

(2) $x \in [\Gamma]$ is decomposed uniquely as follows,

$x = x^+ + x^-$, $x^+ \in [\Lambda]$, $x^- \in [\hat{}\Lambda]$.

We call the decomposition, $x=P(x)+M(x)$, $P(x)=x^+$, $M(x)=x^-$, P-M decomposition of x.

[Cor.] RR=R, LL=L, PP=P, MM=M

[Definition 2.2] norm balance

If $x \in [\Gamma]$ satisfies the condition $|R(x)|=|L(x)|$, then x is called norm balanced, where the norm | | is accounting norm defined on exchange algebras.

[Definition 2.3] norm balanced space

Let $[\Gamma]_{nb} = \{x \mid x \in [\Gamma] \text{ and } |R(x)|=|L(x)|\}$, then $[\Gamma]_{nb}$ is called a norm balanced space.

[Proposition 2.1]

$[\Gamma]_{nb}$ is a sub algebras of an exchange algebras$[\Gamma]$, that is,

(1) if $x, y \in [\Gamma]_{nb}$ then $x+y \in [\Gamma]_{nb}$

(2) if $x \in [\Gamma]_{nb}$ and $a \in N$ then $ax \in [\Gamma]_{nb}$

(3) if $x \in [\Gamma]_{nb}$ then $\hat{}x \in [\Gamma]_{nb}$

(4) if $x \in [\Gamma]_{nb}$ then $\bar{x} \in [\Gamma]_{nb}$

We can explain error checking ability of the book keeping system by using the notion of norm balanced space. The calculations in the bookkeeping system is closed in the sub algebras, norm balanced space, then we can find the error by checking the norm balanced property of the element x which is the result of any calculations on the norm balanced space.

It is possible to extend our algebras to the operator algebras on the space. It will become a functional analysis over the exchange algebras. These extension is necessary for studying the problem of production and decision making. In this manuscript we omit these problems. Instead of it we formulate the notion of economic field.

[Definition 2.4] ECONOMIC FIELD

Let Ω be a set of economic units and $[\Gamma]_{rb}$ be a norm balanced space.

Then $K(\Omega, [\Gamma]_{rb})=\{f \mid f:\Omega\rightarrow[\Gamma]_{rb}\}$ is called an economic field and an element f is called a state of economic field.

We consider that the state description by using economic field is common in any economic systems.

3. STRUCTURE OF ECONOMIC FIELD AND SNA

In this chapter we introduce the definition of simplified national economic field.

[Definition 3.1] Basic Set of Economic Units Ω_0, Basic accounts Λ_0

$\Omega_0=\{a_1, ..., a_n, b_1, ..., b_m,$ Household, Government, Overseas$\}$

Where, $\{a_1, ..., a_n\}$=Set of manufacturing enterprises, $\{b_1, ..., b_m\}$=Set of banking enterprises

Λ_0=Asset accounts_0 $\cup$Liability accounts_0 $\cup$Revenue accounts_0 $\cup$Cost accounts_0

Asset accounts_0=Current assets $\cup$Products $\cup$Service $\cup$ Stock investments

Current assets={Cash, Deposits, Trade accounts receivable, Receivable, Equity securities}

Products=$\{e_1, ..., e_k\}$

Service={Government service}

Stock investments={ Equipment investments, Inventory investments}

Stockholders' equity_0={Capital stock, Reserve for depreciation, Reserve}

Liability accounts_0={Trade accounts payable, Payable}

Revenue accounts_0={Value added, Operating surplus, Wages earned, Taxes revenue, Dividends earned, Interest earned}

Cost accounts_0={Wages expense, Interest expense, Taxes expense, Dividends expense, Consumption expense}

Next we introduce the two types of sets of economic units and map between them.

[Definition 3.2] Sector-Set Ω_1, Integrated-Set Ω_2, Map g1, g2

Ω_1={Industry, Banking enterprise, Household, Government, Overseas}

Ω_2={Nation, Overseas}

Maps g1:$\Omega_0 \rightarrow \Omega_1$, g2:$\Omega_1 \rightarrow \Omega_2$ are defined as follows.

(1) g1:$\Omega_0 \rightarrow \Omega_1$

(a)If $\omega \in$Set of manufacturing enterprises then g1(ω)=Industry

(b)If $\omega \in$Set of banking enterprises then g1(ω)=Banking enterprise

(c)If $\omega \in${Household, Government, Overseas} then g1(ω)= ω

(2) g2:$\Omega_1 \rightarrow \Omega_2$

(a) $\omega \in${Industry, Banking enterprise, Household, Government} then g1(ω)=Nation

(b) $\omega \in${Overseas} then g1(ω)= ω

[Definition 3.3] Internal transaction, External transaction

The transactions in a period are classified as follows.

(1) Internal transaction $Tr_{int}[\omega_i, \Omega, \Gamma \mid a_k] \subseteq F[\Omega, \Gamma]$

Let f be a transaction and f(ω)=0 iff $\omega \neq \omega_i$, then f is called an internal transaction for ω_i.

Internal transactions are made as self-decision making. INT give a classification of the internal transactions as follows.

INT={Production, Equipment investments, Inventory investments, Consumption expense, Operating surplus transfer, Depreciation, Reserve transfer}

a. Production

$Tr_{int}[\omega_i, \Omega, \Gamma \mid$ Production]

f(ω_i)=x_1^a_1+...+$x_n a_n$+za_n+1+yValue added,

where a_1, ..., $a_n \in$Products, a$\in$Products $\cup${Government service}, z=x_1+...+x_n+y.

b. Equipment investments (Industry, Government)

$Tr_{int}[\omega_i, \Omega, \Gamma \mid$ Equipment investments]

f(ω_i)=x_1^e+x_1Equipment investments, e$\in$Products.

c. Inventory investments

$Tr_{int}[\omega_i, \Omega, \Gamma \mid$ Inventory investments]

f(ω_i)=x_1^e+x_1Inventory investments, e$\in$Products.

d. Consumption expense(Household, Government)

$Tr_{int}[\omega_i, \Omega, \Gamma \mid$ Consumption expense]

f(ω_i)=x_1^e+x_1Consumption expense, e$\in$Products $\cup${Government service}.

e. Operating surplus transfer

$Tr_{int}[\omega_i, \Omega, \Gamma \mid$ Operating surplus transfer]

f(ω_i)=x_1^Value added+x_1Operating surplus.

f. Depreciation

$Tr_{int}[\omega_i, \Omega, \Gamma \mid$ Depreciation]

f(ω_i)=x_1^Value added+x_1Reserve for depreciation.

g. Reserve transfer

$Tr_{int}[\omega_i, \Omega, \Gamma$ | Reserve transfer]
f(ω_i)=x_1^Operating surplus+x_1Reserve.
e.t.c.

(2) External transaction $Tr_{ext}[\alpha, \beta, \Omega, \Gamma \mid b_k] \subseteq F[\Omega, \Gamma]$
f:External transaction btween α and β, $\alpha, \beta \in \Omega$ is a transaction as follows.
f(α)≠0, f(β)≠0, f(ω)=0 if $\omega \neq \alpha$ or β.$b_k \in$EXT is classified by External transaction as follows.
EXT={Domestic products transaction, Deposits transaction, Product export transaction, Product import transaction, Interest transaction,
Wages transaction, Taxes transaction, Dividends transaction, Loan transaction, Equity securities transaction, e.t.c.}

a. Domestic products transaction
$Tr_{ext}[\alpha, \beta, \Omega, \Gamma$ | Domestic products transaction]
For example,
f(α)=x_1^e_i+x_1Cash, f(β)=x_1e_i+x_1^Cash, or
f(α)=x_1^e_i+x_1Trade accounts receivable, f(β)=x_1e_i+x_1Trade accounts payable
$e_i \in$Products

b. Deposits transaction
$Tr_{ext}[\alpha, \beta, \Omega, \Gamma$ | Deposits transaction]
f(α)=x_1^Cash+x_1Deposits
f(β)=x_1Cash+x_1Deposits payable
or
f(α)=x_1Cash+x_1^Deposits
f(β)=x_1^Cash+x_1^Deposits payable
$\alpha \in${Household, Government} ∪Set of manufacturing enterprises, $\beta \in$Set of banking enterprises

c. Product export transaction
$Tr_{ext}[\alpha$, Overseas, Ω, Γ | Product export transaction]
f(α)=x_1^e_i+x_1Cash
f(Overseas)=x_1e_i+x_1^Cash
$e_i \in$Products

d.Product import transaction
$Tr_{ext}[\alpha$, Overseas, Ω, Γ | Product import transaction]
f(α)=x_1e_i+x_1^Cash
f(Overseas)=x_1^e_i+x_1Cash
$e_i \in$Products

e.Interest transaction
$Tr_{ext}[\alpha, \beta, \Omega, \Gamma$ | Interest transaction]
f(α)=x_1Cash+x_1Interest earned

$f(\beta)=x_1$^Cash$+x_1$Interest expense

f.Wages transaction

$Tr_{ext}[\alpha$, Household, Ω, Γ | Wages transaction]

$f(\alpha)=x_1$Wages expense$+x_1$^Cash

f(Household)$=x_1$Wages earned$+x_1$Cash

g.Taxes transaction

$Tr_{ext}[\alpha$, Government, Ω, Γ | Taxes transaction]

$f(\alpha)=x_1$Taxes expense$+x_1$^Cash

f(Government)$=x_1$Taxes revenue$+x_1$Cash

h.Dividends transaction

$Tr_{ext}[\alpha, \beta, \Omega, \Gamma$ | Dividends transaction]

$f(\alpha)=x_1$Dividends expense$+x_1$^Cash

$f(\beta)=x_1$Dividends earned$+x_1$Cash

i.Loan transaction

$Tr_{ext}[\alpha, \beta, \Omega, \Gamma$ | Loan transaction]

$f(\alpha)=x_1$Receivable$+x_1$^Cash

$f(\beta)=x_1$Payable$+x_1$Cash

or $f(\alpha)=x_1$^Receivable$+x_1$Cash

$f(\beta)=x_1$^Payable$+x_1$^Cash

j.Equity securities transaction

$Tr_{ext}[\alpha, \beta, \Omega, \Gamma$ | Equity securities transaction]

$f(\alpha)=x_1$Equity securities$+x_1$^Cash

$f(\beta)=x_1$Capital stock$+x_1$Cash

or $f(\alpha)=x_1$Equity securities$+x_1$^Cash

$f(\beta)=x_1$^Equity securities$+x_1$Cash

In this case we ommit the problem of capital gain.

e.t.c.

We denote $\cup\{Tr_{int}[\omega_i, \Omega, \Gamma \mid X] \mid \omega_i \in \Omega\}$by $Tr_{int}[\omega, \Gamma \mid X]$, or simply X. We also define the set of internal transactions as follows.

$INTrS= \cup\{Tr_{int}[\omega, \Gamma \mid X] \mid X \in INT\}$

We denote $\cup\{Tr_{ext}[\alpha, \beta, \Omega, \Gamma \mid X] \mid \alpha, \beta \in \Omega\}$ by $Tr_{ext}[\Omega, \Gamma \mid X]$ or simply X. We also define the set of external transactions as follows.

$EXTrS= \cup\{Tr_{ext}[\Omega, \Gamma \mid X] \mid X \in EXT\}$

Then the set of transactions of ω_i, $TrS[\Omega_i]$, is defined as follows.

$TrS[\omega_i]=EXTrS[\omega_i] \cup INTrS[\omega_i]= \cup\{Pr[\omega_i](f) \mid f \in EXTrS\} \cup \cup\{Pr[\omega_i](f) \mid f \in INTrS\}$,

where, $Pr[\omega_i]$ is a projection of a transaction to the transaction of the economic unit ω_i.

The set of the transactions of the economic units belonging to Ω, $TrS[\Omega]$, is defined as follows.

$TrS[\Omega]= \cup\{Tr[\omega_i] \mid \omega_i \in \Omega\}$.

Notice that our descriptions are different from usual business accounting. It is rather similar to SNA. Instead that SNA consists of statistical notions our notions are algebraic.

In this paper we do not discuss how these transactions are determined. This is a problem of economic decision making and self controle. In this paper we introduce field quantities of economic field which are basis of studying interactons on the field.

[Definition 3.4] Basic-National economic field, Sector-National economic field, Integrated-National economic field

We denote Basic-National economic field by $F[\Omega_0, \Gamma_0]$, Sector-National economic field by $F(\Omega_1, \Gamma_0)$or$F[\Omega_1, \Gamma 1]$and Integrated-National economic field by $F[\Omega_2, \Gamma_0]$.

[Definition 3.5] Simple-Sector-Aggregation map

Simple-Sector-Aggregation map $G_0: F(\Omega_0, \Gamma_0) \rightarrow F(\Omega_1, \Gamma_0)$

Let $f \in F(\Omega_0, \Gamma_0)$ then Simple-Aggregation map G_0 is defined as follows.

For $\omega \in \Omega_2$, $G_0(f)(\omega) = \Sigma\{f(\alpha) \mid \alpha \in g_1^{-1}(\Omega)\}$

In some cases of aggregation it is important to offset the accounts of transactions. Then we introduce the offset function as follows.

[Definition 3.6] Offset function

Let f be transactions between economic units $\alpha 1$ and $\alpha 2$ and AGS={{Trade accounts receivable, Trade accounts payable}, {Deposits, Deposits payable}, {Interest earned, Interest expense}, {Dividends earned, Dividends expense}, {Receivable, Payable}, {Equity securities, Capital stock}}

Then offset function S[A, B], $\{A, B\} \in AGS$ is defined as follows.

For $f(\alpha 1) = xA + \ldots$, $f(\alpha 2) = xB + \ldots$, $\alpha 1, \alpha 2 \in g^{-1}(\omega)$,

$S[A, B](f)(\alpha 1) = \overline{\ }(f(\alpha 1) + x\hat{\ }A)$

$S[A, B](f)(\alpha 2) = \overline{\ }(f(\alpha 2) + x\hat{\ }B)$

§ Sector-National economic field

【Flow-quantities】

[Definition 3.7] Production Vector $f_{pro}[e_i]$, Needs Vector $f_{needs}[e_i]$

Let Products=$\{e_1, \ldots, e_k\}$ and denote $\Sigma\{Pr[e](f) \mid f \in Set\}$, $SET \subseteq F[\Omega, \Gamma]$ by $f_{sum}[e, Set]$. Then the production Vector and the needs Vector are defined as follows.

$f_{pro}[e_i] = f_{sum}[e_i$, Production$] + f_{sum}[e_i$, Product import transaction$]$

$f_{needs}[e_i] = f_{sum}[\hat{\ }e_i$, Production$] + f_{sum}[\hat{\ }e_i$, Industry Equipment investments$] + f_{sum}[\hat{\ }e_i$, Government Equipment investments$] + f_{sum}[\hat{\ }e_i$, Inventory investments$]$

$+ f_{sum}[\hat{\ }e_i$, Product export transaction$] + f_{sum}[\hat{\ }e_i$, Household Consumption expense$] + f_{sum}[\hat{\ }e_i$, Government Consumption expense$]$.

Where, Industry Equipment investments= $\cup\{Tr_{int}[\omega_i, \Omega, \Gamma$ |Equipment investments] | $\omega_i \in$ Industry},
Government Equipment investments=Tr_{int}[Government, Ω, Γ |Equipment investments],
Household Consumption expense=Tr_{int}[Household, Ω, Γ |Consumption expense],
Government Consumption expense=Tr_{int}[Government, Ω, Γ |Consumption expense].
$f_{Interim\ needs}[e_i]=f_{sum}[\hat{}e_i$, Production]

It is possible to express the above formulas by balanced table according as SNA as follow.

$f_{sum}[e_i$, Production]	$f_{Interim\ needs}[e_i]$
$f_{sum}[e_i$, Product import transaction]	$f_{sum}[\hat{}e_i$, Industry Equipment investments]
	$f_{sum}[\hat{}e_i$, GovernmentEquipment investments]
	$f_{sum}[\hat{}e_i$, Inventory investments]
	$f_{sum}[\hat{}e_i$, Product export transaction]
	$f_{sum}[\hat{}e_i$, Household Consumption expense]
	$f_{sum}[\hat{}e_i$, Government Consumption expense]
$f_{pro}[e_i]$	$f_{needs}[e_i]$

[Definition 3.8] Products Integrated-production Vector f_{pro}[Products], Needs Vector f_{needs}[Products]

f_{pro}[Products]= $\Sigma\{ f_{pro}[e_i]$ | $e_i \in$ Products}

f_{pro}[Products]=f_{pro}[Domestic products] + f_{pro}[Overseas products]

f_{pro}[Domestic products]= $\Sigma\{ f_{sum}[e_i$, Production] | $e_i \in$ Products}

f_{pro}[Overseas products]= $\Sigma\{ f_{sum}[e_i$, Product import transaction] | $e_i \in$ Products}

f_{needs}[Products]=f_{needs}[Production] + f_{needs}[Industry Equipment investments] + f_{needs}[GovernmentEquipment investments] + f_{needs}[Inventory investments] + f_{needs}[Product export transaction] + f_{needs}[Household Consumption expense] + f_{needs}[Government Consumption expense]

f_{needs}[Production]= $\Sigma\{ f_{sum}[\hat{}e_i$, Production] | $e_i \in$ Products}

f_{needs}[Industry Equipment investments]= $\Sigma\{ f_{sum}[\hat{}e_i$, Industry Equipment investments] | $e_i \in$ Products}

f_{needs}[GovernmentEquipment investments]= $\Sigma\{ f_{sum}[\hat{}e_i$, GovernmentEquipment investments] | $e_i \in$ Products}

f_{needs}[Inventory investments]= $\Sigma\{ f_{sum}[\hat{}e_i$, Inventory investments] | $e_i \in$ Products}

f_{needs}[Product export transaction]= Σ{ f_{sum}[^e_i, Product export transaction] | e_i∈Products}
f_{needs}[Household Consumption expense]= Σ{ f_{sum}[^e_i, Household Consumption expense] | e_i∈Products}
f_{needs}[Government Consumption expense]= Σ{ f_{sum}[^e_i, Government Consumption expense] | e_i∈Products}

This is also shown by following balanced table.

f_{pro}[Domestic products]	f_{needs}[Production]
f_{pro}[Overseas products]	f_{needs}[Industry Equipment investments]
	f_{needs}[GovernmentEquipment investments]
	f_{needs}[Inventory investments]
	f_{needs}[Product export transaction]
	f_{needs}[Household Consumption expense]
	f_{needs}[Government Consumption expense]
f_{pro}[Products]	f_{needs}[Products]

Next we introduce the vector fields which correspond to the V-table and U-table of SNA.

[Definition 3.9] V Vector , U Vector

(1)V Vector field fv :This is a vector on which manufacturing enterprise, {a_1, ..., an}, the products, {e_1, ..., e_k}, are made. fv(a_i)denote a production by economic unit a_i.

fv(a_i)= Σ{ f_{sum}[e, Pr[a_i](Production)] | e∈Products}

Where Pr[a_i](Production)={Pr[a_i](f) | f ∈Production}.

(2)U Vector field fu :This is a vector on which manufacturing enterprise, {a_1, ..., an}, the products, {e1, ..., ek}, are used for input. fu(a_i)denote a input for production by economic unit a_i.

fu(a_i)= Σ{ f_{sum}[^e, Pr[a_i](Production)] | e∈Products}

It is easy to get U-table and V-table from these vectors and is also easy to get input-output table by using appropriate assumptions.

[Definition 3.10] Industry-sector, Government-sector Value added production and appropriation Vector

$f_{\text{Value added production}}[\omega]$, $f_{\text{Value added appropriation}}[\omega]$

These are vectors of production and appropriation of value added.

$f_{\text{Value added production}}$[Industry]=$f_{sum}$[Value added, Industry Production]

$f_{\text{Value added appropriation}}$[Industry]=$f_{sum}$[Wages expense, Industry Wages transaction] + f_{sum}[Operating surplus, Industry Operating surplus transfer] + f_{sum}[Reserve for depreciation, Industry depreciation]

$f_{\text{Value added production}}$[Government]=$f_{sum}$[Value added, Government production]

$f_{\text{Value added appropriation}}$[Government]=$f_{\text{sum}}$[Wages expense, GovernmentWages transaction] + f_{sum}[Reserve for depreciation, Government depreciation]
Where,
Industry production= $\cup\{Tr_{int}[\omega_i, \Omega, \Gamma$ | Production] | $\omega_i \in$ Industry}
Industry depreciation= $\cup\{Tr_{int}[\omega_i, \Omega, \Gamma$ | Depreciation] | $\omega_i \in$ Industry}
Industry Operating surplus transfer= $\cup\{Tr_{int}[\omega_i, \Omega, \Gamma$ | Operating surplus transfer] | $\omega_i \in$ Industry}
Industry wages transaction= $\cup\{Tr_{ext}$[Household, ω_i, Ω, Γ | Wages transaction] | $\omega_i \in$ Industry}
Government production=Tr_{int}[Government, Ω, Γ | Production]
Government depreciation=Tr_{int}[Government, Ω, Γ | Depreciation]
Government wages transaction=Tr_{ext}[Household, Government, Ω, Γ | Wages transaction]

[Definition 3.11]

Industry, Government sector production Vector $f_{\text{pro}}[\omega]$
Industry, Government sector cost Vector $f_{\text{cost}}[\omega]$
Industry, Government sector needs Vector $f_{\text{needs}}[\omega]$

These are vectors of production and its cost.
f_{pro}[Industry]=f_{pro}[Domestic products]
f_{cost}[Industry]= $\Sigma\{fu(a_i) \mid a_i \in \{a_1, ..., an\}\}$ + f_{sum}[Wages expense, Industry wages transaction] + f_{sum}[Operating surplus, Industry Operating surplus transfer] + f_{sum}[Reserve for depreciation, Industry depreciation]
Where $\Sigma\{fu(a_i) \mid a_i \in \{a_1, ..., an\}\}$is called fInterim input. Then
f_{cost}[Industry]=fInterim input + f_{sum}[Wages expense, Industry wages transaction] + f_{sum}[Operating surplus, Industry Operating surplus transfer] + f_{sum}[Reserve for depreciation, Industry depreciation]
In general fInterim input is smaller than $f_{\text{Interim needs}}[e_i]$.
f_{needs}[Industry]=f_{needs}[Products]
f_{pro}[Government]=f_{sum}[Government service, Government production]
f_{cost}[Government]= $\Sigma\{f_{\text{sum}}[\hat{}e_i$, Government production] | $e_i \in$ Products} + f_{sum}[Wages expense, Government wages transaction] + f_{sum}[Reserve for depreciation, Government depreciation]
Notice: $\Sigma\{f_{\text{sum}}[\hat{}e_i$, Government production] | $e_i \in$ Products} denote the inputs for the production of government service.
f_{needs}[Government]=f_{sum}[^Government service, Government Consumption expense] + f_{sum}[^Government service, Household Consumption expense]
Where
Government Consumption expense=Tr_{int}[Government, Ω, Γ | Consumption expense],
Household Consumption expense=Tr_{int}[Household, Ω, Γ | Consumption

expense]

[Definition 3.12] Sector-incom Vector $f_{incom}[\omega]$,
Output Vector $f_{output}[\omega]$

These are the vector of income and expense (output).

f_{incom}[Industry]=f_{sum}[Operating surplus, Industry Operating surplus transfer] + f_{sum}[Interest earned, Industry interest transaction] + f_{sum}[Dividends earned, Industry Dividends transaction]

Where,

Industry interest transaction= $\cup\{Tr_{ext}[\alpha, \omega_i, \Omega, \Gamma$ |Interest transaction] | $\omega_i \in$Industry, $\alpha \in \Omega\}$,

Industry Dividends transaction= $\cup\{Tr_{ext}[\alpha, \omega_i, \Omega, \Gamma$ |Dividends transaction] | $\omega_i \in$Industry, $\alpha \in \Omega\}$

f_{output}[Industry]=

f_{sum}[Interest expense, Industry interest transaction] + f_{sum}[Dividends expense, Industry Dividends transaction] + f_{sum}[Taxes expense, Industry Taxes transaction] + f_{sum}[Deposits, Industry Deposits transaction] + f_{sum}[Reserve, Industry Reserve transfer]

Notice: the usual notion of deposit of SNA is defined as Reserve + Deposits in our system.

Industry Taxes transaction= $\cup\{Tr_{ext}[\alpha, \omega_i, \Omega, \alpha$ |Taxes transaction] | $\omega_i \in$Industry, $\alpha \in \Omega\}$,

Industry Deposits transaction= $\cup\{Tr_{ext}[\alpha, \omega_i, \Omega, \Gamma$ |Deposits transaction] | $\omega_i \in$Industry, $\alpha \in \Omega\}$

f_{incom}[Banking enterprise]=f_{sum}[Interest earned, Finantial interest transaction] + f_{sum}[Dividends earned, Finantial dividents transaction]

Where,

Finantial interest transaction= $\cup\{Tr_{ext}[\alpha, \omega_i, \Omega, \Gamma$ |Interest transaction] | $\omega_i \in$Banking enterprise, $\alpha \in \Omega\}$,

Finantial dividents transaction= $\cup\{Tr_{ext}[\alpha, \omega_i, \Omega, \Gamma$ |Dividends transaction] | $\omega_i \in$Banking enterprise, $\alpha \in \Omega\}$

f_{output}[Banking enterprise]=f_{sum}[Interest expense, Finantial interest transaction] + f_{sum}[Dividends expense, Finantial dividents transaction] + f_{sum}[Taxes expense, Finantial Taxes transaction] + f_{sum}[Reserve, Finantial Reserve transfer]

Where,

Finantial Taxes transaction= $\cup\{Tr_{ext}[\alpha, \omega_i, \Omega, \Gamma$ |Taxes transaction] | $\omega_i \in$ Banking enterprise, $\alpha \in \Omega\}$

Notice:In this model we ommit the notion of central bank.

f_{incom}[Household]=f_{sum}[Wages earned, Wages transaction] + f_{sum}[Interest earned, Household interest transaction] + f_{sum}[Dividends earned, Household dividents transaction]

Household interest transaction= $\cup$\{Tr_{ext}[α, Household, Ω, Γ | Interest transaction] | $\alpha \in \Omega$\}

f_{output}[Household]= f_{sum}[Consumption expense, Household Consumption expense] + f_{sum}[Interest expense, Household interest transaction] + f_{sum}[Taxes expense, Household wages transaction]
+ f_{sum}[Deposits, HouseholdDeposits transaction] + f_{sum}[Reserve, Household Reserve transfer]

f_{incom}[Government]=f_{sum}[Taxes revenue, Government wages transaction] + f_{sum}[Interest earned, Government interest transaction] + f_{sum}[Dividends earned, Government dividents transaction]

f_{output}[Government]=f_{sum}[Consumption expense, Government Consumption expense] + f_{sum}[Interest expense, Government interest transaction] + f_{sum}[Deposits, Government Deposits transaction] + f_{sum}[Reserve, Government Reserve transfer]

[Definition 3.13] Sector-accumulation Vector

(1)Stock investments Vector $f_{Stock\ investments}$[ω]

(2)Investments resource Vector $f_{Investments\ resource}$[ω]

(3)Deposits-investments difference (scalar quantity)

Difference[ω]= | $f_{Investments\ resource}$[ω] | — | $f_{Stock\ investments}$[ω] |

(4)Current assets increase Vector $f_{Current\ assets}$[ω]

(5)Financial liability increase Vector $f_{Financial\ liability}$[ω]

(6)Short and over (scalar quantity)

Short and over[ω]= | $f_{Current\ assets}$[ω] | — | $f_{Financial\ liability}$[ω] |

$\omega \in$\{Industry, Banking enterprise, Household, Government, Overseas\}=Ω_1 − \{Overseas\}

These are vector and scalar which mean investments resource, stock investments and so on of the economic units Industry, Banking enterprise, Household, Government.

(1) $f_{Stock\ investments}$[Industry]= Σ\{ f_{sum}[^e_i, Inventory investments] | $e_i \in$ Products\} + Σ\{ f_{sum}[^e_i, Equipment investments] | $e_i \in$Products\}

(2) $f_{Investments\ resource}$[Industry]=$f_{sum}$[Deposits, Industry Deposits transaction] + f_{sum}[Reserve for depreciation, Industry depreciation] + f_{sum}[Reserve, Industry Reserve transfer]

(3)Difference[Industry]= | $f_{Investments\ resource}$[Industry] | — | $f_{Stock\ investments}$[Industry] |

(4) $f_{Current\ assets}$[Industry]=$f_{Current\ assets}$[Industry, EXTrS]

$f_{Current\ assets}$[Industry, EXTrS]=

$\overline{\ }$(f_{sum}[Cash, $\cup$\{EXTrS[ω_i] | $\omega_i \in$Set of manufacturing enterprises\} + f_{sum}[^Cash, $\cup$\{EXTrS[ω_i] | $\omega_i \in$Set of manufacturing enterprises\})
+ $\overline{\ }$(f_{sum}[Equity securities, $\cup$\{EXTrS[ω_i] | $\omega_i \in$Set of manufacturing enterprises\} + f_{sum}[^Equity securities, $\cup$\{EXTrS[ω_i] | $\omega_i \in$Set of

manufacturing enterprises})
+ ¯(fsum[Deposits, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises} + fsum[^Deposits, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises})
+ ¯(fsum[Receivable, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises} + fsum[^Receivable, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises})
+ ¯(fsum[Trade accounts receivable, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises} + fsum[^Trade accounts receivable, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises})
(5) fFinancial liability[Industry]=fFinancial liability[Industry, EXTrS]
fFinancial liability[Industry, EXTrS]=
¯(fsum[Payable, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises} + fsum[^Payable, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises})
+ ¯(fsum[Trade accounts payable, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises} + fsum[^Trade accounts payable, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises})
+ fsum[Capital stock, ∪{EXTrS[ω_i] | ω_i∈Set of manufacturing enterprises}
(6)Short and over[Industry]= | fCurrent assets[Industry] | — | fFinancial liability[Industry] |
(7) fOffset Current assets[Industry]=fCurrent assets[Industry, OffsetEXTrS]
OffsetEXTrS=S[Receivable, Payable]·S[Deposits, Deposits payable]·S[Trade accounts receivable, Trade accounts payable]·S[Equity securities, Capital stock]·EXTrS
(8) fOffset Financial liability[Industry]=fFinancial liability[Industry, OffsetEXTrS]

[Definition 3.14] Sector-Overseas Vector
fImport[Industry]=fpro[Overseas products]
fExport[Industry]=fneeds[Product export transaction]

【Stock】
[Definition 3.15] Sector-Initial balance sheet Vector, Sector-Final balance sheet Vector
These vector mean the stock quantities of Sector-economic field.
fTBL[Final, ω]=fTBL[Initial, ω] + fflowL[Interim, ω], $\omega \in \Omega_1$—{Overseas}
fTBR[Final, ω]=fTBR[Initial, ω] + fflowR[Interim, ω]
fTBL[Final, Industry]=fTBL[Initial, Industry] + fflowL[Interim, Industry]
fTBR[Final, Industry]=fTBR[Initial, Industry] + fflowR[Interim, Industry]
fTBL[Initial, Industry]=x_1Equipment investments + x_2Inventory investments + x_3Cash + x_4Deposits + x_5Trade accounts receivable + x_6Receivable + x_7Equity securities
fTBR[Initial, Industry]=y_1Capital stock + y_2Trade accounts payable + y_3Payable + y_4Reserve + y_5Reserve for depreciation

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = y_1 + y_2 + y_3 + y_4 + y_5$

f_{flowL}[Interim, Industry]=$f_{Stock\ investments}$[Industry] + $f_{Current\ assets}$[Industry]

f_{flowR}[Interim, Industry]=f_{sum}[Reserve for depreciation, Industry depreciation] + f_{sum}[Reserve, Industry Reserve transfer] + $f_{Financial\ liability}$[Industry]

§ **Integrated-National economic field and its field quantities**
【Flow-quantities】

[Definition 3.16] Domestic Integrated cost Vector $f_{Domestic\ cost}$,
Domestic Integrated needs Vector $f_{Domestic\ needs}$

$f_{Domestic\ cost}$= f_{cost}[Industry] + f_{cost}[Government]

$f_{Domestic\ needs}$= f_{needs}[Industry] + f_{needs}[Government]

[Definition 3.17] Domestic Integrated production Vector $f_{Domestic\ pro}$,
Integrated output Vector $f_{Domestic\ con}$

$f_{Domestic\ pro}$ = f_{sum}[Wages expense, Industry wages transaction] + f_{sum}[Operating surplus, Industry Operating surplus transfer] + f_{sum}[Reserve for depreciation, Industry depreciation]
+ f_{sum}[Wages expense, Government wages transaction] + f_{sum}[Reserve for depreciation, Government depreciation]
= $\overline{\ }$($f_{Domestic\ cost}$ + $\Sigma\{$^$f_U(a_i)$ $|\ a_i \in \{a_1, ..., an\}\}$ + $\Sigma\{$^f_{sum}[^e_i, Government production] $|\ e_i \in$ Products$\}$)

$f_{Domestic\ con}$= $\overline{\ }$($f_{Domestic\ needs}$ + ^f_{needs}[Production])

[Definition 3.18] National incom Vector $f_{National\ incom}$,
National output Vector $f_{National\ output}$

$f_{National\ incom}$ = f_{sum}[Operating surplus, Industry Operating surplus transfer] + f_{sum}[Wages earned, Wages transaction]

Notice:In this vector the transfers revenue are not included such as interest earned, dividends earned and taxes revenue.

$f_{National\ output}$=
f_{sum}[Deposits, Industry Deposits transaction] + f_{sum}[Reserve, Industry Reserve transfer] + f_{sum}[Reserve, Finantial Reserve transfer] + f_{sum}[Consumption expense, Household Consumption expense] + f_{sum}[Deposits, Household Deposits transaction]
+ f_{sum}[Reserve, Household Reserve transfer] + f_{sum}[Consumption expense, Government Consumption expense] + f_{sum}[Deposits, GovernmentDeposits transaction] + f_{sum}[Reserve, Government Reserve transfer]

[Definition 3.19] National accumulation Vector
(1)National Stock investments Vector $f_{National\ Stock\ investments}$

(2)National Investments resource Vector $f_{\text{National Investments resource}}$

$f_{\text{National Stock investments}}=$

$\cup\{ f_{\text{Stock investments}}[\omega] \mid \omega \in \Omega_1-\{\text{Overseas}\}\}$

$f_{\text{National Investments resource}} =$

$\cup\{ f_{\text{Investments resource}}[\omega] \mid \omega \in \Omega_1-\{\text{Overseas}\}\}$

[Definition 3.20] Overseas operating transaction Vector

$f_{\text{Import}}= f_{\text{pro}}[\text{Overseas products}]$

$f_{\text{Export}}= f_{\text{needs}}[\text{Product export transaction}]$

National operating surplus= $|f_{\text{Export}}| - |f_{\text{Import}}|$

Notice:In this model we only treat transactions of products with oversea.

【Stock】

[Definition 3.21] National Initial balance sheet Vector, National Final balance sheet Vector

$f_{\text{TBL}}[\text{Final, Nation}]=f_{\text{TBL}}[\text{Initial, Nation}] + f_{\text{flowL}}[\text{Interim, Nation}]$

$f_{\text{TBR}}[\text{Final, Nation}]=f_{\text{TBR}}[\text{Initial, Nation}] + f_{\text{flowR}}[\text{Interim, Nation}]$

$f_{\text{TBL}}[\text{Initial, Nation}]=\Sigma\{ f_{\text{TBL}}[\text{Initial}, \omega] \mid \omega \in \Omega_1-\{\text{Overseas}\}\}$

$f_{\text{TBR}}[\text{Initial, Nation}]=\Sigma\{ f_{\text{TBR}}[\text{Initial}, \omega] \mid \omega \in \Omega_1-\{\text{Overseas}\}\}$

$f_{\text{flowL}}[\text{Interim, Nation}]=\Sigma\{ f_{\text{flowL}}[\text{Interim}, \omega] \mid \omega \in \Omega_1-\{\text{Overseas}\}\}$

$f_{\text{flowR}}[\text{Interim, Nation}] =\Sigma\{ f_{\text{flowR}}[\text{Interim}, \omega] \mid \omega \in \Omega_1-\{\text{Overseas}\}\}$

- Reference -

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