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Offer Analysis for the Arbitration Procedure FDOA

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Abstract

When two disputants are ignorant of their opponents' estimates about the arbitrator's fair point, the arbitration procedure FDOA (Final-Double-Offer Arbitration) is better than other procedures, because it can induce the existence of a non-empty contract zone in some cases even though the IG (intrinsic gap) of the estimates is positive. This paper gives offer analysis for FDOA. First, we show that when two disputants are risk neutral, the dsiputants' offers of FDOA are closer than those of FOA. Then we consider the influence of risk attitude to offers, and show that when the disputants increase their risk aversion, their offers converge to their reservation prices respectively, so that they can reach agreement by themselves. We use an example to show that this property does not hold for the arbitration procedure FOA (Final-Offer Arbitration).

Key words: dispute, final-offer arbitration, final-double-offer arbitration, game, offer, risk attitude.

1 Introduction

In real life, disputes arise in politics, economic, sociology and many other fields. Arbitration is very effective in solving these disputes. Since 1966, when Stevens proposed arbitration procedure "Final-Offer Arbitration", which is abbreviated as FOA, the research about arbitration procedures has become more and more important.

The procedure FOA requires that the arbitrator choose one of the offers that is closer to his own fair point. Before FOA was proposed, conventional arbitration, by which the arbitrator takes his fair point as settlement, was the only arbitration method. Although two disputants are also usually required to give their own offers for reference, the arbitrator is allowed to proclaim any settlement, which he thinks fair, as the final result. You can image that this procedure encourages two disputants give greedy offers. The introduction of FOA is path-breaking, because FOA uses one of the disputants' offers, instead of arbitrator's favorite point, as the arbitration result. Anyone of the disputants does not dare to give too greedy offer under FOA, otherwise his opponent's offer will be chosen by the arbitrator, and then becomes the final result. Besides, after all, the arbitrator is an outsider, his fair point may hurt both disputants.

In FOA, if any disputant is afraid that his opponent' offer may be chose as the final result, so that he makes more concession in negotiation and therefore an agreement is reached before the real arbitration, the dispute is soon settled. This time, we say that the disputants have

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reached agreement by themselves. That the procedure FOA is welcome is just because it induces disputants do so. Brams, Kilgour and Weber (1991) cited a comment on FOA by a baseball arbitrator: "I'm starting to feel like the atomic bomb. The deterrent effect of me as an arbitrator is enough". Thus when we evaluate arbitration procedure, its deterrent force is an important factor. However, as analyzed in Brams and Merrill (1983), the disputants' offers will not compatible if they ask to realize arbitration. We cannot expect that two disputants can reach agreement while giving offers. Because the arbitrator is not allowed to give a compromise settlement, the final result will certainly hurt one of the disputants. Therefore the closeness of the disputants' offers is another important criterion.

Of course, two disputants do not know exactly where the arbitrator's favorite point is, otherwise the disputants have to let their offers be equivalent to it. Up to now, while analyzing arbitration procedure like FOA, most literature suppose that two disputants share the same estimates about the arbitrator's favorite point, therefore the authors can use the effective tool — Nash equilibrium to analyze offers. Samuelson (1991) get rid of this assumption, but he supposes that two disputants know that the arbitrator chooses the mid-point of something like reservation prices — private information of disputants, which are supposed to be known by the arbitrator. Usually, two disputants are ignorant of their opponents' estimates about the arbitrator's favorite point, let alone their equivalence. Noting this fact, in this paper, we consider "prudent offer", supposing that the disputants use prudent strategies when they give offers. We have another reason to do so. Wittman (1986) used the concept of "Cournot-Nash" equilibrium. Under some conditions, there exist(s) (some) equilibrium (equilibria). The conditions are very strong in the sense that, even the simplest case of the model Brams and Merrill (1983) used, the uniform estimate does not satisfy the conditions Wittman proposed. In addition, when there are more than one Cournot-Nash equilibria, we have to suppose that two disputants image the same equilibrium. Therefore. when we consider the general model, under the supposition that the two estimates may be different, it is not bad to neglect the existence of Nash equilibrium. Thus we use prudent offer in this paper. This also provides us the possibility to deal with pure strategy only. Arbitration is different from usual game. Mixed strategies is meaningful if the game is multiply played, but arbitration is absolutely once only.

In Zeng Ohnishi and Ibaraki (1992), we think that the main reason that disputants resort to arbitration is that their estimates have positive "intrinsic gap", the contract zones of FOA and many other procedures are empty if positive intrinsic gap appears, in other words, the procedures cannot overcome this gap. Therefore we present a new arbitration procedure FDOA and we show that FDOA is better than FOA and some other procedures in the sense that it can overcome the intrinsic gap under certain conditions. This paper further analyzes the procedure FDOA.

As stated above, when we analyze arbitration procedure, two factors are considered. Zeng Ohnishi and Ibaraki (1992) have used the concept of "contract zone" to analyze the deterrent force of FDOA. This paper considers the second factor. In Section 2, we derive some necessary conditions for disputants' prudent offers in FDOA and FOA. In Section 3, we show that when the disputants are risk neutral, their prudent offers of FDOA are closer than that of FOA, therefore FDOA is better than FOA by the above mentioned criterion. Second, we examined FDOA for the case that disputants may be risk averse. We show that FDOA is strong enough to induce risk averse disputants to reach agreement.

Farber and Katz (1979) and Wittman (1986) give some results about the effect of increased risk aversion by one of the disputants, for conventional arbitration and FOA respectively. Their results show that the "contract zone" becomes larger when disputants become more risk averse. Wittman (1986) gives a result about the convergence of the disputants' offers when the disputants become infinitely risk-seeking, but he has not given any result about the convergence of the offers when the disputants become infinitely risk averse. In our opinion, the latter case is more important, because usually, people are risk averse. In the final part of Section 4, we use an example to show that FOA is not strong enough to induce any disputant to give offer that converges to his "reservation price" when he is infinitely risk averse.

2 Arbitration Procedure FDOA

As usually, we use the buyer-seller model to analyze arbitration. Two parties, referred to as seller and buyer, must jointly decide on a determinate value of some continuous variable x in $[L, H]$, where L (H) is a finite lower (upper) bound. Disputant s (seller) wants the value to be high — the higher the better — whereas b (buyer) wants the value to be low — the lower the better. L (H) can be explained as reservation price, which represents the very minimum (maximum) the party s (b) will settle for.

As everyone knows, the buyer may often be an individual which is usually risk averse, and the seller may often be a firm, which is usually risk neutral. Since we will consider the risk attitude effect in Section 4, we mainly examine the buyer part when we give proofs for theorems, if the similar conclusions hold for the seller part.

In Zeng Ohnishi and Ibaraki (1992), we propose arbitration FDOA. In FDOA, disputants s and b are requested to give pairs of two offers (x_s, y_s), and (x_b, y_b) respectively. The first components x_s and x_b are their real offers, which may become the final result as in FOA. The second components y_s and y_b are their estimates of the arbitrator's fair point. Although their estimates may be probabilistic, y_s and y_b are required to be a single point.

As illustrated in the flow chart of Figure 1, the arbitrator will first compare y_s, y_b with his fair point x_a. If $y_s > y_b$, their estimates are not compatible. This time, as in FOA, the arbitrator will choose x_s (x_b or $(x_s + x_b)/2$) as the final result if y_s is closer (farther or equal) to x_a than y_b to x_a. By this rule, we discourage the disputants to give an estimate that is too optimistic. If $y_s \leq y_b$, their estimates are compatible. This time, the arbitrator will compare $|x_s - y_s|$ with $|x_b - y_b|$. If $|x_s - y_s| < |x_b - y_b|$, s has made more concession and thus s's offer x_s will be the final result. By this rule, we encourage the disputants to make their offers x_s and x_b close to their estimates y_s and y_b respectively, therefore neither player can make an arbitrarily extreme real offer.

Let us describe disputant s's (b's) estimate on the arbitrator's fair point by probabilistic density function $f_s(x)$ $(f_b(x))$ and distribution function $F_s(x)$ $(F_b(x))$ over $[L, H]$. We suppose that $f_s(x)$ and $f_b(x)$ are continuous functions with means E_s, E_b, medians m_s, m_b respectively. As usual, we use $U_s(x)$ and $U_b(x)$ to denote the utility functions of disputants s and b respectively, which are supposed to be differentiable.

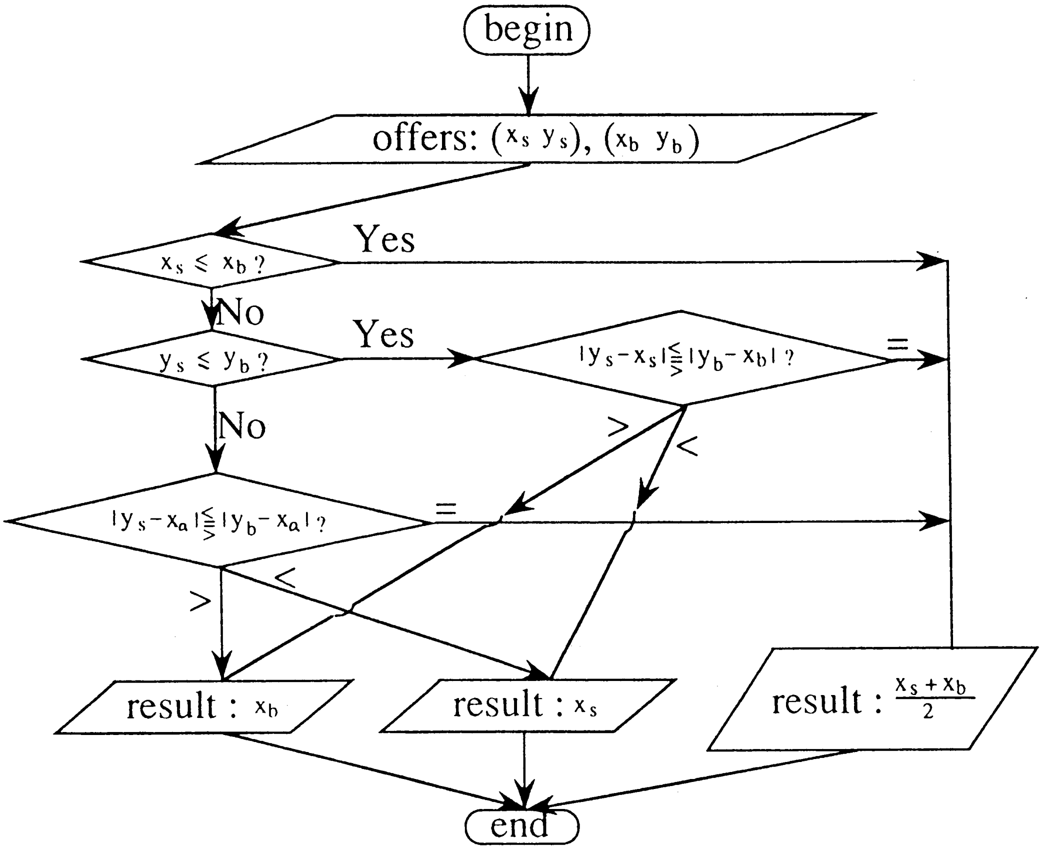

Fig. 1: flow chart of FDOA procedure

At first, we use two axioms to modify the activity of our disputants.

Axiom 2.1 *The disputants give offers in* $[L, H]$.

If one of the disputants give any offer which is outside $[L, H]$, the arbitrator will judge that his opponent wins.

Axiom 2.2 *For any positive number* ϵ, $\int_L^{L+\epsilon} f_s(x)dx > 0$, $\int_{H-\epsilon}^{H} f_b(x)dx > 0$.

This axiom assumes that any disputant cannot affirm that the arbitrator will give result that is better than his reservation price for ϵ. This assumption can simplify our discussion without much loss of generality. Because, if this axiom is violated, say for some ϵ and disputant s, we can use $L + \epsilon$ to replace L.

Usually, disputants s and b do not share a common estimate about the arbitrator's fair settlement. On the contrary, they may not even know their opponents' estimates. Therefore we suppose that they use a prudent offer which is defined below:

Definition 2.1 *(Definition 2.2 of Zeng Ohnishi and Ibaraki (1992)) With the density function $f_s(x)$ ($f_b(x)$) of estimate, value x_s^* (x_b^*) is a prudent offer of s (b) under an arbitration procedure P if and only if*

$$\inf_{x_b\in[L,H]} g_s(x_s^*, x_b|P) = \sup_{x_s\in[L,H]} \inf_{x_b\in[L,H]} g_s(x_s, x_b|P)$$

$$\left(\inf_{x_s\in[L,H]} g_b(x_s, x_b^*|P) = \sup_{x_b\in[L,H]} \inf_{x_s\in[L,H]} g_b(x_s, x_b^*|P)\right)$$

where $g_s(x_s, x_b|P)$ ($g_b(x_s, x_b|P)$) denotes the expected profit of s (b) under the procedure P if s's offer is x_s and b's offer is x_b.

Of course, this definition makes sense only under the condition that the maximum and the minimum can be arrived. In this paper, we assume that the prudent offers always exist. Besides, the above definition can be easily adapted for the case that players give more than one offer, such as in procedure FDOA.

Lemma 2.1 *Let (x_s^*, y_s^*), (x_b^*, y_b^*) be the prudent offers of s and b in procedure FDOA. Then*

$$\left\{\begin{array}{l} U_s(x_s^*)\int_{y_s^*}^{H} f_s(t)dt = U_s(2y_s^* - x_s^*) \\ 2U_s'(x_s^*)\int_{y_s^*}^{H} f_s(t)dt = U_s(x_s^*)f_s(y_s^*) \end{array}\right. \tag{2.1}$$

and

$$\left\{\begin{array}{l} U_b(x_b^*)\int_{L}^{y_b^*} f_b(t)dt = U_b(2y_b^* - x_b^*) \\ -2U_b'(x_b^*)\int_{L}^{y_b^*} f_b(t)dt = U_b(x_b^*)f_b(y_b^*) \end{array}\right. \tag{2.2}$$

hold.

For arbitration procedure FOA, it is easy to prove the following necessary condition:

Lemma 2.2 *In FOA, s's prudent offer $x_s^\sharp$ and b's prudent offer $x_b^\sharp$ satisfy the following equations (2.3) and (2.4) respectively:*

$$\left\{\begin{array}{l} U_s'(y_s^\sharp)F_s(\frac{x_s^\sharp+y_s^\sharp}{2}) = \frac{U_s(x_s^\sharp)-U_s(y_s^\sharp)}{2}f_s(\frac{x_s^\sharp+y_s^\sharp}{2}) \\ \left[U_s'(x_s^\sharp)+U_s'(y_s^\sharp)\right]F_s(\frac{x_s^\sharp+y_s^\sharp}{2}) = U_s'(x_s^\sharp) \end{array}\right. , \tag{2.3}$$

where $y_s^\sharp$ is b's best reply to s's offer $x_s^\sharp$, and

$$\left\{\begin{array}{l} U_b'(x_b^\sharp)F_b(\frac{x_b^\sharp+y_b^\sharp}{2}) = \frac{U_b(y_b^\sharp)-U_b(x_b^\sharp)}{2}f_b(\frac{x_b^\sharp+y_b^\sharp}{2}) \\ \left[U_b'(x_b^\sharp)+U_b'(y_b^\sharp)\right]F_b(\frac{x_b^\sharp+y_b^\sharp}{2}) = U_b'(y_b^\sharp) \end{array}\right. , \tag{2.4}$$

where $y_b^\sharp$ is s's best reply to b's offer $x_b^\sharp$.

As special case, when the disputants are risk neutral, their prudent offers have the following forms:

Lemma 2.3 *(Theorem 3.1 of Zeng Ohnishi and Ibaraki (1992)) Assume that $f_s'(m_s)$ and $f_b'(m_s)$ exist and $f_s(m_s) > 0$, $f_b(m_b) > 0$ hold. Then, in FOA,*

$$x_s^\sharp = m_s + \frac{1}{2f_s(m_s)}, \qquad x_b^\sharp = m_b - \frac{1}{2f_b(m_b)}$$

are prudent offers of risk neutral disputants s and b respectively, if

$$f_s(x) \le f_s(m_s) + 4f_s^2(m_s)|x - m_s| \quad \text{for } x \text{ such that } |x - m_s| \le \frac{1}{4f_s(m_s)},$$

$$f_b(x) \le f_b(m_b) + 4f_b^2(m_b)|x - m_b| \quad \text{for } x \text{ such that } |x - m_b| \le \frac{1}{4f_b(m_b)}$$

and there exist $c_{s1} \in [L, m_s]$, $c_{s2} \in [m_s, H]$, $c_{b1} \in [L, m_b]$ and $c_{b2} \in [m_b, H]$ such that

$$\begin{array}{lcll} f_s(x) & \ge & f_s(m_s)exp(-2f_s(m_s)|x - m_s|) & \text{for } c_{s1} \le x \le c_{s2}, \\ f_s(x) & \le & f_s(m_s)exp(-2f_s(m_s)|x - m_s|) & \text{for } x \le c_{s1} \text{ and } x \ge c_{s2}, \\ f_b(x) & \ge & f_b(m_b)exp(-2f_b(m_b)|x - m_b|) & \text{for } c_{b1} \le x \le c_{b2}, \\ f_b(x) & \le & f_b(m_b)exp(-2f_b(m_b)|x - m_b|) & \text{for } x \le c_{b1} \text{ and } x \ge c_{b2}. \end{array}$$

3 Comparison of FDOA with FOA

In this section, we discuss the case when two disputants are risk neutral. We prove that the prudent offer of s in FDOA is smaller than that in FOA, and the prudent offer of b in FDOA is larger than that in FOA, therefore FDOA is better than FOA in the sense that the two disputants' offers are closer. In order to use the convenient expression of the disputants' prudent offers in FOA, we supposes that the functions $f_s(t)$ and $f_b(t)$ satisfy the conditions in Lemma 2.3.

Theorem 3.1 *For two risk neutral disputants s and b, if $f_s(x)$ is concave and symmetrical in $[L, H_s]$ with median $m_s = \frac{L+H_s}{2}$, $f_b(x)$ is concave and symmetrical in $[L_b, H]$ with median $m_b = \frac{L_b+H}{2}$ then*

$$x_s^\sharp > x_s^*, \quad x_b^\sharp < x_b^*,$$

where $x_s^\sharp$ and $x_b^\sharp$ are the prudent offers in FOA of s and b respectively, x_s^* and x_b^* are the first components of the prudent offers in FDOA of s and b respectively.

Proof[1]: We only prove the case for b, i.e.,

$$x_b^\sharp < x_b^*. \tag{3.1}$$

[1] To conserve space, details of the proofs of other theorems, lemmas and consequences, which are not presented in this paper, are available from the authors.

By Lemmas 2.1 and 2.3, we have

$$x_b^\sharp = m_b - \frac{1}{2f_b(m_b)}$$

and x_b^* with parameter y solves the following equations:

$$(H - y)f_b(y) = F_b(y)(1 + F_b(y)) \tag{3.2}$$

$$(H - x_b^*)f_b(y) = 2F_b(y). \tag{3.3}$$

It is easy to check that $f_b(y) > 0$. Therefore we can rewrite (3.3) as

$$H - x_b^* = \frac{2F_b(y)}{f_b(y)} = \frac{2(H - y)}{1 + F_b(y)} \tag{3.4}$$

where the second equality is from (3.2).

First, we consider the case $y \geq m_b$. If $f_b(y) \leq f_b(m_b)/2$, as $f_b(x)$ is concave, $H - y \leq (H - m_b)/2$. From the second equality of (3.4), $H - x_b^* \leq 2(H - y) \leq H - m_b$, therefore $x_b^\sharp < m_b \leq x_s^*$. If $f_b(y) > f_b(m_b)/2$, by the first equality of (3.4) and equation (3.2), we have

$$y - x_b^* = \frac{F_b(y)[1 - F_b(y)]}{f_b(y)} = \frac{\frac{1}{4} - [F_b(y) - \frac{1}{2}]^2}{f_b(y)} < \frac{1}{2f_b(m_b)},$$

therefore

$$x_b^* > y - \frac{1}{2f_b(m_b)} \geq m_b - \frac{1}{2f_b(m_b)} = x_b^\sharp.$$

Now we consider the case $y \in (L_b, m_b)$. As f_b is concave and symmetrical around m_b, $f_b(m_b) = \max_{t \in [L_b, H]} f_b(t)$, therefore $(H - L_b)f_b(m_b) \geq F_b(H) = 1$, i.e.,

$$f_b(m_b) \geq 1/(H - L_b); \tag{3.5}$$

Let $A(t) = 2m_b F_b(t) - (t - L_b)$, we have $A(m_b) = A(L_b) = 0$, $A''(t) = 2m_b f_b'(t) \geq 0$ for $t \in (L_b, m_b)$ thus

$$F_b(t) \leq \frac{t - L_b}{H - L_b} \quad \text{for} \quad t \in (L_b, m_b)\ ; \tag{3.6}$$

Similar to (3.6), as f_b is concave, we have

$$f_b(t) \geq \frac{2(t - L_b)}{H - L_b} f_b(m_b) \quad \text{for} \quad t \in (L_b, m_b)\ . \tag{3.7}$$

We conclude that the equality in (3.7) does not hold for $t \in (L_b, m_b)$. In fact, as $f_b(t)$ is concave, $f_b(m_b) \geq f_b(t) \geq 0$ for all $t \in (L_b, m_b)$. If this equality holds for some $t^* \in (L_b, m_b)$, then it holds for all $t \in (L_b, m_b)$. Thus $f_b(m_b) = 1/(m_b - L_b)$, $f_b(t) = 2(t - L_b)/(H - L_b)^2$, $F_b(t) = (t - L_b)^2/(H - L_b)^2$ for $t \in [L_b, m_b]$. Under these conditions , (3.2) does not hold for all $y \in (L_b, m_b)$. Therefore the inequality in (3.7) holds strictly for all $t \in (L_b, m_b)$.

By the above three inequalities, we have

$$H - x_b^* = \frac{2F_b(y)}{f_b(y)} < \frac{1}{f_b(m_b)} \leq H - m_b + \frac{1}{2f_b(m_b)} = H - x_b^\sharp\ , \tag{3.8}$$

Which leads to (3.1).

It is easy to show that $x_b^* > L_b$, thus we can conclude our proof. □

Remark 3.1 *Note that we have not suppose any order relation between L_b and L, H_s and H. According to the Axiom 2.2, if $[L_b, H] \subseteq [L, H]$ and $[L, H_s] \subseteq [L, H]$, the intrinsic gap (Zeng, Ohnishi and Chen (1992)) between the two estimates is zero. However, our results are not restricted to this case. They hold even if the intrinsic gap is positive.*

In Brams and Merrill (1991), the authors propose an improvement for the procedure FOA, which is called "Final–Offer Arbitration with a Bonus". They have also shown that the offers of this procedure are closer than those of FOA, but that the final result of this procedures are the same as under FOA. Differently, because FDOA's result is also one of the offers or the average, just as the same of FOA, the above theorem is strong, it tells us that FDOA is intrinsically better than FOA.

4 Risk Attitude Effect for FDOA

For simplicity, we now assume that each party has a utility function in the following form:

$$U(p, \alpha) = p^{\frac{1}{\alpha}} \quad \text{for} \quad p \geq 0, \tag{4.1}$$

where p is the profit and α is a parameter which is supposed to be positive. By Axiom 2.1, we do not consider the case that $p < 0$.

This is a convenient functional form because, the risk preferences of the parties are completely determined by α. He is risk averse, risk neutral or risk loving as α is respectively, greater than, equal to, or less than one. And he becomes more risk averse as α becomes larger.

Lemma 2.1 has the following corollary:

Corollary 4.1 *In FDOA, when the disputant b's utility function is in the form of (4.1), then his prudent offer with parameter α is*

$$x_b(\alpha) = H - \frac{2F_b(y_b(\alpha))}{\alpha f_b(y_b(\alpha))} \tag{4.2}$$

where $y_b(\alpha)$ satisfies the following equation:

$$\alpha(H - y_b(\alpha))f_b(y_b(\alpha)) = F_b(y_b(\alpha))\,(1 + F_b^{\alpha}(y_b(\alpha)))\,. \tag{4.3}$$

Now we consider the problem how the offers are changed when the disputants become more and more risk averse. We only consider disputant b in this section, therefore we omit the subscript b, and denote b's estimate density function as $f(t)$, b's estimate distribution function as F(t), his prudent offer in FDOA as $(x(\alpha), y(\alpha))$, if there is no other illustration.

For simplicity, we suppose that $f(t)$ is concave and symmetrical in $[L_b, H]$.

Lemma 4.1 *The offer $y(\alpha)$ is a strictly increasing function of α, furthermore*

$$\lim_{\alpha \to \infty} y(\alpha) = H. \tag{4.4}$$

Corollary 4.2

(4.5) $$\lim_{\alpha\to\infty} x(\alpha) = H.$$

By Lemma 4.1, $\lim_{\alpha\to\infty} F(y(\alpha)) = 1$, therefore

(4.6) $$\lim_{\alpha\to\infty} (F(y(\alpha)))^{\frac{1}{1-F(y(\alpha))}} = \frac{1}{e}.$$

thus when α converges to ∞, $F^{\alpha}(y(\alpha))$ converges if and only if $\alpha(1 - F(y(\alpha)))$ converges.

As $f(t)$ is concave and symmetrical in $[L_b, H]$ for $t \in [m_b, H]$, $(H-t)f(t)/2 \le 1 - F(t) \le (H-t)f(t)$. According to (4.3), for α which is large enough,

(4.7) $$\begin{aligned}&\frac{F(y(\alpha))[1+F^{\alpha}(y(\alpha))]}{2}\\ \le\ & \alpha(1-F(y(\alpha))) \le F(y(\alpha))[1+F^{\alpha}(y(\alpha))].\end{aligned}$$

Combining (4.6) and (4.7), we can conclude that $\alpha(1 - F(y(\alpha)))$ and $F^{\alpha}(y(\alpha)))$ cannot simultaneously converge to ∞ and therefore they are bounded. Hence there are convergent sequences:

$$A = \lim_{\alpha_n\to\infty} F^{\alpha_n}(y(\alpha_n))), \qquad B = \lim_{\alpha_n\to\infty} \alpha_n F(y(\alpha_n)).$$

Lemma 4.2 *The above limit A is bounded:* $e^{-2} < A < e^{-1/2}$.

Lemma 4.3 *There exists* α^* *such that when* $\alpha > \alpha^*$, $x(\alpha)$ *is also a strictly increasing function of* α.

Proof: Differentiate (4.2) with respect to α, the following equality holds:

$$\alpha f(y(\alpha))x'(\alpha) = [-2f(y(\alpha)) + \alpha(H - x(\alpha))f'(y(\alpha))]y'(\alpha) + (H - x(\alpha))f(y(\alpha)).$$

It is easy to check that

$$-2f(y(\alpha)) + \alpha(H - x(\alpha))f'(y(\alpha)) < 0.$$

Therefore what we want to prove is that when α is large enough,

$$[2f(y(\alpha)) - \alpha(H - x(\alpha))f'(y(\alpha))]y'(\alpha) < (H - x(\alpha))f(y(\alpha)),$$

which is equivalent to

(4.8) $$\begin{aligned}&2\alpha(H - y(\alpha))f^2(y(\alpha))\\ >\ & [-\ln F^{\alpha}(y(\alpha))]F^{\alpha+1}(y(\alpha))\left(\frac{2f(y(\alpha))}{\alpha} - (H - x(\alpha))f'(y(\alpha))\right).\end{aligned}$$

The equivalence is due to the equations (4.2) and (4.3). According to Lemma 4.1, there exist α_1 such that when $\alpha > \alpha_1$, $y(\alpha) > m_b$ thus without loss of generality, we suppose that $f'(y(\alpha)) < 0$

in (4.8), therefore $-f'(y(\alpha)) \le f(y(\alpha))/(H-y(\alpha))$ as $f(t)$ is concave and decreasing in $[m_b, H]$. So the following inequality can assure (4.8).

$$\begin{aligned} &2F(y(\alpha))[1+F^\alpha(y(\alpha))] \\ > \;& [-\ln F^\alpha(y(\alpha))]F^{\alpha+1}(y(\alpha))\left(\frac{2}{\alpha}+\frac{2}{1+F^\alpha(y(\alpha))}\right). \end{aligned} \tag{4.9}$$

Let the left part of (4.9) be $L(\alpha)$ and the right part be $R(\alpha)$, then by Lemma 4.2, for any positive number $\epsilon < [(1+e^{-2})^2 - 2e^{-1/2}]/(1+e^{-2})$, there exists α^* such that when $\alpha > \alpha^*$, $L(\alpha) > 2(1+e^{-2}) - \epsilon > 4e^{-1/2}/(1+e^{-2}) + \epsilon$, and $R(\alpha) < 4e^{-1/2}/(1+e^{-2}) + \epsilon$, and therefore (4.9) is right for $\alpha > \alpha^*$. □

As $y(\alpha)$ is strictly increasing, it is easy to show that $\lim_{\alpha\to 0+} y(\alpha) = L_b$, and therefore $\lim_{\alpha\to 0+} x(\alpha) = L_b$. But $x(\alpha)$ is not an increasing function for all α. For example, when $f(t) = 1$ for $t \in [0,1] = [L_b, H]$, if $x(\alpha)$ is decreasing, by (4.8), we have

$$(1+y^\alpha(\alpha)) > -y^\alpha(\alpha)\ln y(\alpha) \tag{4.10}$$

while (4.3) turns to be

$$\alpha(1-y(\alpha)) = y(\alpha)(1+y^\alpha(\alpha)). \tag{4.11}$$

By (4.10),

$$\frac{1+y^{\alpha_y}}{y^{\alpha_y}} > \ln\frac{1}{y}. \tag{4.12}$$

where we denote the α which satisfies (4.11) by α_y while y is given. As the right part of (4.12) converges to $+\infty$ when y converges to $+0$, (4.12) tells us that

$$\lim_{y\to +0} y^{\alpha_y} = 0, \tag{4.13}$$

therefore, according to (4.11),

$$\lim_{y\to +0} \frac{\alpha_y(1-y)}{y} = 1. \tag{4.14}$$

Thus

$$\lim_{y\to +0} y^{\alpha_y} = \lim_{y\to +0} y^{\frac{y}{1-y}} = \lim_{y\to +0} (y^y)^{\frac{1}{1-y}} = 1$$

which contradicts to (4.13).

The above Lemmas lead to the following theorem.

Theorem 4.1 *If $f_s(t)$ ($f_b(t)$) is concave and symmetrical in $[L, H_s] \subseteq [L, H]$ ($[L_b, H] \subseteq [L, H]$), then*

a) *$y_s(\alpha)$ ($y_b(\alpha)$) is strictly decreasing (increasing) function of α;*

b) *There exist α^*, such that when $\alpha > \alpha^*$, $x_s(\alpha)$ ($x_b(\alpha)$) is strictly decreasing (increasing) function of α;*

c) $\lim_{\alpha\to\infty} y_s(\alpha) = \lim_{\alpha\to\infty} x_s(\alpha) = L$*;* $\lim_{\alpha\to\infty} y_b(\alpha) = \lim_{\alpha\to\infty} x_b(\alpha) = H$.

This theorem tells us that when disputants s and b are risk averse enough, s's offers x_s will be less than b's offer x_b so that they can reach agreement by themselves.

We have examples which show that the above property does not hold for arbitration procedure FOA, b's prudent offer does not converges to H while he becomes more and more risk averse with expression (4.1) as his utility function form.

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