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Alternating-Move Preplays and $vN-M$ Stable Sets in Two Person Strategic Form Games

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1 Introduction

This is a summary of the paper , "Alternating-Move Preplays and $vN-M$ Stable Sets in Two Person Strategic Form Gemes," CentER Discussion Paper No.9371, Center for Economic Research, Tilburg University, 1993. Motivation and basic definitions are fully described; but results are only briefly mentioned. Refer to the original paper for details.

An alternating-move preplay negotiation procedure for two-person games was proposed by Bhaskar [1989] in the context of a price-setting duopoly. The preplay proceeds as follows. One of the players, say player 1, first announces the price that he intends to take; and then player 2 announces his price. Player 1 is now given the opion of changing his price. If he does so, player 2 can change his price. The process continues in this manner; and it comes to an end when one of the two players chooses not to change his price. Bhaskar succeeded in showing that through this process only the monopoly price pair can be attained in equilibrium where the equilibrium is the subgame perfect equilibrium with undominated strategies.

One of the aims of this paper is to examine the validity of the alternating-move preplay process in other two-person games. In addition to the conditions that Bhaskar imposed on equilibria, we require that strategies in equilibrium be Markov (or stationary). It will be shown that the preplay process works well in typical 2×2 games such as the prisoner's dilemma and a pure coordination game. The pair of (Cooperation, Cooperation) and a Pareto optimal strategy pair are obtained as the unique equilibrium outcome, respectively. Further in the price-setting duopoly it will be shown that the monopoly price pair can be reached even if the preplay starts from any price pair. The preplay, however, does not always work well. In fact, a sort of the Folk Theorem is shown to hold in the prisoner's dilemma with continous strategy spaces: in the game every individual rational outcome can be attained as an equilibrium outcome.

Another objective of this paper is to study the von Neumann and Morgenstern $(vN-M)$ stable sets in two-person strategic form games. Recently Greenberg [1990] proposed a way to apply $vN - M$ stable sets, or at least its spirit, to strategic form games by appropriately introducing a dominance relation on the space of strategy combinations. Later studies, Chwe [1992] and Muto and Okada [1992], however, revealed that a modification of the dominance relation is desirable as Harsanyi [1974] already pointed out in his study of the $vM - N$ stable set in characteristic function form games. Following Harsanyi's discussion, we will study relations between $vN - M$ stable sets in strategic form games and equilibria in their extended games with preplays.

2 The Extended Game with Alternating-Move Preplays

Throughout the paper we will work on the following two-person game:

$$G = (N = \{1,2\}, \{X_i\}_{i=1,2}, \{u_i\}_{i=1,2})$$

where $N = \{1,2\}$ is the set of players, X_i, $i = 1,2$, is player i's action set and u_i, $i = 1,2$, is player i's payoff function, i.e., a real valued function on $X = X_1 \times X_2$. We assume u_i takes nonnegative values.

The alternating-move preplays, proposed by Bhaskar [1989], proceed as follows. One of the players, say player 1, moves first and announces the action $x_1 \in X_1$ that he intends to take. The first player to move is determined in advance of the preplays. Then player 2 announces an action $x_2 \in X_2$. Player 1 now has the option of changing his action to x_1'. If he does so, player 2 can change his action to x_2' and so on. The preplay process comes to an end when any of the two players chooses not to change.

Let $x^k = (x_1^k, x_2^k)$ be the action combination at the end of the kth period. For convenience let $x^1 = x_1^1$: x_1^1 is the action that player 1 announces at the 1st period. Suppose the preplay process ends at the Kth period with player i's turn; thus $x^{K-1} = x^K$. Then since player i chooses not to move, he is satisfied with his action $x_i^K = x_i^{K-2}$ against j's action $x_j^K = x_j^{K-1}$. Further player j's action x_j^{K-1} is his response to player i's x_i^{K-2}. Thus both players are satisfied with the action combination x^K. Player i will be paid $u_i(x^K)$, $i = 1,2$. If the equality $x^K = x^{K-1}$ never arises, then the game will go on indefinitely. In this event, we define the players' payoffs are zero.

Hereafter, we will call this alternating-move game the extended game of G.

3 Formal Description of the Extended Game

In the following we describe the extended game in which player 1 moves first. Thus in the following player 1 (player 2, resp.) moves in odd (even, resp.) number of periods. The game in which player 2 moves first is described in the same manner.

3.1 Strategies and Payoffs

Take the kth period, and suppose actions announced up to the $(k-1)$st period are $x_1^1, x_2^2, x_1^3, \ldots, x_i^{k-1}$ where i is the player who moved at the $(k-1)$st period. Then the action combination x^l at the end of the lth period is given by

$$x^l = \begin{cases} (x_2^{l-1}, x_1^l) & \text{if } l \text{ is odd and } l \geq 3 \\ (x_1^{l-1}, x_2^l) & \text{if } l \text{ is even.} \end{cases}$$

The history up to the $(k-1)$st period is written as $h^{k-1} = (x^1, x^2, \ldots, x^{k-1})$. Let the set of all possible h^k by H^k, and let $H = \bigcup_{k=0}^{\infty} H^k$ where $H^0 = \{e\}$ and e denotes the empty history. Players' strategies, denoted by σ_1 for player 1 and σ_2 for player 2, are maps such that

$$\sigma_1 : \bigcup_{k=0}^{\infty} H^{2k} \rightarrow X_1$$

and

$$\sigma_2 : \bigcup_{k=0}^{\infty} H^{2k+1} \rightarrow X_2.$$

A strategy combination (σ_1, σ_2) is denoted by σ. The set of all starategies of player 1 (player 2, resp.) is denoted by $\Sigma_1(\Sigma_2$, resp.). The outcome (action combination) path induced by a strategy combination σ is denoted by $\pi(\sigma)$.

Player i's payoff under a strategy combination σ is given by

$$f_i(\sigma) = \begin{cases} u_i(z) & \text{if } \pi(\sigma) \text{ is of finite length, i.e., if the game} \\ & \text{ends after a finite number of periods:} \\ & z \text{ is the final outcome, i.e., } z = x^K \text{ when the game ends at the } Kth \text{ period} \\ 0 & \text{otherwise.} \end{cases}$$

3.2 Subgames

The extended game is a game with perfect information; and thus games starting from each move of players are subgames. Let h be a history up to the $(k-1)$st period, and denote by $\Gamma(h)$ the subgame starting from the kth period after the history h. Let $\sigma_i(h), i = 1, 2$, be player i's strategy in $\Gamma(h)$, and let $\sigma(h) = (\sigma_1(h), \sigma_2(h))$. Denote by $\pi(\sigma(h))$ the outcome path in $\Gamma(h)$ induced by $\sigma(h)$. Player i's payoffs in $\Gamma(h)$ under $\sigma(h)$ are given by

$$f_i^h(\sigma(h)) = \begin{cases} u_i(z) & \text{if } \pi(\sigma(h)) \text{ is of finite length:} \\ & z \text{ is the final outcome in the path } \pi(\sigma(h)) \\ 0 & \text{otherwise.} \end{cases}$$

3.3 Equilibrium

Similarly to Bhaskar [1989], we require equilibrium strategies to be subgame perfect and also require that in equilibria the strategies played after any history should not be weakly dominated. The latter is defined in the following manner. Take a subgame $\Gamma(h)$, and take player i's two strategies $\sigma_i(h)$ and $\sigma_i'(h)$ in $\Gamma(h)$. We say that $\sigma_i(h)$ weakly dominates $\sigma_i'(h)$ in $\Gamma(h)$ if (1) $f_i^h((\sigma_i(h), \sigma_j(h))) \geq f_i^h((\sigma_i'(h), \sigma_j(h)))$ for all player j's strategies $\sigma_j(h)$ in $\Gamma(h)$, and (2) $f_i^h((\sigma_i(h), \sigma_j(h))) > f_i^h((\sigma_i'(h), \sigma_j(h))$ for at least one $\sigma_j(h)$ in $\Gamma(h)$. The second condition that Bhaskar imposed requires that if $\sigma = (\sigma_1, \sigma_2)$ is the equilibrium, then the following hold for both players $i = 1, 2$: in each subgame $\Gamma(h)$, there is no strategy of player i which dominates $\sigma_i \mid h$ in $\Gamma(h)$ where $\sigma_i \mid h$ is the restriction of σ_i to the subgame $\Gamma(h)$.

In addition to the two conditions, we require equilibrium strategies to be Markov (or stationary) and conservative.

A player's strategy is called Markov if each action induced by the strategy depends only on a prevailing action combination. Thus player 1's (player 2's, resp.) Markov strategy is a function from $\{e\} \bigcup X$ to X_1 (from $X_1 \bigcup X$ to X_2, resp.). We will hereafter use ρ_1 and ρ_2 to denote Markov strategies of players 1 and 2.

The restriction to Markov strategies greatly simplifies the analysis since interactions of players' strategies are kept as simple as possible. But a more important reason for imposing the Markov property comes from one of the objectives of the paper; that is, the study of the $vN-M$ stable set or its variants in strategic form games from the viewpoint of equilibria in their extended games with preplays. Since the $vN-M$ stable set is a static solution concept, we want the stability being independent of the history of preplay negotiations.[1]

A mathematical justification of restricting to the Markov strategy was given in Harsanyi [1974, Lemmas 6 and 7]. That is, if $\rho = (\rho_1, \rho_2)$ is a Nash equilibrium when players are restricted to using the Markov strategies, then ρ is still an equilibrium even if each player is free to use any strategy in Σ_i (not necessarily Markov).

The conservativeness, initially defined by Harsanyi [1974], assumes that each player never moves unless he will positively benefit from this move. The assumption arises also from the study of the $vN - M$ stable set: it assumes such conservativeness in its definition. Formally the conservativeness is defined in the following manner. Take a strategy combination ρ^*, and a subgame $\Gamma(h^k)$ which follows the history $h^k = (x^1, x^2, \ldots, x^k)$ up to the kth period. ρ^* is called conservative in $\Gamma(h^k)$ if the following hold. Let z be the final outcome in $\Gamma(h^k)$ under the restriction of ρ^* to this subgame: z may be an infinite sequence of outcomes. Then (1) $z = x^k$ or (2) If $x^{k+1}, x^{k+2}, \ldots (i^{k+1}, i^{k+2}, \ldots$, resp.) is the sequence of outcomes (of corresponding players, resp.) under ρ^*, then

$$u_{i^l}(z) > u_{i^l}(x^{l-1}) \text{ for all } l = k+1, k+2, \ldots$$

except for $l = K$ or $K - 1$ where K is the period that the game ends.

Since payoffs are nonnegative and further in case the game never ends they are zero, the game must end after a finite number of steps if a pair of players' strategies is conservative.

A strategy combination $\rho = (\rho_1, \rho_2)$ is called a conservative Markov perfect equilibrium, denoted by CMPE hereafter, of the extended game if it satisfies the four conditions above, i.e.,

1. ρ is subgame perfect;
2. ρ_1, ρ_2 are not weakly dominated in each subgame;
3. ρ_1, ρ_2 are Markov strategies; and
4. ρ is conservative in each subgame.

[1] Other defenses of assuming the Markov property, in particular, in analyzing duopoly markets, are found in Maskin and Tirole [1988].

The restriction to Markov strategies makes it possible to describe subgames in a simpler way. That is, it is sufficient to make clear starting action combination x and player i to move first. Thus subgames will hereafter be denoted by $\Gamma(x,i), x \in X, i = 1,2$. $\Gamma(e,1)(\Gamma(e,2)$, resp.) is the whole extended game starting from the move of player 1 (player 2, resp.).

We say $\rho = (\rho_1, \rho_2)$ is a CMPE in the subgame $\Gamma(x,i)$ if ρ_i's are Markov and ρ satisfies (1),(2),(4) above in each subgame of $\Gamma(x,i)$.

4 Applications

Consider first the following prisoner's dilemma and its extended games.

1 \ 2	C	D
C	4,4	0,5
D	5,0	1,1

Proposition 4.1: *Let ρ^* be a CMPE in $\Gamma(e,i), i = 1$ or 2. Then the following must hold.*

$$\rho_1^*(CC) = C, \quad \rho_1^*(CD) = D, \quad \rho_1^*(DC) = D, \quad \rho_1^*(DD) = D,$$
$$\rho_2^*(CC) = C, \quad \rho_2^*(CD) = D, \quad \rho_2^*(DC) = D, \quad \rho_2^*(DD) = D.$$

That is, player 1 (player 2, resp.) changes his action only at the outcome $CD(DC$, resp.). Figure 4.1 depicts ρ_1^*, ρ_2^* and the induced movements.

Therefore the subgames $\Gamma(CC,1)$ and $\Gamma(CC,2)$ end at the outcome $CC; \Gamma(CD,2)(\Gamma(DC,1)$, resp.) ends at $CD(DC$, resp.); and $\Gamma(CD,1), \Gamma(DC,2), \Gamma(DD,1)$ and $\Gamma(DD,2)$ end at DD.

On the basis of Proposition 4.1, we may show that in the whole game every CMPE produces the unique outcome CC.

Proposition 4.2: *Take the whole game $\Gamma(e,i), i = 1$ or 2. Then every CMPE in $\Gamma(e,i)$ induces CC as its final outcome.*

In a similar manner, it is shown in the pure coordination game that the Pareto superior payoff pair is produced as the unique final outcome. In the battle of the sexes, Pareto efficient outcomes are also obtained; but the so-called second mover advantage appears: the player who moves first gets less.

Consider next the following symmetric duopoly. Two firms 1,2 are producing homogeneous goods with the same marginal cost c. For simplicity, let $c = 0$ in what follows. Consumers' demands are represented by a demand function $D(p)$. $D(p)$ is decreasing in p, and there exists a price $\tilde{p}$ such that $D(p) = 0$ for all $p \geq \tilde{p}$. The market profit at price p is $\pi(p) = pD(p)$, and $\pi(0) = \pi(\tilde{p}) = 0$. Suppose $\pi(p)$ is continuous and strictly concave. Then there is a unique price p^m, called the monopoly price, which maximizes $\pi(p)$. Denote firm 1's (firm 2's, resp.) price level by $p^1(p^2$, resp.). If their prices are equal, they split even the market profit; otherwise all sales go to a lower pricing firm. This duopoly market is written as the following two-person game:

$$G^B = (N = \{1,2\}, \{X_i\}_{i=1,2}, \{u_i\}_{i=1,2})$$

where $X_i = [0, \tilde{p}]$ for $i = 1, 2$,

and

$$u_i : X = X_1 \times X_2 \rightarrow R_+ \text{ (nonnegative reals) defined by}$$

$$u_i(p_i, p_j) = \begin{cases} \pi(p_i) & \text{if } p_i < p_j \\ \pi(p_i)/2 & \text{if } p_i = p_j \quad \text{for } i, j = 1, 2, \ i \neq j \\ 0 & \text{if } p_i > p_j. \end{cases}$$

We assume that player (firm) 1 is first to move. But, needless to say,the same results hold even if player 2 moves first because of their symmetry.

The first proposition shows that under every CMPE, the following holds: in each price pair other than the pair of the monopoly prices, at least one player has an incentive to move, and his move induces a sequential movement of prices which eventually reaches the monopoly price pair.

Proposition 4.3: *Let $\rho = (\rho_1, \rho_2)$ be a CMPE of $\Gamma(e, 1)$ and take a price pair (p_1, p_2). Then the subgames $\Gamma((p_1, p_2), i), i = 1, 2$, has the final outcome (p^m, p^m) under ρ, except when $p^{m-} \leq p_i \leq p^{m+}$ and $p_i < p_j$; and if this is the case the subgame ends at (p_1, p_2).*

The following two propositions then follow which show that the momopoly price pair is the unique final outcome.

Proposition 4.4: *Let $\rho = (\rho_1, \rho_2)$ be a CMPE of $\Gamma(e, 1)$. Take p_1 with $p^{m-} \leq p_1 \leq p^{m+}$. Then $\rho_2(p_1) \leq p_1$ must hold, and the subgame $\Gamma(p_1, 2)$ has the final outcome (p^m, p^m) under ρ.*

Proposition 4.5: *Let $\rho = (\rho_1, \rho_2)$ be a CMPE of $\Gamma(e,1)$. Then player 1's choice $\rho_1(e)$ in the first period can be arbitrary; and the game ends at the monopoly price pair (p^m, p^m) irrespective of his choice.*

So far we have shown that the alternating-move preplay process works well in various examples. It is shown, however, that in the mixed extension of prisoner's dilemma every individually rational outcome could be attained as a final outcome of a CMPE: a sort of Folk theorem holds.

5 Final Outcomes under CMPE and Stable Sets

We next examine relations between stable outcomes under CMPE and the $vN - M$ stable set.

Similarly to Greenberg [1990], Chwe [1992] and Muto and Okada [1992], we define a binary relation, called the dominance relation, on the outcome space in the following manner. Take two outcomes $x = (x_1, x_2)$ and $y = (y_1, y_2) \in X$. We say that x is induced from y by player i, denoted $x \leftarrow\!|_i\, y$, if $x_j = y_j$, for $i, j = 1, 2, i = j$.

Definition 5.1 (Domination): For $x, y \in X$ and player $i = 1, 2$, x dominates y via i, denoted by $x dom_i y$ if (1) $x \leftarrow\!|_i\, y$ and (2) $u_i(x) > u_i(y)$. We simply say x dominates y, denoted $x dom y$, if $x dom_1 y$ or $x dom_2 y$.

Definition 5.2 (The $vN - M$ stable set w.r.t. *dom*): A set $V \subseteq X$ is a stable set w.r.t. *dom* if the following two conditions are satisfied. (1) For any two outcomes x, y in V, neither $x dom y$ nor $y dom x$; and (2) for any z not in V, there exists $x \in V$ such that $x dom z$. (1) and (2) are called internal and external stability, respectively.

Muto and Okada [1992] applied the $vN - M$ stable set w.r.t. *dom* to the price-setting duopoly; and they showed that unreasonable outcomes may be included in the stable set. They claimed that, to remove out these outcomes, one must take into account not only a direct domination but also a sequence of players' reactions that may ensue after a player changes his action. Harsanyi [1974] already pointed out the necessity of this indirect domination in the context of cooperative characteristic function form games. On the basis of Harsanyi's idea, we define the following indirect dominance relation on the outcome space.

Definition 5.3 (Indirect domination): For $x, y \in X, x$ indirectly dominates y, denoted by $xidomy$, if there exist a sequence of pairs of actions $y = x^0, x^1, \ldots, x^m = x$ and the corresponding sequence of players $i^1, \ldots, i^m$ such that for all $k = 1, 2, \ldots, m, i^k \neq i^{k-1}, x^k \leftarrow|_{i^k} x^{k-1}$ and $u_{i^k}(x) > u_{i^k}(x^{k-1})$.

Since there may exist various sequences of action pairs, Harsanyi proposed to pick up a particular one which may be supported by an equilibrium of an appropriately constructed noncooperative bargaining game. The game models players' negotiation on how to distribute the amount that the grand coalition can gain. In parallel with the Harsanyi's approach, we consider the extended game with preplays, and pick up a particular sequence of indirect domination which is supported by a CMPE.

Definition 5.4 (Effective domination): Take a CMPE ρ of the extended game $\Gamma(e, 1)$ or $\Gamma(e, 2)$. For $x, y \in X, x$ effectively dominates y under ρ, denoted $xedom(\rho)y$, if (1) $xdomy$, or (2) $xidomy$ with a sequence of action pairs $y = x^0, x^1, \ldots, x^m = x$ and a sequence of players $i^1, \ldots, i^m$ such that $x^k = \rho_{i^k}(x^{k-1})$ for $k = 2, \ldots, m$.

Definition 5.5(Effectively stable set) A set $V(\rho) \subseteq X$ is an effectively stable set under ρ if the following two conditions are satisfied. (1) For any two outcomes x, y in $V(\rho)$, neither $xedom(\rho)y$ nor $yedom(\rho)x$; and (2) for any z not in $V(\rho)$, there exists $x \in V(\rho)$ such that $xedom(\rho)z$. (1) and (2) are called internal effective stability and external effective stability, respectively.

In general, $K(\rho)$ always satisfies the internal effective stability as the next proposition shows.

Proposition 5.1: *Let ρ be a CMPE, and take the set $K(\rho)$ of its stable outcomes under ρ: $K(\rho)$ is the set of action pairs in which neither player moves under ρ. Then $K(\rho)$ satisfies the effective internal stability.*

However, $K(\rho)$ may not always satisfy the external stability. One sufficient condition for $K(\rho)$ to satisfy the external effective stability is given in the next proposition.

Proposition 5.2: *Let ρ be a CMPE and $K(\rho)$ be the set of its stable outcomes under ρ. Suppose there is no sequence (cycle) of outcomes $x^0, x^1, \ldots, x^m = x^0$ such that $x^k = \rho_{i^k}(x^{k-1})$ for $k = 1, \ldots, m$, $i^k \neq i^{k-1}, k = 1, \ldots, m-1$, and $i^1 = i^m$. Then $K(\rho)$ satisfies also external effective stablity, and thus it is an effectively stable set.*

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