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Transient Analysis of Nonlinear Transmission Lines by Hybrid Harmonic Balance Method

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Abstract- **We discuss a transient analysis of transmission lines terminated by nonlinear subnetworks. In this analysis, the circuit is partitioned into the transmission line and nonlinear subnetwork by using substitution source. The transmission line is solved by the phasor technique or perturbation method, and the nonlinear subnetwork by a numerical integration technique. We calculate the the substitution source giving rise to the transient response by application of the** *relaxation hybrid harmonic balance method.*

1 Introduction

For a computer-aided design of nonlinear systems, it is very important to calculate the transient responses. Especially, for the high operating frequency, VLSI chips and printed circuit boards must be considered as distributed circuits. For the linear transmission lines, the transient responses are usually calculated by the *numerical inverse Laplace transformations* [1-4] and/or frequency-domain approach [5-7]. Especially, Wang and Wing [6] have proposed a bilevel waveform relaxation algorithm to get the transient responses of transmission lines terminated by nonlinear loads.

In this paper, we present an efficient and simple frequency-domain method for getting the transient responses of the linear and/or nonlinear transmission line terminated by nonlinear subnetwork [8]. For the first step, the distributed circuit is partitioned into the transmission line and nonlinear subnetwork with a substitution source. The response of the transmission line can be calculated by a frequency-domain technique such as phasor or perturbation technique. On the other hand, the response of nonlinear subnetwork to the substitution source can be calculated by a time-domain numerical integration technique. The substitution source giving rise to the transient response is evaluated by an iterational method. We call our method a *relaxation hybrid harmonic balance method.* Especially, in the case of the linear transmission line, the variational value at each iteration is calculated by the application of a relaxation technique to the time-invariant sensitivity circuit having *an associated current source* corresponding to the residual error at the partitioning point [8].

For the nonlinear transmission line, it is equivalently replaced by the linear transmission line containing the distributed sources decided by the perturbational technique. Therefore, the response can be calculated by the application of the two-point boundary value problem.

In section 2, we show the idea of our relaxation hybrid harmonic balance method to the linear transmission lines. In section 3, we show an algorithm for solving the nonlinear transmission lines.

2 Linear transmission line terminated by nonlinear subnetwork

To understand the idea of our method, we consider a simple circuit in Fig.1(a), where N_L is a linear transmission line and N_N is a nonlinear subnetwork. Using the substitution theorem, let us partition the circuit into two subnetworks as shown in Fig.1(b).

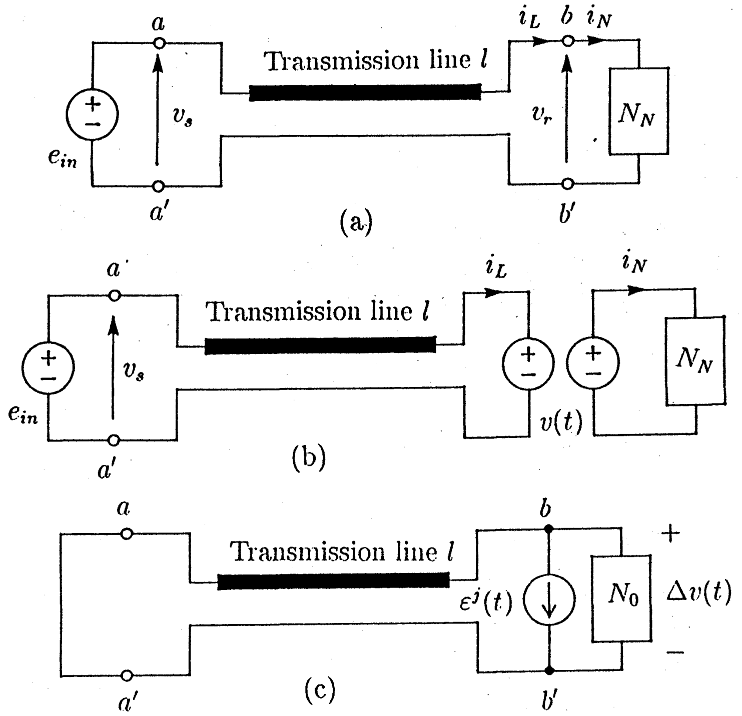

Fig. 1(a) A transmission line terminated by a nonlinear subnetwork
(b) Partition the circuit into linear and nonlinear subnetwork using a substitution source $v(t)$
(c) Sensitivity circuit for getting the variational value $\Delta v(t)$

where the input impulse is described by the Fourier expansion as follows:

$$e_{in}(t) = E_0 + \sum_{k=1}^{M} (E_{2k-1} \cos k\omega t + E_{2k} \sin k\omega t) \tag{1}$$

where $\omega = \frac{2\pi}{T}$. For the linear transmission line, the circuit equation is described by

$$\begin{pmatrix} E \\ I_s \end{pmatrix} = \begin{pmatrix} \cosh \theta l & Z_0 \sinh \theta l \\ \frac{1}{Z_0} \sinh \theta l & \cosh \theta l \end{pmatrix} \begin{pmatrix} V \\ I_L \end{pmatrix} \tag{2}$$

where Z_0 and θ are the *characteristic impedance* and *propagation constant*, respectively. Hence, the current through partitioning point is given by

$$I_L = \frac{E - V \cosh \theta l}{Z_0 \sinh \theta l} \tag{3}$$

Thus, we can easily calculate the response of linear transmission line $i_L(t)$ with phasor technique.

On the other hand, the response of the nonlinear subnetwork to the substitution voltage source $v(t)$ is calculated by a numerical integration technique such as *backward difference formula.* The substitution theorem says that the transient response $v(t)$ satisfies

$$F(v(t)) = i_L(v) - i_N(v) = 0 \tag{4}$$

where $i_N(v)$ is a response of the nonlinear subnetwork in Fig.1(b).

Let us calculate the steady-state periodic response satisfying (4) with an iterational method. Assume the waveform at jth iteration by

$$v^j(t) = V_0^j + \sum_{k=1}^{M} \left(V_{2k-1}^j \cos k\omega t + V_{2k}^j \sin k\omega t \right) \tag{5}$$

In this case, the response $i_L^j(t)$ of the linear transmission line given by (3) can be calculated by the superposition theorem to each frequency component of (1) and (5). On the other hand, for the nonlinear subnetwork, we can calculate the steady-state periodic response $i_N^j(t)$ from a time-domain approach. Note that when the damping term is sufficiently large, we can directly calculate the steady-state response by simply numerical integration technique. To estimate the solution at the $j+1$st iteration, put the solution

$$v^{j+1}(t) = v^j(t) \; + \; \Delta v(t) \tag{6}$$

where $\Delta v(t)$ is a variational voltage waveform described by

$$\Delta v(t) = \Delta V_0 + \sum_{k=1}^{M} (\Delta V_{2k-1} \cos k\omega t + \Delta V_{2k} \sin k\omega t) \tag{7}$$

Substituting v^{j+1} from (6) into (4), we obtain

$$\begin{aligned} F(v^j + \Delta v) &= i_L(v^{j+1}) - i_N(v^{j+1}) \\ &\simeq \mathcal{Y}_L^j(\Delta v) - \mathcal{Y}_{N,t}^j(\Delta v) + \varepsilon^j(t) = 0 \end{aligned} \tag{8}$$

It is not easy to solve the time-varying circuit given by (8) even if it is a linear. Hence, we introduce the following approximate time-invariant system:

$$\mathcal{Y}_L^j(\Delta v) - \mathcal{Y}_{N,0}^j(\Delta v) + \varepsilon^j(t) = 0 \tag{9}$$

where the *residual error* $\varepsilon^j(t)$ is defined by

$$\varepsilon^j(t) \equiv i_L(v^j) - i_N(v^j) \tag{10}$$

The symbols $\mathcal{Y}_L^j(\Delta v)$, $\mathcal{Y}_{N,t}^j(\Delta v)$ and $\mathcal{Y}_{N,0}^j(\Delta v)$ in (8) and (9) denote *linear operators* which transform Δv into the time-domain response of the associated variational subnetwork, where the subscript "t" denotes the time-varying operator and "0" the time-invariant operator, respectively.

The solution of (9) is calculated in the following manner: i.e., After getting the steady-state response to jth substitution source $v^j(t)$, every nonlinear element in the subnetwork N_N is

replaced by a linear time-invariant element as follows:
For nonlinear resistors $i_G = \hat{i}_G(v_G)$:

$$G_0^j = \frac{1}{T}\int_0^T \frac{\partial \hat{i}_G}{\partial v_G} dt \bigg|_{v_G = v_G^j} \tag{11}$$

For nonlinear capacitors $q_C = \hat{q}_C(v_C)$:

$$C_0^j = \frac{1}{T}\int_0^T \frac{\partial \hat{q}_C}{\partial v_C} dt \bigg|_{v_C = v_C^j} \tag{12}$$

For nonlinear inductors $\phi_L = \hat{\phi}_L(i_L)$:

$$L_0^j = \frac{1}{T}\int_0^T \frac{\partial \hat{\phi}_L}{\partial i_L} dt \bigg|_{i_L = i_L^j} \tag{13}$$

Observe that G_0^j, C_0^j and L_0^j are equal to the average values in the period $[0, T]$ at the "j"th substitution voltage.

Now, consider the equivalent circuit for determining $\Delta v(t)$ using algorithm (9). It has the same circuit configuration as the original one, except that the voltage source is short-circuited and all of the nonlinear elements are replaced by time-invariant elements defined by (11)-(13). Furthermore, at the partitioning point, it has a current source equal to the residual error $\varepsilon^j(t)$ given by (10). Thus, we have the equivalent circuit as shown in Fig.1(c). The variational voltage $\Delta v(t)$ can be independently calculated by the applications of the *superposition theorem* to each frequency component of $\varepsilon^j(t)$.

The iteration is continued until the variation satisfies $\| \Delta V \| < \epsilon$ for a given small ϵ. The residual error after convergence of the iteration is given by

$$\epsilon^j = \frac{1}{T}\int_0^T [i_L(t) - i_N(t)]^2 dt \tag{14}$$

If the residual error is not small enough, we must choose the more frequency components given by (1) and repeat again the same iteration.

3 Nonlinear transmission line terminated by nonlinear subnetwork

3.1 Perturbational technique

Now, consider a nonlinear transmission line whose circuit equation is described by the following partial differential equation:

$$-\frac{\partial v}{\partial x} = \frac{\partial \phi_L}{\partial t} + v_R \qquad -\frac{\partial i}{\partial x} = \frac{\partial q_C}{\partial t} + i_G \tag{15}$$

Assume that the transmission line is uniform for the distance, and the nonlinear characteristics per unit length are functions of voltage $v(x)$ and current $i(x)$ as follows:

$$\phi_L = L_0 i + \epsilon\hat{\phi}_L(i), \quad v_R = R_0 i + \epsilon\hat{v}_R(i) \tag{16}$$
$$q_C = C_0 v + \epsilon\hat{q}_C(v), \quad i_G = G_0 v + \epsilon\hat{i}_G(v) \tag{17}$$

where ϵ means a small constant.

Let us apply a perturbation method to the analysis of the nonlinear transmission line. As shown in section 2, we partition the circuit into the transmission line and the nonlinear subnetwork with the substitution source. Applying an iterational technique, we try to find out the substitution source $v^j(t)$ giving rise to the same responses at the partitioning point. Assume the waveform as follows:

$$v^j(x,t) = V_0^j(x) + \sum_{k=1}^{M}\left(V_{2k-1}^j(x)\cos k\omega t + V_{2k}^j(x)\sin k\omega t\right) \tag{18}$$
$$i^j(x,t) = I_0^j(x) + \sum_{k=1}^{M}\left(I_{2k-1}^j(x)\cos k\omega t + I_{2k}^j(x)\sin k\omega t\right) \tag{19}$$

Applying the *perturbational technique* to the nonlinear transmission line, we describe (16)-(17) as follows:

$$\phi_L(i^j) \simeq L_0 i^j + \epsilon\hat{\phi}_L(i^{j-1}) \quad v_R(i^j) \simeq R_0 i^j + \epsilon\hat{v}_R(i^{j-1}) \tag{20}$$
$$q_C(v^j) \simeq C_0 v^j + \epsilon\hat{q}_C(v^{j-1}) \quad i_G(v^j) \simeq G_0 v^j + \epsilon\hat{i}_G(v^{j-1}) \tag{21}$$

Substituting (20)-(21) into (15) and applying the *harmonic balance method*, we have the following relations.

For dc component:

$$-\frac{dV_0^j(x)}{dx} = R_0 I_0^j(x) + \epsilon V_{R,0}^{j-1}(x) \tag{22}$$
$$-\frac{dI_0^j(x)}{dx} = G_0 V_0^j(x) + \epsilon I_{G,0}^{j-1}(x) \tag{23}$$

For the kth harmonic component:

$$-\frac{dV_{2k-1}^j(x)}{dx} = k\omega L_0 I_{2k}^j(x) + R_0 I_{2k-1}^j(x) + \epsilon V_{2k-1}^{j-1}(x) \tag{24}$$
$$-\frac{dV_{2k}^j(x)}{dx} = -k\omega L_0 I_{2k-1}^j(x) + R_0 I_{2k}^j(x) + \epsilon V_{2k}^{j-1}(x) \tag{25}$$
$$-\frac{dI_{2k-1}^j(x)}{dx} = k\omega C_0 V_{2k}^j(x) + G_0 V_{2k-1}^j(x) + \epsilon I_{2k-1}^{j-1}(x) \tag{26}$$
$$-\frac{dI_{2k}^j(x)}{dx} = -k\omega C_0 V_{2k-1}^j(x) + G_0 V_{2k}^j(x) + \epsilon I_{2k}^{j-1}(x) \tag{27}$$
$$k = 1, 2, \cdots, M$$

Therefore, the partial differential equations (15) are transformed into a linear ordinary differential equations.

Hence, it can be uniquely solved only if the boundary conditions are given. Note that the conditions at the input terminal are given from (1) as follows:

$$V_0^j(0) = E_0, \quad V_{2k-1}^j(0) = E_{2k-1}, \quad V_{2k}^j(0) = E_{2k} \tag{28}$$

Now, consider the responses of nonlinear subnetwork which is partitioned by a substitution source $v^j(t)$ given by (5). Assume that the steady-state response to the substitution source $v^{j-1}(l,t)$ is described by the Fourier expansion as follows:

$$i_N^{j-1}(t) = \hat{I}_{N,0}^{j-1}(x) + \sum_{k=1}^{M} \left(\hat{I}_{N,2k-1}^{j-1}(x) \cos k\omega t + \hat{I}_{N,2k}^{j-1}(x) \sin k\omega t \right) \tag{29}$$

We approximate $i_N^j(t)$ by the following relation:

$$i_N^j(t) \simeq \mathcal{Y}_{N,0}^{j-1}(v^j) + \left\{ i_N^{j-1}(t) - \mathcal{Y}_{N,0}^{j-1}(v^{j-1}) \right\} \tag{30}$$

where the symbol $\mathcal{Y}_{N,0}^{j-1}(v^j)$ is a linear operator defined in section 2 which transforms $v^j(t)$ to the corresponding time-domain response. Observe that if $\mathcal{Y}_{N,0}^{j-1}$ were the sensitivity circuit at $j-1$th iteration, the algorithm would be exactly equal to the *Newton method.* Therefore, we can hope the large convergence ratio for the weakly nonlinear subnetwork because the operator $\mathcal{Y}_{N,0}^{j-1}$ will be a good approximation of $\mathcal{Y}_{N,t}^{j-1}$ from the sensitivity circuit. Let us describe the second term of (30) in the Fourier series as follows:

$$i_N^{j-1}(t) - \mathcal{Y}_{N,0}^{j-1}(v^{j-1}) = I_{N,0}^{j-1} + \sum_{k=1}^{M} \left(I_{N,2k-1}^{j-1} \cos k\omega t + I_{N,2k}^{j-1} \sin k\omega t \right) \tag{31}$$

Therefore, from (18), (19) and (31), we have another boundary conditions as follows:

$$G_N^{j-1}(0)V_0^j(l) + I_{N,0}^{j-1} = I_0^j(l) \tag{32}$$

$$G_N^{j-1}(k\omega)V_{2k-1}^j(l) + B_N^{j-1}(k\omega)V_{2k}^j(l) + I_{N,2k-1}^{j-1} = I_{2k-1}^j(l) \tag{33}$$

$$G_N^{j-1}(k\omega)V_{2k}^j(l) - B_N^{j-1}(k\omega)V_{2k-1}^j(l) + I_{N,2k}^{j-1} = I_{2k}^j(l) \tag{34}$$

where the input impedance of the sensitivity subnetwork is assumed by

$$Y_{N,0}^{j-1}(k\omega) = G_N^{j-1}(k\omega) + jB_N^{j-1}(k\omega)$$

Observe that eqs.(28) and (32)-(34) are the boundary conditions for solving the differential equations (22)-(23) and (24)-(27). Thus, we can estimate the Fourier coefficients with the *two-point boundary value problem.* The algorithm is a kind of perturbation method so that it is efficiently applied to the relatively weakly nonlinear transmission lines.

3.2 Two-point boundary value problem

Now, we consider to solve the *differential equations* (22)-(23) and (24)-(27) under the *boundary conditions* (28) and (32)-(34). Let us rewrite their relations for dc component and kth frequency component simply:

For dc component:

$$\frac{dy^j}{dx} = Ay^j + f(x, y^{j-1}), \quad y \in (V_0, I_0) \tag{35}$$

$$y_n^j(0) = E_n, \qquad n = 1(E_1 = E_0) \tag{36}$$

$$\sum_{i=1}^{2} \alpha_{n,i}^{j-1} y_i^j(l) = I_n^{j-1}, \ n = 2(I_2^{j-1} = I_{N,0}^j) \tag{37}$$

For kth frequency component:

$$\frac{dy^j}{dx} = Ay^j + f(x, y^{j-1}), \quad y \in (V_{2k-1}, V_{2k}, I_{2k-1}, I_{2k}) \tag{38}$$

$$y_n^j(0) = E_n, \qquad n = 1, 2(E_1 = E_{2k-1}, \ E_2 = E_{2k}) \tag{39}$$

$$\sum_{i=1}^{4} \alpha_{n,i}^{j-1} y_i^j(l) = I_n^{j-1}, n = 3, 4 \ (I_3^{j-1} = I_{N,2k-1}^j, I_4^{j-1} = I_{N,2k}^j) \tag{40}$$

For the case of generality, put $N_i = 2$, and $N_f = 4$. Let us solve the two-point boundary value problem (38)-(40). Consider the adjoint system to (38) as follows:

$$\frac{dw}{dx} = -A^T w \tag{41}$$

From (38) and (41), we have [9]

$$\frac{d}{dx} \sum_{i=1}^{N_f} w_i(x) y_i^j(x) = \sum_{i=1}^{N_f} w_i(x) f_i(x, y^{j-1}) \tag{42}$$

On integrating (42) over $[0, l]$, we have

$$\sum_{i=1}^{N_f} w_i(l) y_i^j(l) - \sum_{i=1}^{N_f} w_i(0) y_i^j(0) = \int_0^l \sum_{i=1}^{N_f} w_i(x) f_i(x, y^{j-1}) dx \tag{43}$$

This gives the relation between $y_i^j(x)$ of the original equation and $w_i^j(x)$ of the adjoint equation at the two end points $x = 0$ and $x = l$. Let us integrate the adjoint equations backward from $x = l$ with the terminal conditions as follows:

$$w_m^n(l) \ = \ \alpha_{n,m}^{j-1}, \quad m = 1, \cdots, N_f, n = N_i + 1, \cdots, N_f \tag{44}$$

Then, the fundamental identity (43) gives

$$\sum_{i=1}^{N_f} \alpha_{n,i}^{j-1} y_i^j(l) - \sum_{i=1}^{N_f} w_i^n(0) y_i^j(0) = \int_0^l \sum_{i=1}^{N_f} w_i^n(x) f_i(x, y^{j-1}) dx$$
$$n = N_i + 1, \cdots, N_f \tag{45}$$

Substituting (40) and (44) into (45), we have

$$\sum_{i=N_i+1}^{N_f} w_i^n(0) y_i^j(0) = I_n^{j-1} - \sum_{i=1}^{N_i} w_i^n(0) E_i - \int_0^l \sum_{i=1}^{N_f} w_i^n(x) f_i(x, y^{j-1}) dx \tag{46}$$

for $n = N_i + 1, \cdots, N_f$. Thus, combining (39) and (46), we can estimate the initial condition $y_i^j(0)$ of kth frequency component at jth iteration.
The iteration is continued until the variational value satisfies

$$\sum_{i=N_i+1}^{N_f} \| y_i^j(0) - y_i^{j-1}(0) \| \leq \epsilon \tag{47}$$

for every frequency component.

4 Illustrative examples

4.1 Transient response of linear circuit

To understand the efficiency of our frequency-domain analysis, we consider a simple circuit as shown in Fig.2(a). The response at distance y from the terminal load Z_L is given as follows:

$$V(y) = V_s \frac{Z_L \cosh \theta y + Z_0 \sinh \theta y}{Z_L \cosh \theta l + Z_0 \sinh \theta l} \tag{48}$$

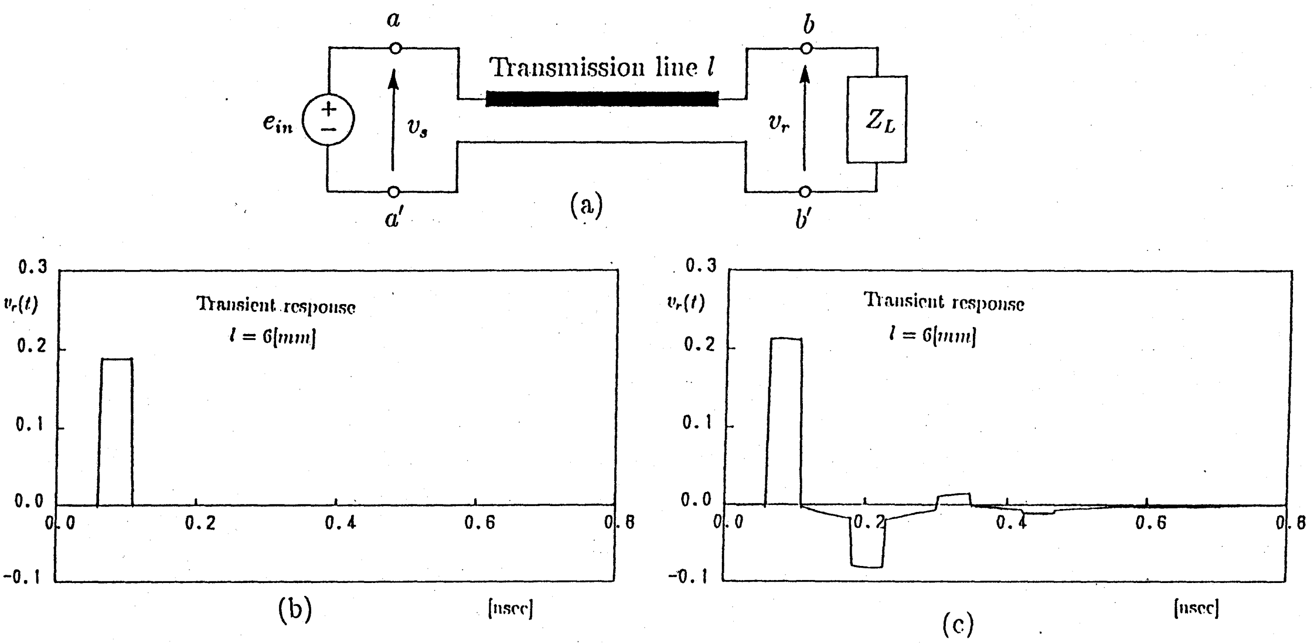

Fig.2(a) Transmission line terminated by a linear load Z_L
(b) No distortion waveform at end terminal $Z_L = 1[k\Omega]$
Parameters of transmission line per unit length $[mm]$:
$R = 0.01[k\Omega]$, $L = 0.01[\mu H]$, $C = 0.01[\mu F]$, $G = 0.01[mS]$
(c) Transient response of transmission line terminated by
by a linear load $Z_L = R_L + sL_L$: $R_L = 0.5[k\Omega]$, $L_L = 0.5[\mu H]$
Parameters of transmission line per unit length $[mm]$:
$R = 0.2[k\Omega]$, $L = 0.01[\mu H]$, $C = 0.01[\mu F]$, $G = 0.01[mS]$

$$I(y) = \frac{V_s}{Z_0} \frac{Z_0 \cosh \theta y + Z_L \sinh \theta y}{Z_L \cosh \theta l + Z_0 \sinh \theta l} \tag{49}$$

where the characteristic impedance is Z_0 and the propagation constant θ.
Now, consider a no-distortion transmission line $l = 6[mm]$. Then, the load impedance must be matching to the characteristic impedance Z_0, and is given by $Z_L = 1k\Omega$ in this example. Assume the impulse waveform of $E = 2$ and period $T = 1[nsec]$. The waveform is described by Fourier series with 1024 frequency components. The transient response by our frequency-domain is shown in Fig.2(b). The amplitude at terminal load is slightly decreased, but the waveform is exactly the same shape as the input.
Next, consider a circuit terminated by R-L series load ($R_L = 0.5[k\Omega]$ and $L_L = 0.5[\mu H]$). The response at terminal point is given by Fig. 2(c). The transient phenomena continues much longer than the above example, and has remarkable reflection phenomena from the two end points.
Note that we can get these responses in a second by SUN SPARC station IPX. We found from two examples that the frequency components for this kind of problem is enough at most 1024 frequency components. The algorithm can be applied to much larger systems such as multi-conductor transmission lines.

4.2 Analysis of a transmission line terminated by a diode

Conside an application of our relaxation hybrid harmonic balance method to a stiff circuit as shown in Fig. 3(a). Note that we can not neglect the nonlinear capacitor in the high frequency domain, so that their characteristics are given as follows: Nonlinear resistor :

$$i_d = I_s[exp(\lambda v_d) - 1] \tag{50}$$

Nonlinear capacitor :

$$q_d = T_F I_s[exp(\lambda v_d) - 1] - \frac{p_d C_{jd}}{1 - m_d}[(1 - \frac{v_d}{p_d}^{(1-m_d)} - 1] \tag{51}$$

where $I_s = 10^{-16}[A]$, $\lambda = 40$, $T_F = 50[psec]$, $C_{jd} = 0.1[pF]$, $m_d = 0.4$, $p_d = 0.8$. The both characteristics are sharply changed around $v_d = 0.7 - 0.8[V]$. For this example, our hybrid harmonic balance method has never converged because of the stiff nonlinearity. Hence, we have introduced the compensation resistor R_c, and it is partitioned into the linear transmission line and nonlinear subnetwork by the substitution voltage $v(t)$ as shown in Fig.3(b). For the steady-state analysis of nonlinear subnetwork, we used the backward difference formula. The impulse response at the partitioning point is given by Fig.3(d), where the input amplitude is $E = 5[V]$. The convergence ratios for the different compensation resistors (see Appendix) are shown in Fig. 3(e). We found that the iteration algorithm is stable for wide range of the compensation resistor value.

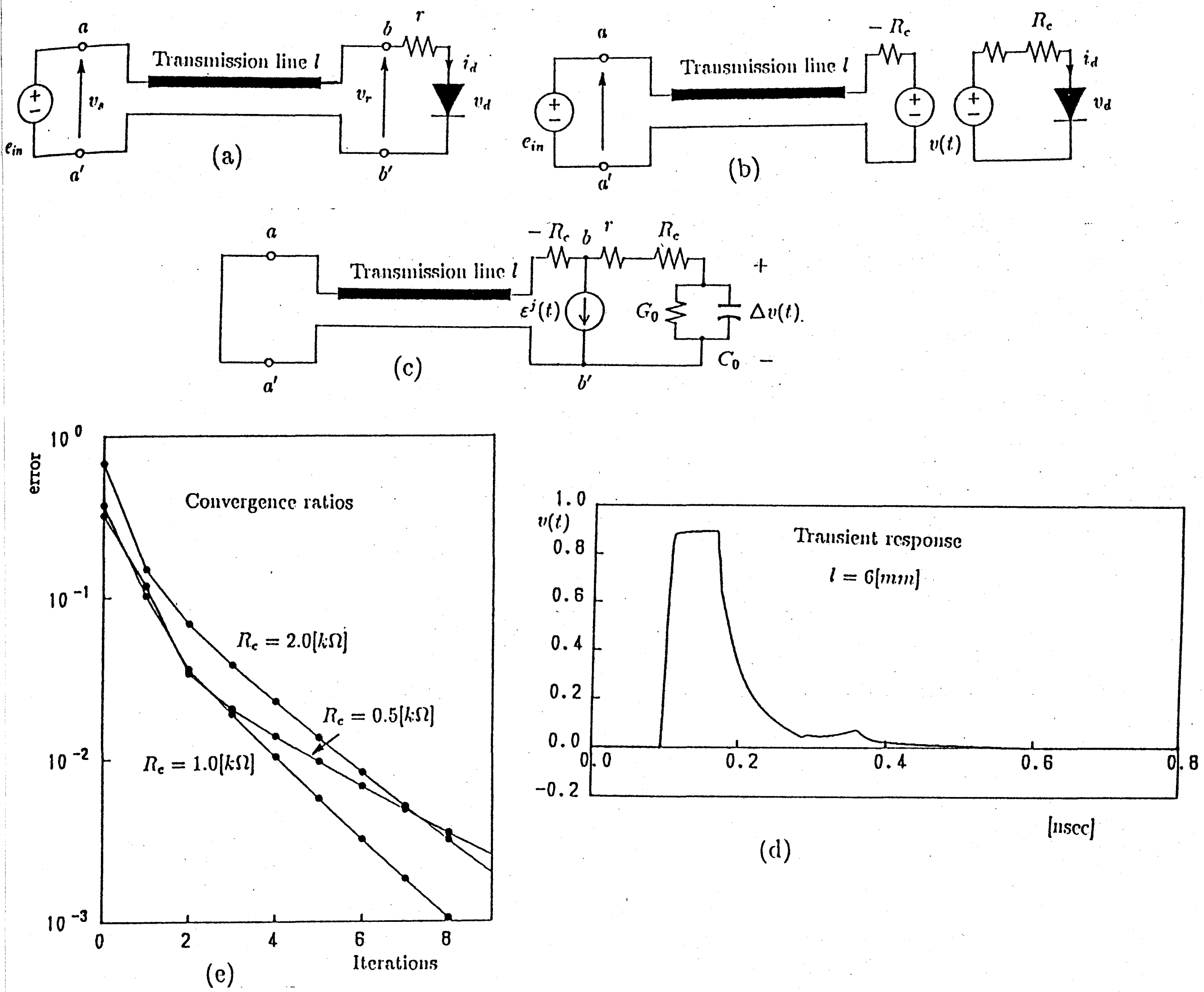

Fig.3(a) Transmission line terminated by a diode and $r = 0.2[k\Omega]$
Parameters of transmission line per unit length $[mm]$:
$R = 0.5[k\Omega]$, $L = 0.01[\mu H]$, $C = 0.01[\mu F]$, $G = 0.01[mS]$
(b) Partition using substitution theory, where R_c, $-R_c$ are compensation resistors
(c) The sensitivity circuit for getting variational value $\Delta v(t)$, where $G_0 = 1[mS]$ and $C_0 = l[\mu F]$
(d) Transient response of diode circuit
(e) Convergence ratios for different compensation resistors

5 Conclusions and remarks

We have presented an algorithm for calculating transient response of a transmission line terminated by nonlinear subnetworks, which belongs to a class of frequency-domain technique. At the first step, a circuit is partitioned into the transmission line and the nonlinear subnetwork with the substitution source. At the second step, the transmission line is solved by the phasor technique and/or perturbational technique. On the other hand, the nonlinear subnetwork is solved by a numerical integration formula. The variation at each iteration is calculated by the phasor method to the linear transmission line and the two-point boundary value problem to the nonlinear transmission line. The both algorithms are very simple and can be applied wide classes of transmission line terminated by nonlinear subnetworks. We also have shown an importance of introducing the compensation elements to the stiff nonlinear subnetworks. The technique is usefully applied to the circuits containing transistors and diodes.

Acknowlegedment

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Appendix

Consider the convergence condition of the compensation technique in the example 4.2, which has two compensation resistor R_c and $-R_c$ as shown in Fig. 4. For the simplicity, we assume that the nonlinear subnetwork at jth and $j+1$ iteration has the same sensitivity circuit.
Now, set the residual current at jth iteration (9) by

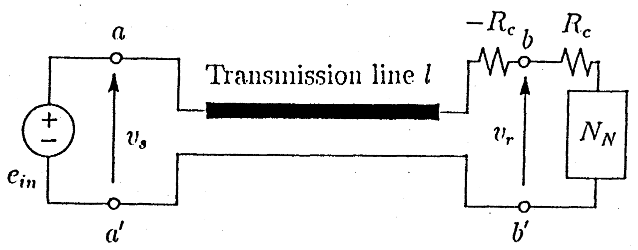

Fig.4 Compensation technique

$$\varepsilon^j(t) = \varepsilon_0^j + \sum_{k=1}^{M} \left(\varepsilon_{2k-1}^j \cos k\omega t + \varepsilon_{2k}^j \sin k\omega t\right) \tag{52}$$

and the impedances of the linear and the nonlinear sensitivity circuit at kth frequency component

$$Z_{L,k} = R_{L,k} + jX_{L,k} \tag{53}$$
$$Z_{N,k}^j = R_{N,k}^j + jX_{N,k}^j \tag{54}$$

Then, the total impedance at the partitioning point is given

$$Z_k^j = \frac{(R_{L,k} - R_c + jX_{L,k})(R_{N,k}^j + R_c + jX_{N,k}^j)}{R_{L,k} + R_{N,k}^j + j(X_{L,k} + X_{N,k}^j)} \tag{55}$$

where R_c is the compensation resistor.
Hence, the variational voltage at jth iteration is given by

$$\Delta V_{2k-1} + j\Delta V_{2k} = \bar{Z}_k^j(\varepsilon_{2k-1}^j + j\varepsilon_{2k}^j) \tag{56}$$

where $\bar{Z}$ denotes the complex conlugate of Z. Therefore, the variational current at $j+1$th iteration is given by

$$\varepsilon_{2k-1}^{j+1} + j\varepsilon_{2k}^{j+1} = \frac{R_{N,k}^j - R_{L,k} + 2R_c - j(X_{N,k}^j - X_{L,k})}{R_{L,k} + R_{N,k}^j - j(X_{L,k} + X_{N,k}^j)}(\varepsilon_{2k-1}^j + j\varepsilon_{2k}^j) \tag{57}$$

Hence, the relaxation hybrid harmonic balance method will converge if the compensation resistor R_c satisfies

$$\sqrt{\frac{(R_{N,k}^j - R_{L,k} + 2R_c)^2 + (X_{N,k}^j - X_{L,k})^2}{(R_{L,k} + R_{N,k}^j)^2 + (X_{L,k} + X_{N,k}^j)^2}} < 1 \tag{58}$$

for $k = 0, 1, \cdots, M$.