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# Continuation of Real Analytic Solutions of Partial Differential Equations up to Convex Conical Singularities

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In his talk at the RIMS Seminar in December 1985, Kaneko gave the following conjecture (*cf.* [Kn3]):

**Kaneko's Conjecture.** *Let $P = D_t^2 - \Delta$ be the wave operator on the Euclidean $n$ space $\mathbf{R}^n$. Let $\Gamma$ be a closed convex proper cone of $\mathbf{R}^n$ with vertex at the origin, sharp enough in a certain direction; i.e., $\Gamma$ is contained in $\{x_1 \geq C|x_2|\}$ for a Euclidean coordinate $(x_1, \cdots, x_n)$ of $\mathbf{R}^n$, for a large $C > 0$. Let $R > 0$ and set $K = \{(x,t) \in \mathbf{R}^n \times \mathbf{R} \mid x \in \Gamma,\ |t| \leq R|x|\}$. Then any real analytic solution to the wave equation $Pu = 0$ defined outside $K$ can be analytically continued up to the origin $(0,0)$ of $\mathbf{R}^n \times \mathbf{R}$.*

We give an answer to this conjecture in a general context.

**Definition.** *Let $K$ be a closed subset of a real analytic manifold $M$ of dimension $n$. $K$ is said to be $C^\alpha$-convex at $x \in M$ $(1 \leq \alpha \leq \omega)$ if there exist a neighborhood $U$ of $x$ and an open $C^\alpha$-immersion $\phi : U \to \mathbf{R}^n$ such that $\phi(U \cap K)$ is convex in $\mathbf{R}^n$. $K$ is said to have a conical singularity at $x$ if $x \in K$ and the tangent cone $C_x(K)$ is a closed proper cone of $T_xM$.*

**Theorem 0.1.** *Let $K$ be a $C^1$-convex closed subset of a real analytic manifold $M$, having a conical singularity at $x$. Let $P = P(x, D)$ be a second order differential operator with analytic coefficients defined in a neighborhood of $x$. Assume that $P$ is of real principal type and is not elliptic. Then any real analytic solution to the equation $Pu = 0$ defined outside $K$ is analytically continued up to $x$.*

In order to state a similar result for overdetermined systems of differential equations, we first recall the notion of a virtual bicharacteristic manifold of a system $\mathcal{M}$ of differential equations.

Let $V = \mathrm{Char}(\mathcal{M})$; $V^c$ denotes the complex conjugate of $V$ with respect to $T_M^*X$. Let $p \in V \cap (T_M^*X \setminus M)$. Assume the following:

(b.1) $V$ is nonsingular at $p$.
(b.2) $V$ and $V^c$ intersect cleanly at $p$; i.e., $V \cap V^c$ is a smooth manifold and

$$T_pV \cap T_pV^c = T_p(V \cap V^c).$$

(b.3) $V \cap V^c$ is regular; i.e., $\omega|_{V \cap V^c} \neq 0$, with $\omega$ being the fundamental 1-form on $T^*X$.
(b.4) The generalized Levi form of $V$ has constant rank in a neighborhood of $p$.

Then one can define the virtual bicharacteristic manifold $\Lambda_p$ of $\mathcal{M}$ passing through $p$ (*cf.* [SKK, Ch.III, Sect.2.4]). we assume

(b.5) $d\pi(T_p\Lambda_p) \neq \{0\}$.

**Theorem 0.2.** *Let* $(K,\, x)$ *be as in Theorem 0.1. Let* $\mathcal{M}$ *be a system of differential equations defined in a neighborhood of* $x$*. Assume that* $\mathrm{Char}(\mathcal{M}) \cap \pi^{-1}(x)$ *has codimension* $\geq 2$ *in* $\pi^{-1}(x)$ *and that* $V = \mathrm{Char}(\mathcal{M})$ *satisfies conditions* (b.1)—(b.5) *at each point* $p$ *of* $V \cap (T^*_M X \setminus M) \cap \pi^{-1}(x)$*. Then any real analytic solution to* $\mathcal{M}$ *defined outside* $K$ *is analytically continued up to* $x$*.*

**Corollary.** *Let* $(K,\, x)$ *be as in Theorem 0.2. Let* $\mathcal{M}$ *be an elliptic system of differential equations and assume that* $\mathrm{Char}(\mathcal{M}) \cap \pi^{-1}(x)$ *has codimension* $\geq 2$ *in* $\pi^{-1}(x)$*. Then any solution* $u$ *of* $\mathcal{M}$ *defined outside* $K$ *can be analytically continued up to* $x$*.*

*Remark.* Cf. [Kw], theorems 4 and 5, for general results on analytic continuation of the solutions of overdetermined systems of differential equations.

The following theorem is a generalization of Theorem 0.1 to higher order differential equations for $K = \{x_0\}$. *Cf.* Theorem 17 and Corollary 22 of [Kn2].

**Theorem 0.3.** *Let* $P = P(x,\, D)$ *be a differential operator of real principal type. Assume that the polynomial* $f(x_0; \zeta)$ *in* $\zeta$ *has no elliptic factors. Then any real analytic solution to the equation* $Pu = 0$ *defined in a neighborhood of* $x_0$ *except* $x_0$ *can be analytically continued on the whole of a neighborhood of* $x_0$*.*


[Kn1] A. Kaneko — On continuation of regular solutions of linear partial differential equations, Publ. RIMS, Kyoto Univ., 12, Suppl., 1977, 113-121.

[Kn2] A. Kaneko — On continuation of real analytic solutions of linear partial differential equations, Astérisque 89-90, Soc. Math. France (1981), 11-44.

[Kn3] A. Kaneko — On continuation of real analytic solutions of linear partial differential equations up to convex conical singularities, Sûriken-Kôkyûroku 592, RIMS Kyoto Univ. (1986), 149-172 (Japanese).

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