数理解析研究所講究録
第 825 巻 1993 年 118-120

The pattern of rheological phase transition by the VERD diagram

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Synopsis

We are concerned with rheolosis. A continuously deformable object is called a rheoloid. A mixture consisting of various materials is a typical rheoloid (e.g. water-oil, earth-sand, polymer, blood); it is comprehensible to imagine a emulsive colloid. Rheolosis is analysis and synthesis of rheoloids. The flowing appearance of a rheoloid changes drastically near a critical condition. This is the rheo-phase transition and there are infinitely many patterns of rheo-phase transition. The purpose of this note is to classify rationally the patterns of rheo-phase transitions by introducing the VERD diagram.

1. Space-time-information manifold

<u>In the first three sections</u>, we explane <u>expositorily</u> the rheo-phase transition by introducing the information manifold $(\Lambda, \mathbf{K})$ and the conception VERD. Dynamics is classified into discrete dynamics and continuum dynamics. Continuum dynamics is classified into fluiddynamics, electromagnetic dynamics, powder dynamics (i.e. micromeritics) etc. according to the types of parity and the types of stress tensor on the space-time manifold. In this note, we confine ourselves to the Galilei parity. The 3-dimensional rotation group is the parity on the space manifold, and the group of Galilei transforms is the parity between the space manifold and the time manifold. Thus the parity $\mathcal{G}$ on the space-time manifold is the group generated by these two groups.

Rheology is the condensed matter-theoretic continuum dynamics regarding a continuum as a rheoloid. In this note, a rheoloid is identified with a emulsive mixture of various materials, and a character of a rheoloid is also regarded as a material. The interaction between two materials is determined by various infromation on the set Λ of materials λ and its structure $\mathbf{K}$. The pair $(\Lambda, \mathbf{K})$ is an information manifold. This is an internal space proper to a rheoloid. The triple $\mathrm{STI} = (V \times [0, \infty) \times \Lambda, \mathcal{G}, \mathbf{K})$ is a space-time-information manifold (STI manifold), where V is a given rheoloid domain in the 3-dimensional Euclidean space $\mathbf{R}$.

2. VERD (viscosity, elasticity, rotation, distortion)

The set Λ is decomposed into four parts Λ_σ ($\sigma = v, e, r, d$) so that each material $\lambda \in \Lambda_\sigma$ is σ-ic. Here "v, e, r, d" is the head letters of viscosity, elasticity, rotation and distortion. Now we show the meanings of these four conceptions and introduce a unified conception VERD.

A rheoloid particle is an infinitesimally small part of a rheoloid. The Euler-Lagrange acceleration of a rheoloid particle in flow is a vector field (v.f.) and regarded as a tensor field (t.f.). Forming a relation between the acceleration and the stress t.f. is called Hooke's principle. Flow is determined by Hooke's principle. The standard strain t.f. generated by a velocity field is a symmetric t.f.. Viscosity is to form a relation between the acceleration and this strain t.f. by Hooke's principle. Viscous flow is a v.f. deduced from viscosity, and a viscous colloid (v-ic colloid) is a material for which viscosity fits. The rotation v.f. (i.e. the vortex) is the unique vector invariant associated with the displacement t.f. of a velocity field, and this is identified with a skew-symmetric stress t.f.. Rotation is to form a relation between the acceleration and this stress t.f. by Hooke's principle. Rotational flow is a v.f. deduced from rotation, and a rotational colloid (r-ic colloid) is a material for which rotation fits. A rotational colloid is an imaginary material, however, it is convenient to introduce such materials; recall that an actual turbulence grows from a small germ of turbulence, which suggests this imagination. Viscocity and rotation are two conceptions based on Hooke's principle by the stress t.f. generated by a velocity field.

A velocity field is considered as the Galilei derivative of a position v.f. (cf. Section **5**). Elasticity is to form a relation between the acceleration and the standard strain t.f. generated by this position v.f., and distortion is to form a relation between the acceleration and the rotation t.f. generated by this position v.f.. Thus elasticity and distortion are two conceptions based on Hooke's principle by the stress t.fs. generated by a position v.f.. The meanings of elastic flow, distortional flow, elastic colloid and distortional colloid are analogous as above. Four conceptions are based on Hooke's principle. It is logically possible to form analogous relations by the strain t.fs. of higher degrees of Galilei primitives and Galilei derivatives, however, the other relations are neglected.

Once four conceptions "v, e, r, d" have been defined, it is natural to introduce a unified conception VERD. VERD is to form a system of relations between a family of the accelerations and a family of the stress t.fs. for rheoloid particles consisting of materials $\lambda \in \Lambda$. Since four conceptions exist, we decompose Λ as above. Note that a mixture of two σ-ic colloid is not necessary one uniform σ-ic colloid (like water and oil). Thus Λ_σ is also a set of materials.

VERD flow is a v.f. deduced from VERD and a VERD colloid is a set of materials for which VERD fits. A VERD colloid particle is an infinitesimally small part of a VERD colloid.

3. Actual rheo-phase transitions of VERD flow

It is convenient to consider that even water consists of viscous colloids and rotational colloids. Hagen-Poiseuille's law for slow-speed laminer flow (Reynolds number ≈ 100) is a character of viscous colloids, and Blasius's law for high-speed turbulent flow (Reynolds number $\approx 10^5$) is a character of rotational colloids. This rheo-phase transition comes from the change of Reynolds number. Flow of a mixture of water and oil reduces to either flow of a uniform colloid or flow consisting of a water-shed and an oil-shed. This phase transition is related to the change of a parameter on a structure of an information manifold. Slow-speed flow of polymer (e.g. polyisopren, silicone fluid) shows the elastic recoil phenomenon. The picture taken by polarized light shows the existence of many distortional lines. On the other hand, high-speed flow of polymer is near to viscous flow. This rheo-phase transition is caused by the change of a paramenter on the time manifold. The control of turbulence by polymer is an application of a rheo-phase transition. An avalenche and a landslide are typical rheo-phase transitions from the state at rest to the violent motion. These come from the change of a parameter on the static pressure. Blood is an important VERD colloid and the precipitation to various materials is a rheo-phase transition related to both Torricelli's theorem and the principle of sandglass. Several parameters are necessary to understand this rheo-phase transition.